The Resource Valuation and Optimisation Model: Real Impact from Real Options

Geoff Evatt

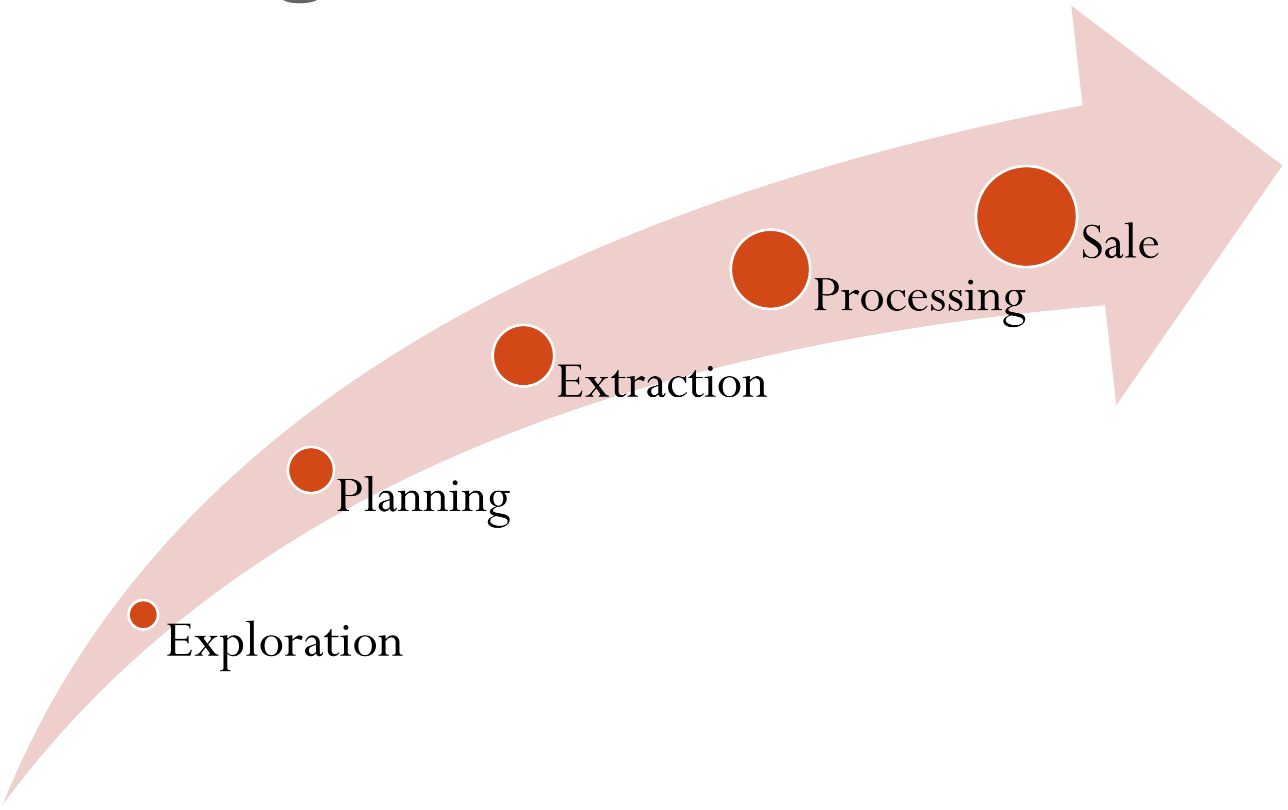
Paul Johnson, Peter Duck, Syd Howell & John Moriarty

