Stochastic Programming with Recourse for Aircraft Separation under Uncertainty

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INTRODUCTION

CONTEXT AND MOTIVATION

DETERMINISTIC MODEL

MODELING AND SOLVING THE STOCHASTIC PROBLEM

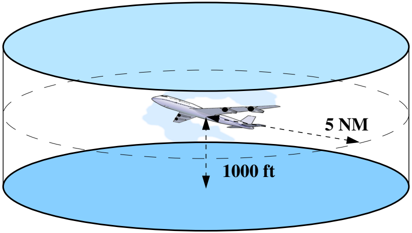
RESULTS

DISCUSSION

CONTEXT

AIR TRAFFIC MANAGEMENT

- Shared civilian airspace
- Double need for safety

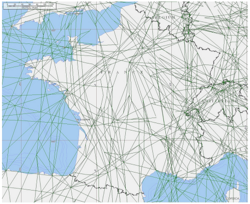


- ... and economical efficiency : time and fuel optimal trajectories

As a consequence...

Air traffic management (ATM) is necessary

- Airspace is organized in routes and flight levels (1000 ft)



- Flight plans have to be followed
- Short term air traffic control (ATC) on control sectors (15 min)
- Sectors' capacities ⇒ delays and suboptimal flight levels

Automation of ATC

Official forecasts plan a continuous growth of air traffic

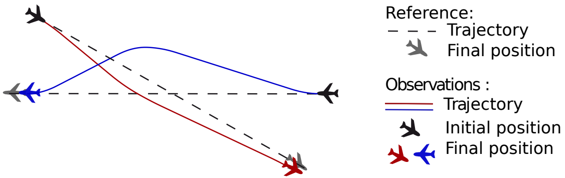
- Airspace capacity is already saturated
- Major cost impact of capacity on delay costs [Lehouillier et al., 2014]

Automation of aircraft separation can increase capacity:

- Find suitable options in really difficult situations
- Explicitly introduce uncertainties

Two-dimensional separation with recovery

- En-route control: not in the vicinity of an airport
- Altitude is stabilized $\Rightarrow$ 2D problem
  - Heading and speed changes
- Recovery of reference trajectory after maneuvers

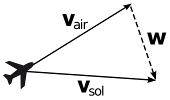


- Minimization of a criterion representing delay and fuel consumption

Errors in tracking trajectories

Only consider environmental factors in this study

- The reference trajectory is an initial prediction
- Only consider environmental sources of uncertainties
  - Simplified effect of the wind: $\mathbf{v}_{ground} = \mathbf{v}_{air} + \mathbf{w}$



  - Flight management systems ensure only lateral tracking
    Errors on wind predictions ⇒ longitudinal errors
- Precision of the speed measures are impacted by variations in temperature and pressure

Deterministic model

Constraints on the aircraft's motions

Set $\mathcal{A}$ of aircraft, time horizon T
Set $\mathcal{C}$ of pairs of aircraft in potential conflict
Variables for $A_i \in \mathcal{A}$: position $\mathbf{p}_i$, speed $\mathbf{v}_i$, acceleration $\mathbf{u}_i$

- Bounds on the norm of speed

$$V_i^{\min} \leq \|\mathbf{v}_i(t)\| \leq V_i^{\max}, \forall A_i \in \mathcal{A}, \forall t \in [0\,;T]$$

- Comfort limits on variations of speed V_i and heading χ_i

$$\left|\dot{V}_i(t)\right| \leq U^{\max} \text{ et } |\dot{\chi}_i(t)| \leq \omega^{\max}, \forall A_i \in \mathcal{A}, \forall t \in [0\,;T]$$