School of Mathematics

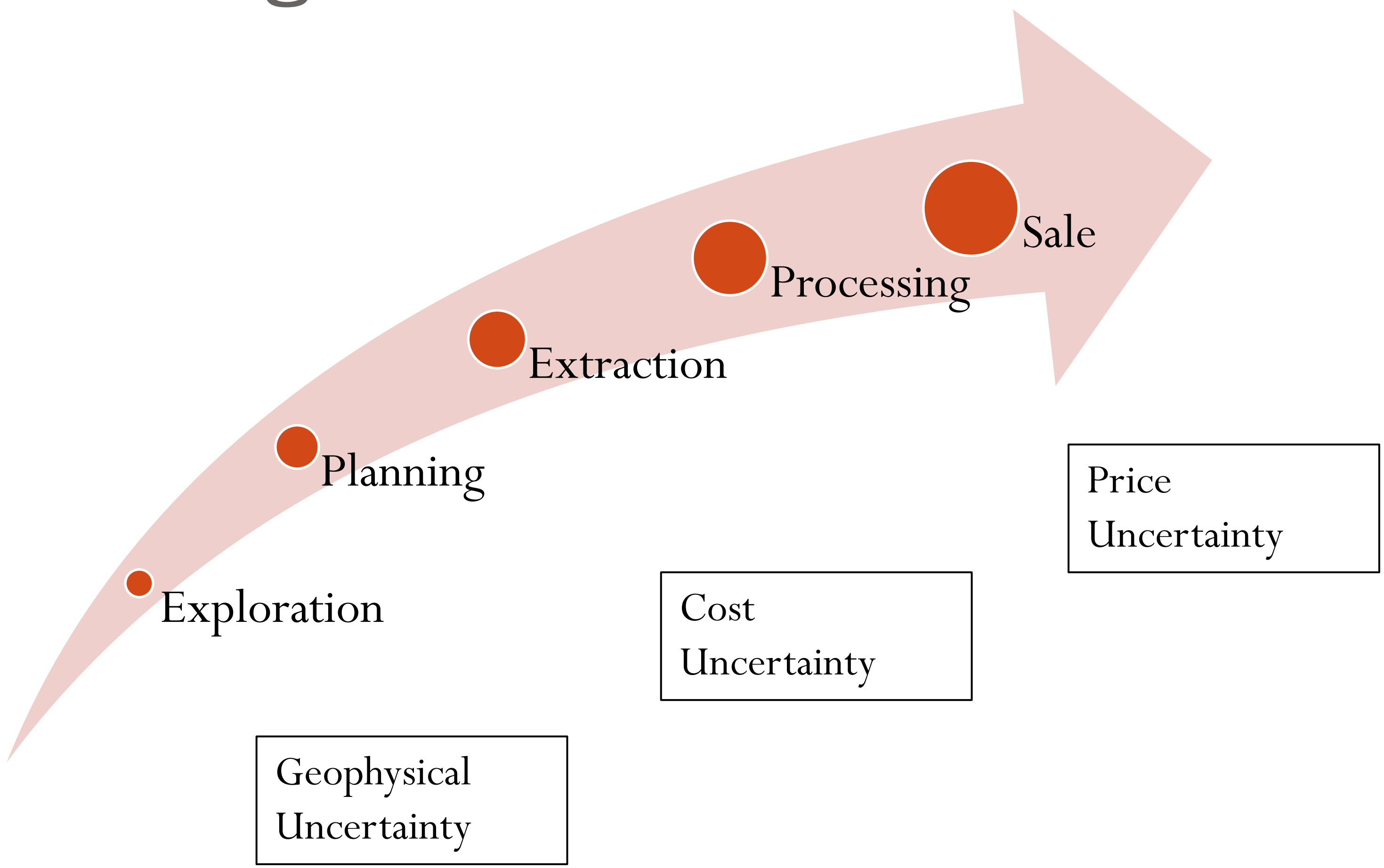
The University of Manchester

Kindly supported by the Engineering and Physical Sciences Research Council, UK.

Basic Mining Overview



Basic Mining Overview

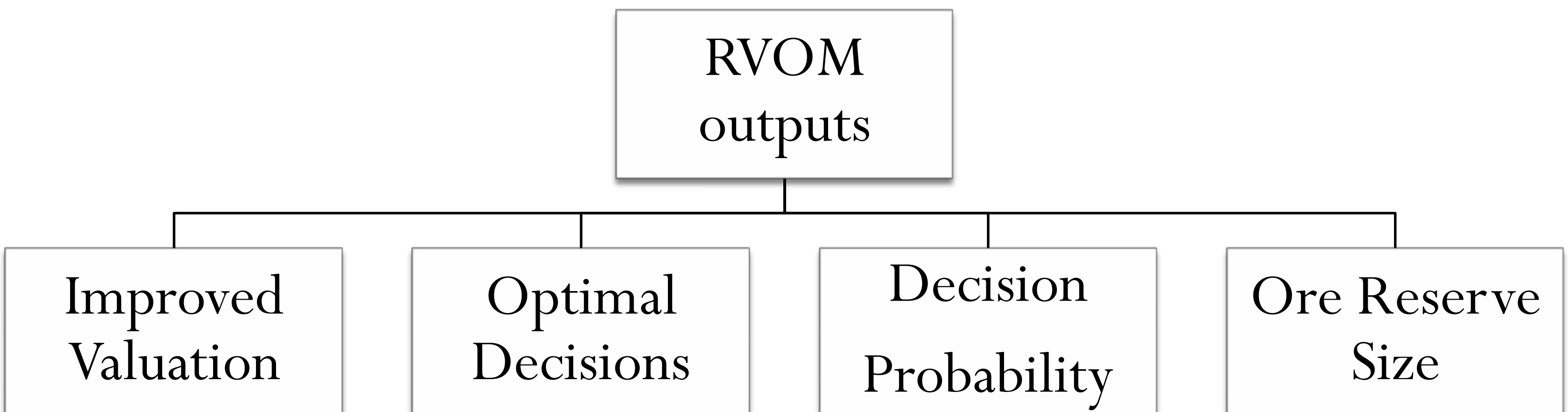


What is the RVOM?

The RVOM is a real options software package that optimises a projects decision making under future, uncertain, commodity prices

What is the RVOM?

The RVOM is a real options software package that optimises a projects decision making under future, uncertain, commodity prices.



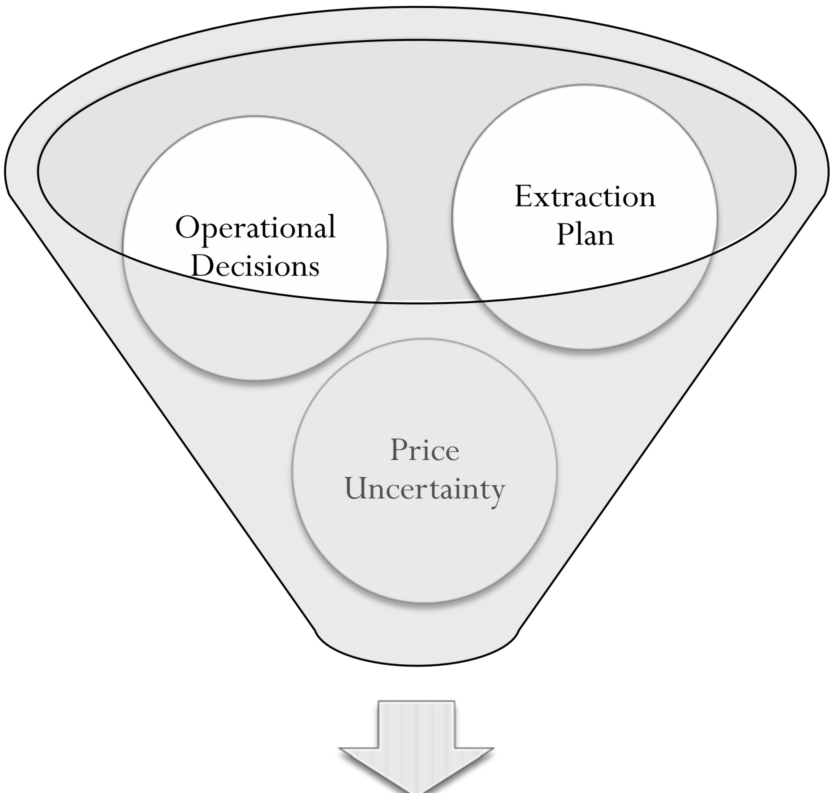
Mining & Operational Decisions

Out of 285 North American gold mines who operated during 1988 and 1997:

- More mines closed due to economic deterioration than depleted their reserves*
- 30% of mines closed due to a fall in gold price*
- Of those that closed, at least 12% reopened when gold prices rose again*