- **Separation according to the required horizontal distance**

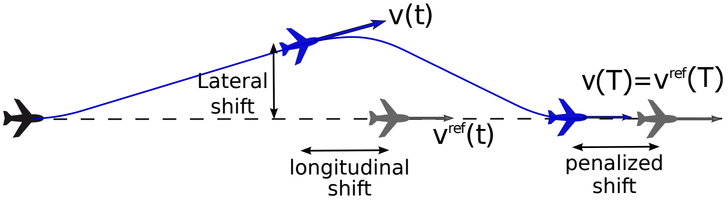
$$\left\|\mathbf{p}_j(t) - \mathbf{p}_i(t)\right\| \geq D, \forall (A_i, A_j) \in \mathcal{C}, \forall t \in [0\,;T]$$

Objective function and trajectory recovery

- Fuel consumption on the time horizon
  Assuming constant mass and altitude:

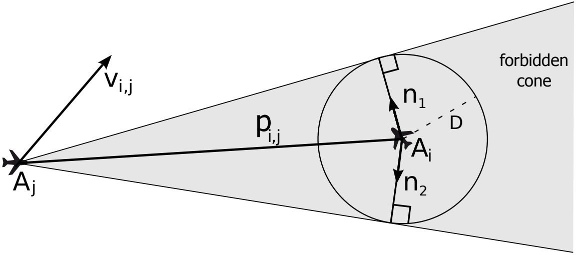
$$C_{t,i}(t) = c_{1,i}\left(1 + \frac{V_i(t)}{c_{2,i}}\right)\left(\alpha_{F_{T,i}} V_i(t)^2 + \frac{\beta_{F_{T,i}}}{V_i(t)^2}\right)$$

- Recovery of the reference trajectory
  - Recovery of reference planned speed
  - No lateral deviation at time T
  - Longitudinal gap at time T is penalized



LINEAR MODEL WITH ONE SPEED VECTOR CHANGE

- Conflict resolution with one set of simultaneous maneuvers
  $\Rightarrow$ separation constraint is simplified



- One disjunction per conflict with two linear constraints

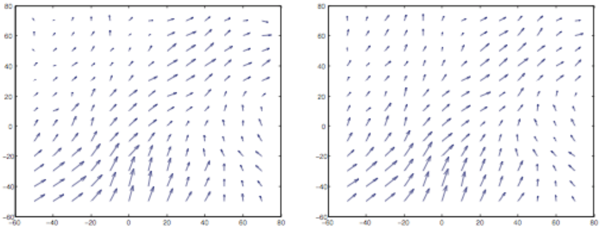
$$\langle \mathbf{v}_{ij} | \mathbf{n}_1 \rangle \geq M\delta_{ij} \quad \text{and} \quad \langle \mathbf{v}_{ij} | \mathbf{n}_2 \rangle \geq M(1 - \delta_{ij})$$

Stochastic separation

Modeling the uncertainties due to wind

Errors on wind prediction can be quantified [Cole, 1998]

- Gaussian isotropic wind field $\mathbf{w}(\mathbf{p}, t)$
- Zero mean and strongly correlated in time and space



Wind map at t (left) and $t + 15$ min (right)

ERRORS ON SPEED MEASURES

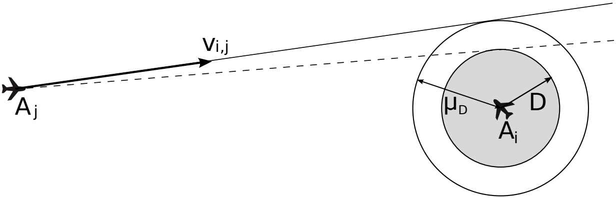
One continuous random variable per aircraft: ϵ_i

- Zero mean, normal distribution with standard deviation σ_ϵ
- Error is constant on the time horizon
- Independence between aircraft