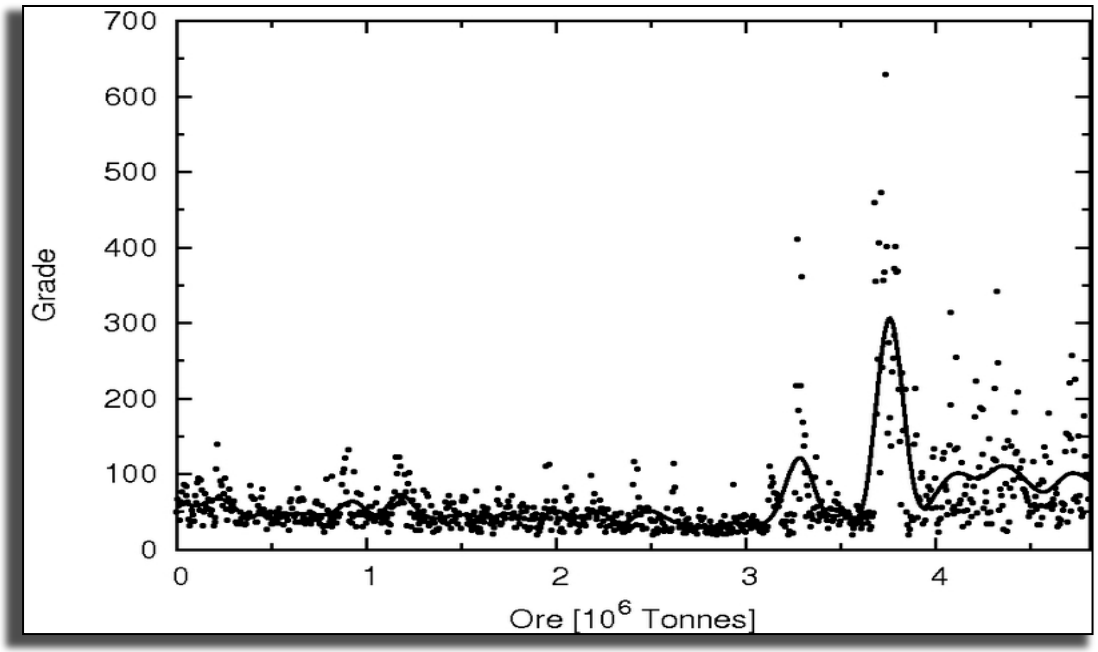
*Moel and Tufano. When are Real Options Exercised? An Empirical Study of Mine Closings. 2002.

The RVOM ingredients



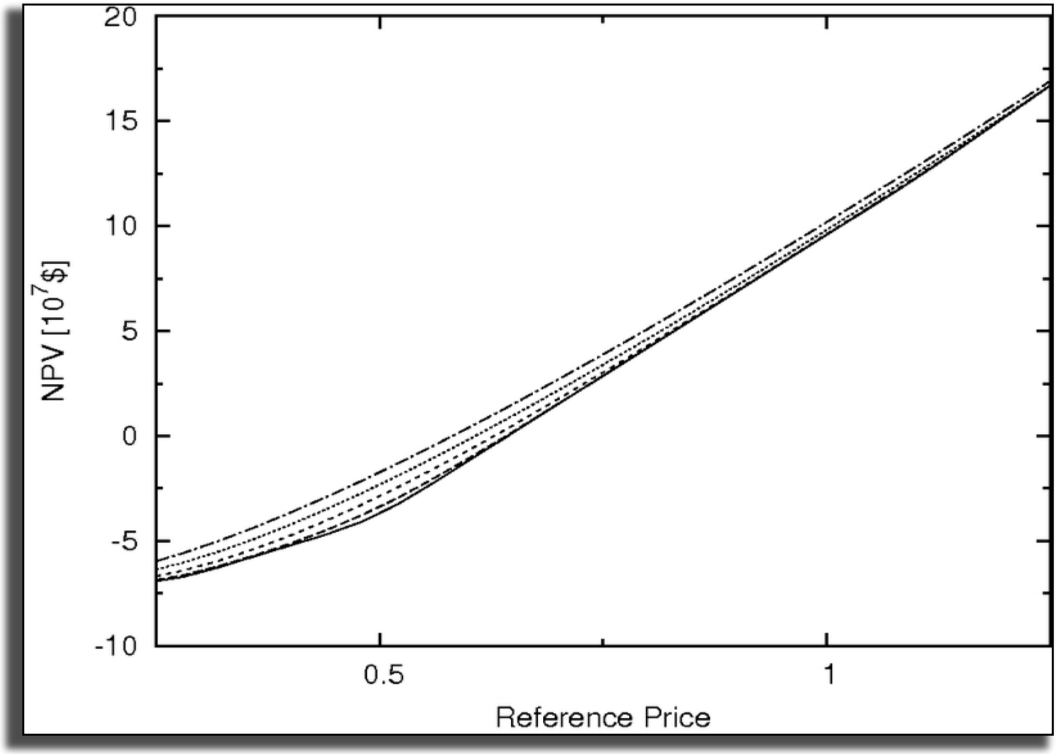
Real Options Analysis

Input: Extraction Schedule



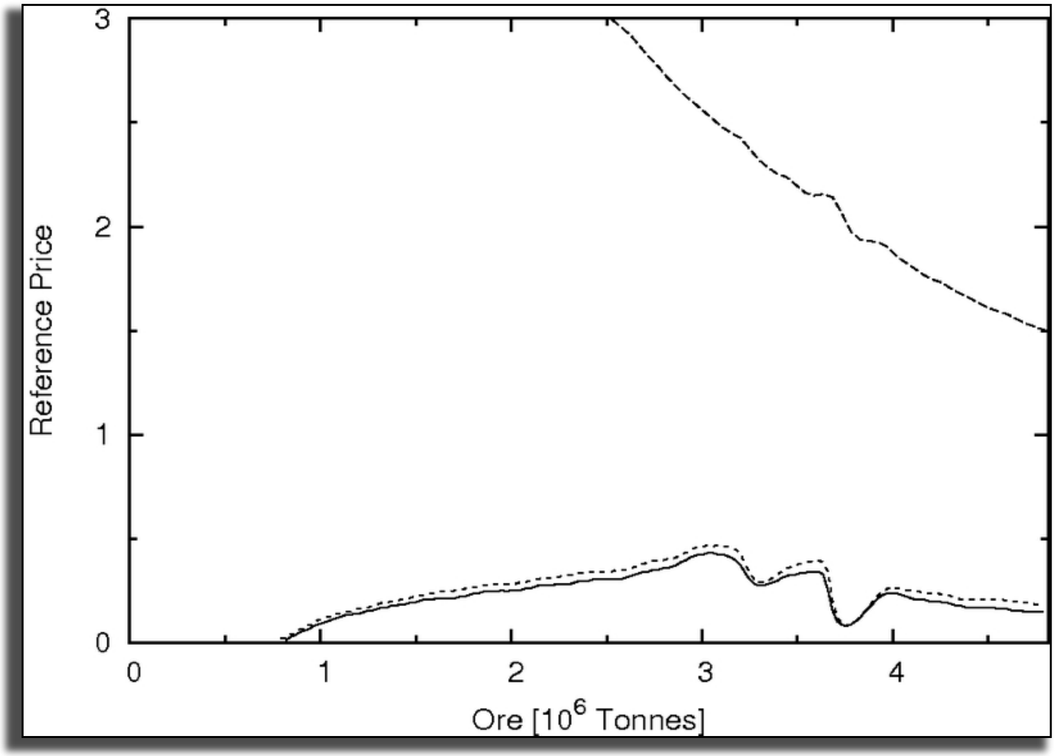
Mine operator holds the options to Expand or Abandon

Output: Valuation



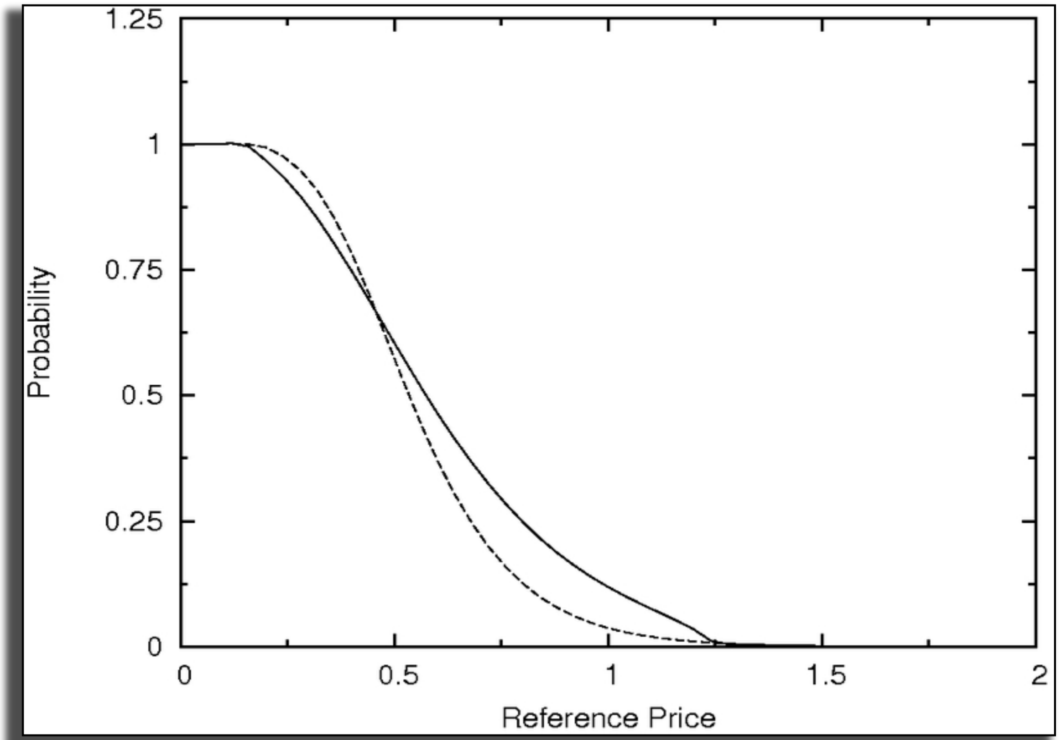
The Resource Valuation and Optimisation Model.
The University of Manchester & Gemcom Software

Output: Optimal Decision

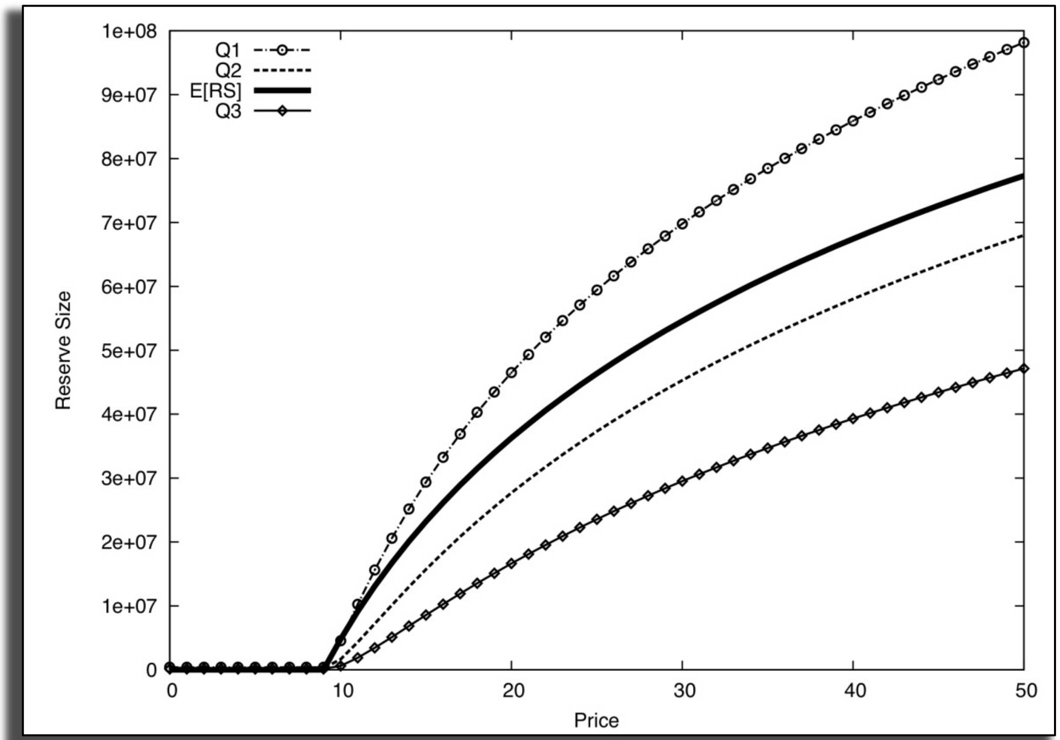


The Resource Valuation and Optimisation Model.
The University of Manchester & Gemcom Software