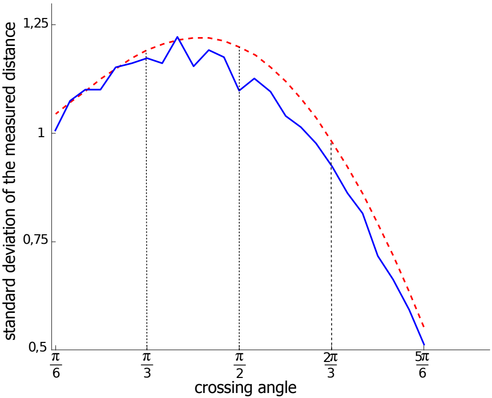
Robust separation

Compute a margin on separation distance to ensure a low probability of conflict
$\Rightarrow$ require an analytical expression of conflict probability $\mathbb{P}_C$

- Done with gaussian and independent errors [Irvine, 2003]
- Approach $\mathbb{P}_C$ with $\mathbb{P}(d(\tau_{ij}) < D)$
  - τ_{ij}: time when $d_{\min}$ is reached without errors
  - Short term $\Rightarrow$ assumption of constant and uniform wind
    $\Rightarrow d(\tau_{\text{ij}}) \sim \mathcal{N}(\mu_d, \sigma_d)$: μ_d = minimal distance without errors
- Set μ_d such that $\mathbb{P}_C = 1\%$



QUALITY OF THE APPROXIMATION



How to implement a control with uncertainties?

- The starting time of the maneuvers reflects a compromise
  - The robust required distance grows with anticipation
  - The amplitude of maneuvers follows the opposite tendency
- The number of control instructions has to remain small

Tested strategies

1. One maneuver per aircraft with a fixed anticipation time equal to 7 minutes
2. Multiple maneuvers computed with a receding horizon
   - Sampling period = 1 minute, prediction horizon = 15 minutes
   - Errors are taken into account as they occur
3. Two maneuvers, the second being corrective
   - First maneuver with a 5 to 10 minutes anticipation
   - Second recourse maneuver as late as possible

   ⇒ Model in the framework of two-stage stochastic programming with recourse

STOCHASTIC OPTIMIZATION WITH RECOURSE

How to model a recourse action?

- Consider the possibility of future actions while computing actions that should be started now
- Future is uncertain $\rightarrow$ expected value of the cost of probable recourse actions

$$\min_{\mathbf{x} \in \mathcal{X}} \mathbf{c}_x^T \mathbf{x} + \mathbb{E}\left(Q(\mathbf{x}, \boldsymbol{\xi})\right)$$

Stochastic linear programming model

- Sample the continuous distribution of error with N Monte-Carlo simulations
- Recourse maneuvers should be only corrective, so:
  - No binary variable in second stage for the separation
  - The recourse is linearized assuming small modifications

$$\begin{aligned} \min\ & \mathbf{c}^T\mathbf{x} + \frac{1}{N}\sum_{n=1}^{N} \mathbf{q}^T\mathbf{y}^n \\ \text{sous}\ & \mathbf{A}\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \in \mathcal{X} \\ & \mathbf{T}^n\mathbf{x} + \mathbf{W}\mathbf{y}^n = \mathbf{h}^n, \forall n \\ & \mathbf{y}^n \geq 0, \forall n \end{aligned} \quad \rightarrow$$

Constraint matrix

$$\begin{pmatrix} \mathbf{A} & \mathbf{0} & \dots & \dots & \dots \\ \mathbf{T}^1 & \mathbf{W} & \mathbf{0} & \dots & \dots \\ \mathbf{T}^2 & \mathbf{0} & \mathbf{W} & \mathbf{0} & \dots \\ \vdots & \vdots & \ddots & \ddots & \\ \mathbf{T}^N & \mathbf{0} & \dots & \mathbf{0} & \mathbf{W} \end{pmatrix}$$

$\Rightarrow$ Benders decomposition is natural for such structure

BENDERS DECOMPOSITION

Master problem

$$\min_{\mathbf{x}\in\mathcal{X}} \quad \mathbf{c}^T\mathbf{x} + \theta$$

$$\text{sous} \quad \mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\theta \geq \sum_{n=1}^{N} \boldsymbol{\pi}_l^n(\mathbf{h}^n - \mathbf{T}^n\mathbf{x}), \forall l$$