Output: Decision Probability



Output: Reserve Size Estimation



The RVOM Flexibility

- Advanced algorithms mean running times are fast
- Handle multiple ore prices
- Unlimited number of operational decisions
- Easily vary economic parameters
- Compatible with the Gemcom Whittle™

Underpinning Science

- The RVOM has risen out of other mathematically similar scientific investigations (Fluid Dynamics, Computational Finance & Optimal Stochastic Control)
- Partial Differential Equations allow for fast computations, high accuracy and sensitivity analysis
- Prices are modelled as stochastic diffusions (e.g. GBM, CIR)
- Broad team expertise
- 'The Expected Lifetime of an Extraction Project', Proceedings of the Royal Society A, 2011

The Feynman-Kac Framework

$$u(x) = E_x \left[e^{-c\tau} f(X_\tau) + \int_0^\tau e^{-cu} g(X_u)\, \mathrm{d}u \right], \tag{2.1}$$

where f and g are functions to be defined, c is a constant, X_t is an Ito diffusion in $\mathbb{R}^n$ such that

$$\mathrm{d}X = b(X)\,\mathrm{d}t + \sigma(X)\,\mathrm{d}B, \tag{2.2}$$

The Feynman-Kac Framework

$$\left.\begin{aligned} Lu(x) - cu(x) &= -g(x) && \text{in } H \\ \lim_{x \to y} u(x) &= f(y) && \text{for } y \in \partial H, \end{aligned}\right\}$$

$$L \equiv \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial}{\partial x_i}$$

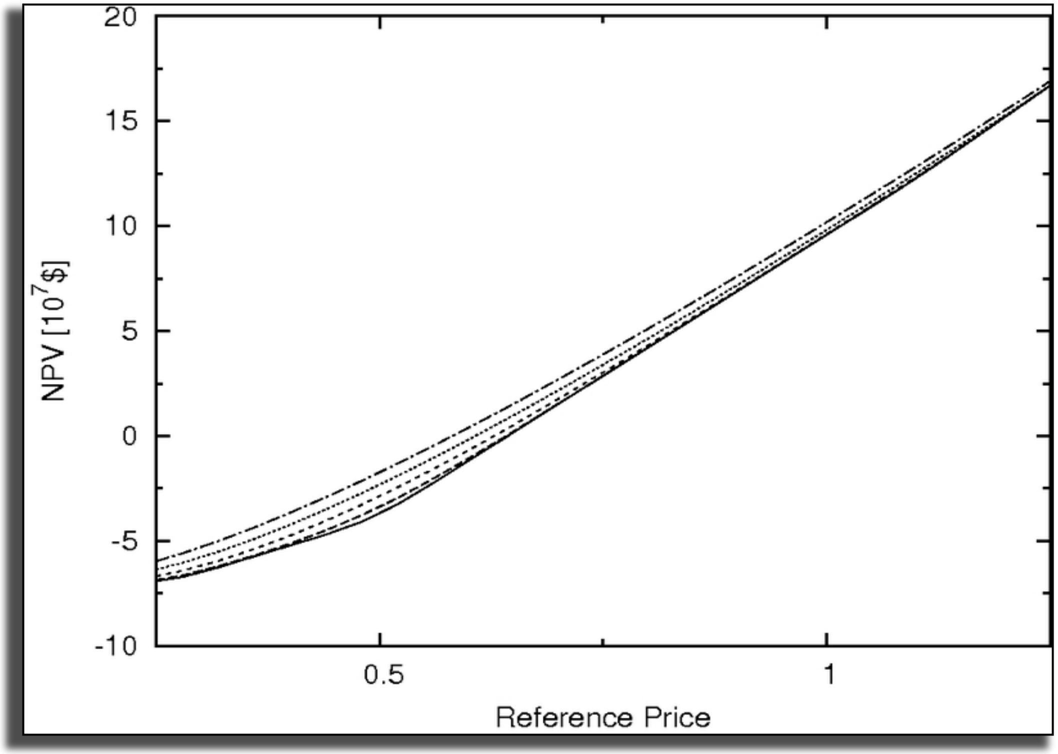
Mine Valuation

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - \frac{\partial V}{\partial \tau} - q\frac{\partial V}{\partial Q} + \mu S \frac{\partial V}{\partial S} - rV + qGS - \epsilon_P - \epsilon_M = 0,$$

V=-C on S =S*,
V=0 on t=T,
V=0 on Q=0,
V ~S as S→∞
dV/dS =0 on S=S*

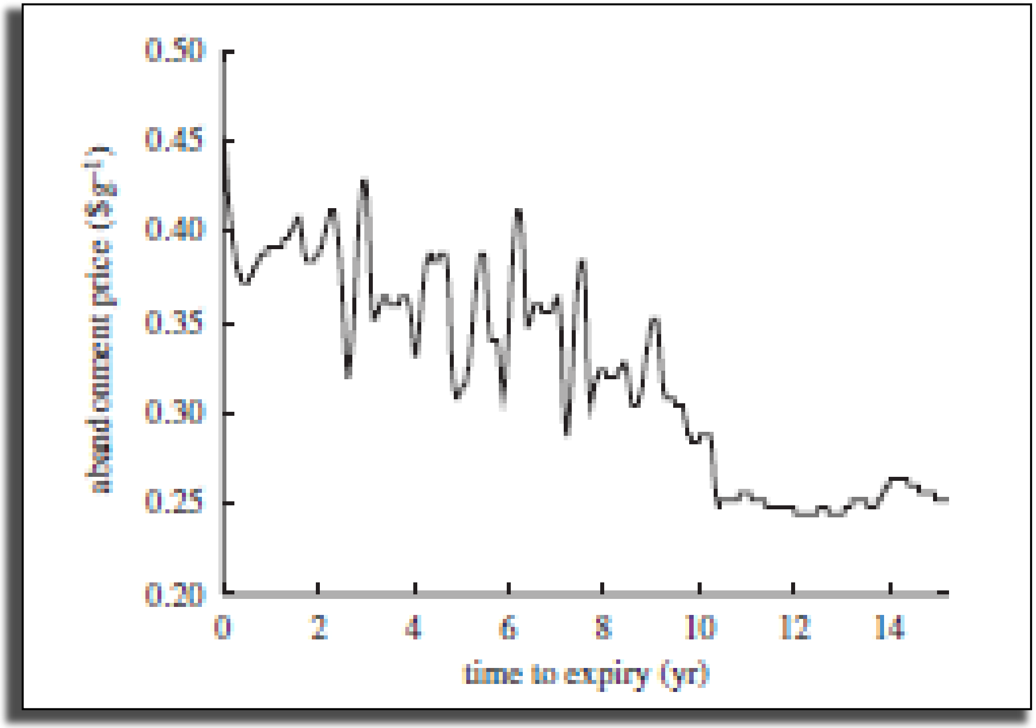
The Resource Valuation and Optimisation Model.
The University of Manchester & Gemcom Software

Output: Valuation



The Resource Valuation and Optimisation Model.
The University of Manchester & Gemcom Software

Output: Optimal Decision

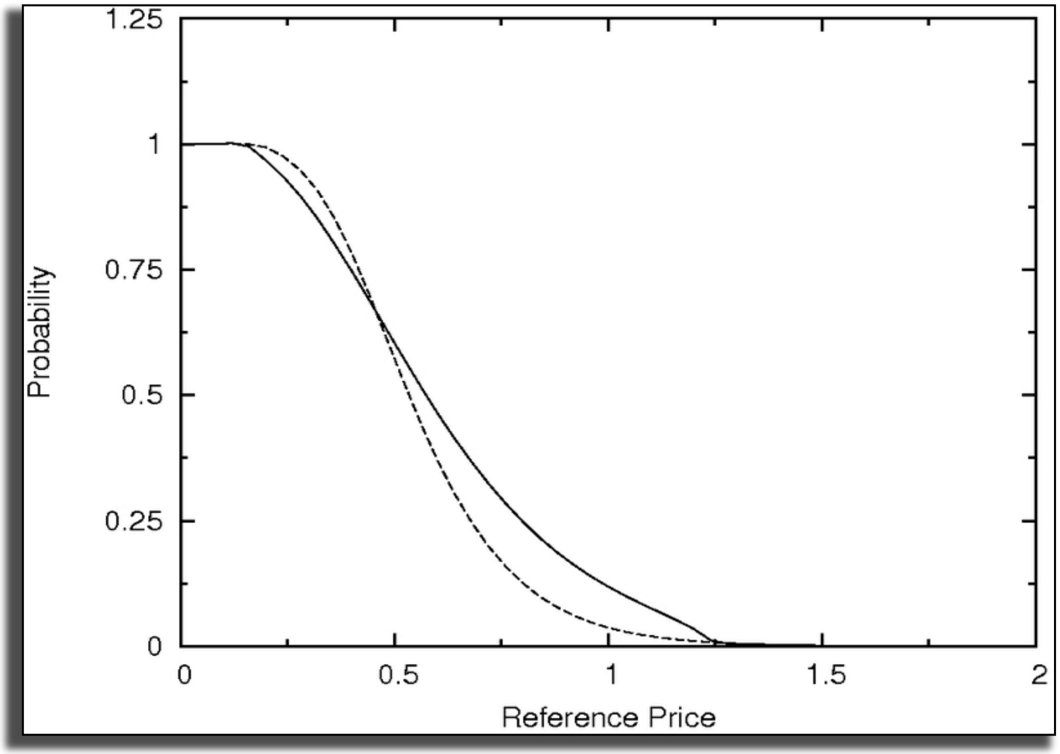


Decision Probabilities

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P}{\partial S^2} - \frac{\partial P}{\partial \tau} - q\frac{\partial P}{\partial Q} + \mu S \frac{\partial P}{\partial S} = 0,$$

$$\left.\begin{array}{ll} P = 0 & \text{on } S = S^*, \\ P = 1 & \text{when } \min\{Q, \tau\} = 0 \\ P \to 1 & \text{as } S \to \infty. \end{array}\right\}$$