Sub-problems

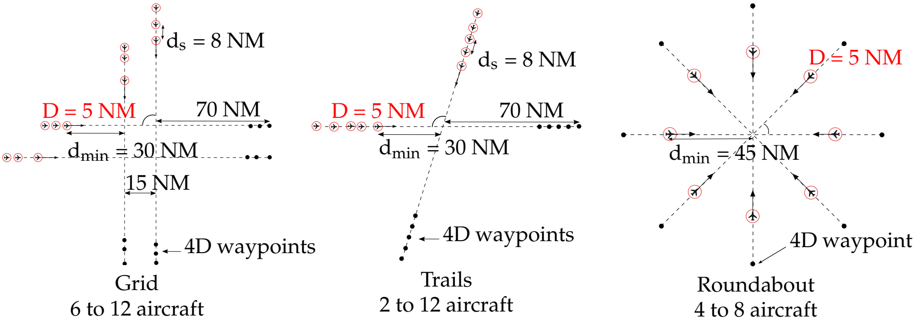
$$\min_{\mathbf{y}\geq 0} \quad \frac{1}{N}(\mathbf{q})^T\mathbf{y}$$

$$\text{sous} \quad \mathbf{W}\mathbf{y} = \mathbf{h}^n - \mathbf{T}^n\hat{\mathbf{x}}_L$$

- Solution of N sub-problems $\rightarrow$ one aggregated cut in MP
- One additional cut reflecting that the cost should be greater than in the deterministic scenario

Results

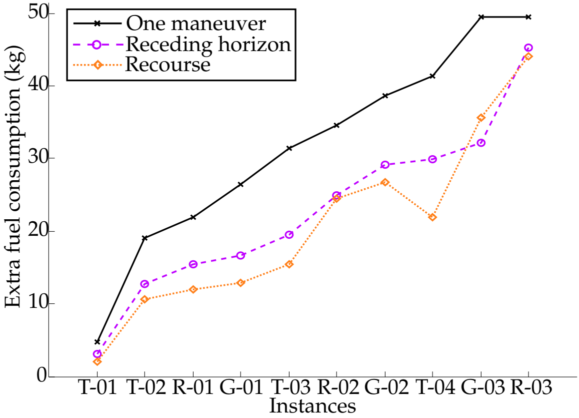
Desription of the benchmark



- Monte-Carlo sampling is done with 100 scenarios
- Results are averaged over 1000 simulations per instance

Computational results

- Every instance is solved in less than 20 seconds
- The probability of unsolved conflict is respected



Discussion

1. Benders decomposition is efficient for the recourse model
   ⇒ The two-stage model seems to be a good compromise
2. Current operational context: only one instruction
   ⇒ Use Monte-Carlo sampling in a robust program
   - Find one maneuver per aircraft
   - Identify the optimal starting time for maneuvers
3. The literature on the sources of uncertainty is very thin
   - Add "human-in-the-loop" uncertainties with simple assumptions
   - Include errors in the realization of maneuvers
4. Run the tests on dynamic situations
   - Aircraft constantly enter and leave control sectors

Cole, Richard, Kim et Bailey
An assessment of the 60 km Rapid Update Cycle (RUC) with near real-time aircraft reports
Project Report NASA/A-1, 1998

Irvine
Target miss distance to achieve a required probability of conflict
FAA/Eurocontrol R&D Seminar, 2003

Chaloulos et Lygeros
Effect of wind correlation on aircraft conflict probability
AIAA Journal of Guidance, Control, and Dynamics, 2007

Vela et al.
A two-stage stochastic optimization model for air traffic conflict resolution under wind uncertainty
IEEE/AIAA Digital Avionics Systems Conference, 2009