Output: Decision Probability



Mining Decisions & Connected Graphs

	Normal	Abandon
Normal	XX	$10m
Abandon	XX	XX

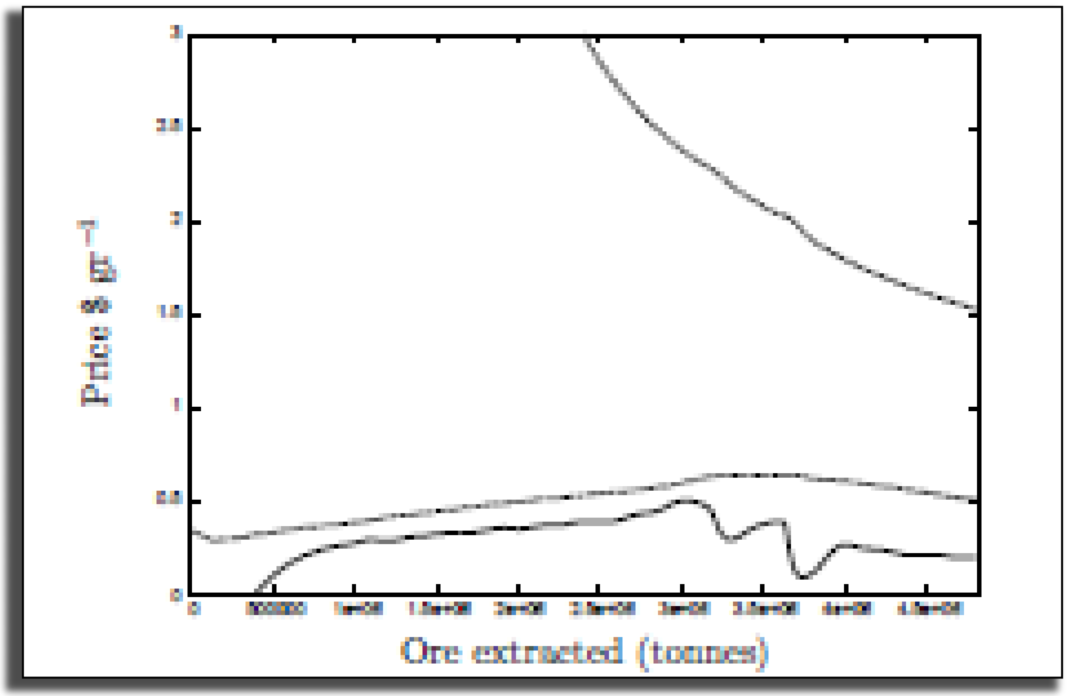
Mining Decisions & Connected Graphs

	Normal	Abandon	Expand
Normal	XX	$10m	$15m
Abandon	XX	XX	XX
Expand	$5m	$20m	XX

Mining Decisions & Connected Graphs

	Normal	Abandon	Expand	Mothball
Normal	XX	$10m	$15m	$10m
Abandon	XX	XX	XX	XX
Expand	$5m	$20m	XX	$10m
Mothball	$5m	$8m	$10m	XX

Overlapping Domains



Hitting Probabilities with Hysteresis

$$\frac{1}{2}\sigma^2 s^2 \frac{\partial^2 P^k}{\partial s^2} - \frac{\partial P^k}{\partial \tau} - q_k \frac{\partial P^k}{\partial q} + \mu s \frac{\partial P^k}{\partial s} = 0 \qquad \text{on } L_k,$$

$$P^k(x) = 0 \quad \text{on} \quad q = 0,$$

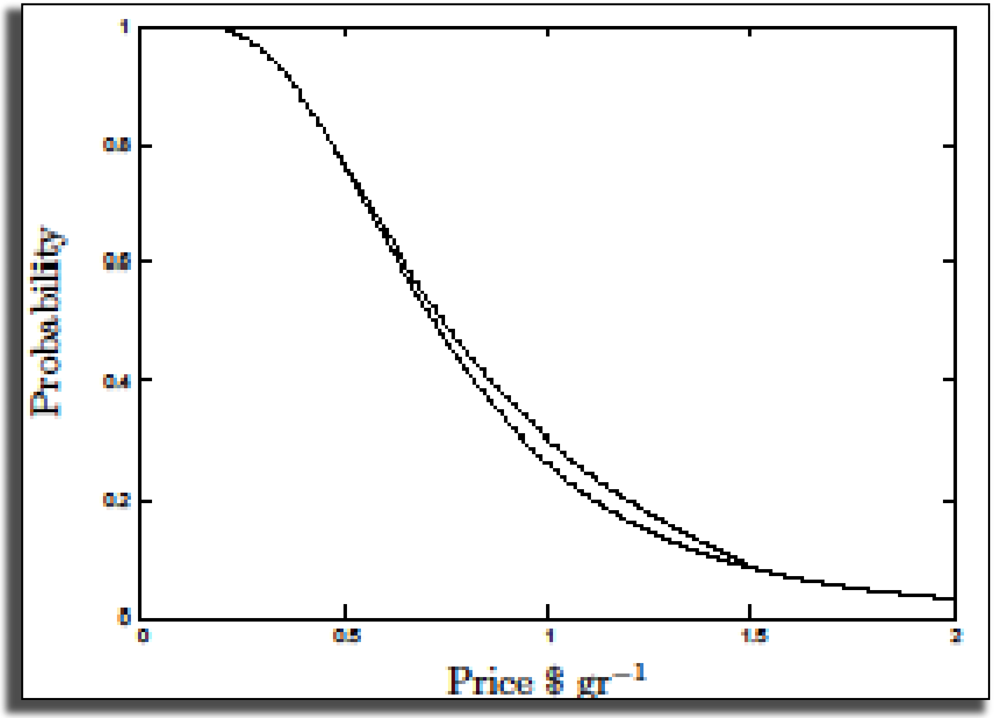
$$P^k(x) = 0 \quad \text{when} \quad t = T,$$

$$P^k(x) \to 0 \quad \text{as} \quad s \to \infty,$$

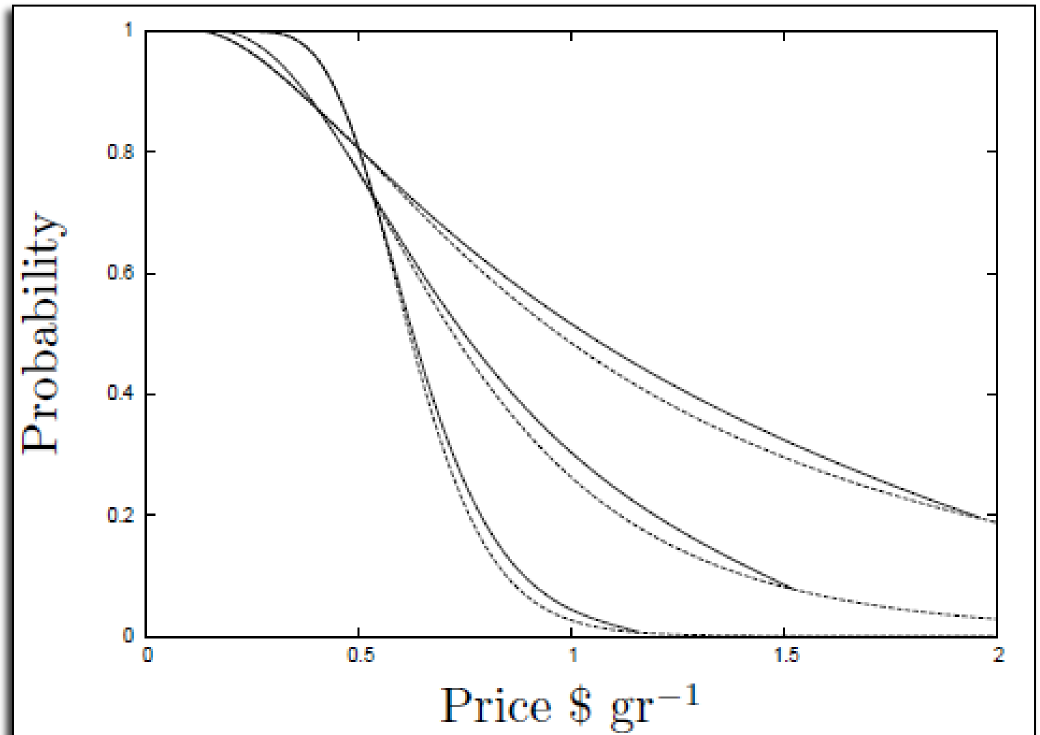
$$P^k(x) = 1 \quad \text{when} \quad x \in A,$$

$$P^k(x) = P^{1-k}(x) \quad \text{when} \quad x \in \partial L_k \cap L_{1-k},$$

Hitting Probabilities with Hysteresis



Hitting Probabilities and Volatility



The Resource Valuation and Optimisation Model.
The University of Manchester & Gemcom Software

Optimal Regulation

Objectives:

- Regulator wants to reduce the probably of a mining company terminating an operation early (World Bank report, Otto 2010)
- The company wants to maximise their valuation

Optimal Regulation

Objectives:

- Regulator wants to reduce the probably of a mining company terminating an operation early (World Bank report, Otto 2010)
- The company wants to maximise their valuation

Controls:

- Regulator sets the size of the early termination fee
- The company controls when it abandons

Optimal Regulation

Objectives:

- Regulator wants to reduce the probably of a mining company terminating an operation early (World Bank report, Otto 2010)
- The company wants to maximise their valuation

Controls:

- Regulator sets the size of the early termination fee, K
- The company controls when it abandons, S*

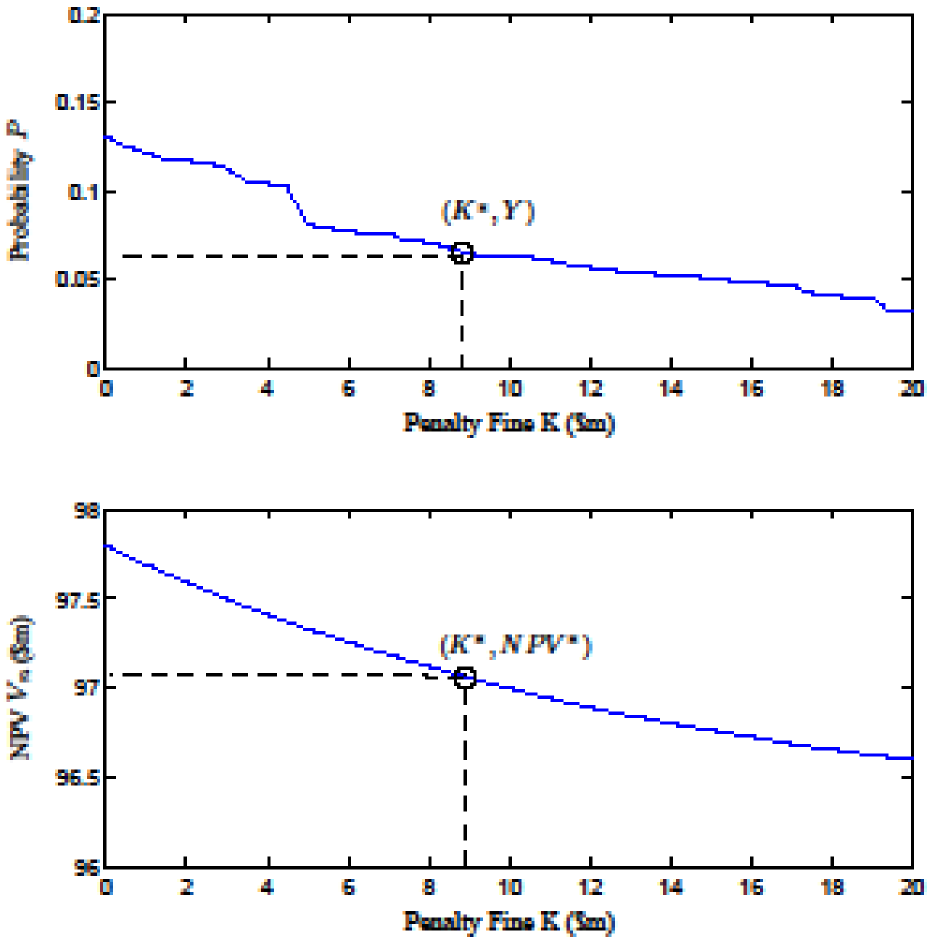
$$V = -C - K \text{ on } S = S^*$$

Optimal Regulation

Three point process to finding this Nash Equilibria

1. Impose a level of fine, and find corresponding optimal company strategy
2. With this strategy, calculate the probability of entering the undesirable state
3. If the probability meets the criteria, finish. If not, update the value of fine, and return to 1.

Optimal Regulation



The Resource Valuation and Optimisation Model.
The University of Manchester & Gemcom Software

Summary

- The RVOM is an advanced Real Options software package
- Aids in mine scheduling, public reporting, risk analysis and increasing profits

Summary

- The RVOM is an advanced Real Options software package
- Aids in mine scheduling, public reporting, risk analysis and increasing profits

Constructed of:

- Brennan & Schwartz (1985)
- Feynman-Kac formula
- Semi-Lagrangian numerical technique

References

- Evatt et.al. *The Resource Valuation and Optimisation Model: Real Impact from Real Options*. Application of Computers in the Minerals Industry (APCOM). 2011.
- Evatt et.al. *The Expected Lifetime of an Extraction Project*. Proceedings of the Royal Society A. 2011
- Evatt at.al. *Regulating Industries under Exogenous Uncertainty*. In review 2011. (Google it and 'Evatt').

Thank you

GEMCOM

MANCHESTER 1824