Open Pit Block Scheduling with Exposed Ore Reserve

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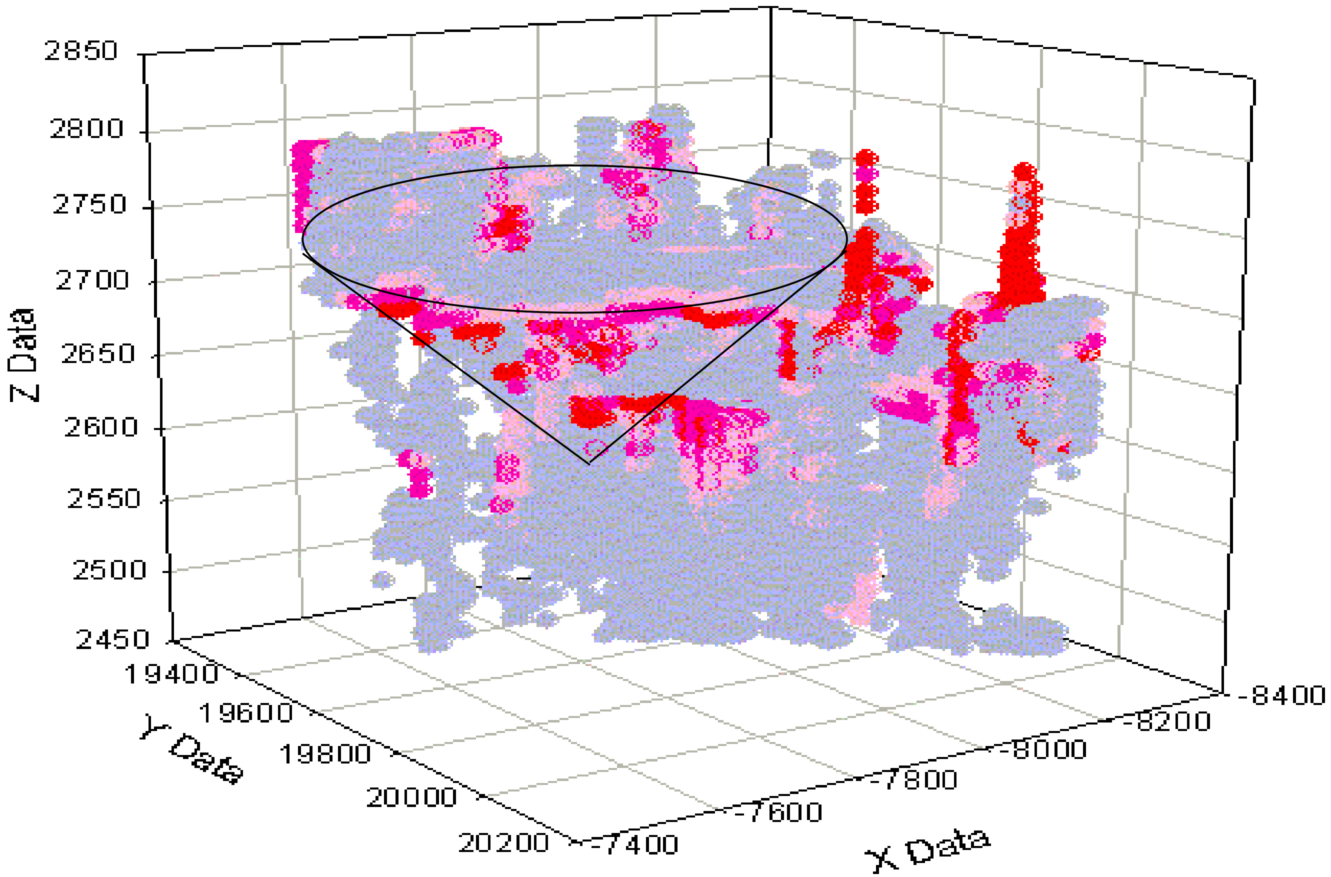
The Chuquicamata open pit is 5 x 3 km large and 1 km deep, is the largest metal mine of the world.

Marmato (Caldas, Colombia).



The information (given an initial profile of the mine) in the data base for our purposes is essentially:

- **coordinates *(x, y, z)***
- **grade at each block (% of Copper/Total mass) and**
- **other characteristics**

THE PROBLEM

Given an **estimation** of the value distribution in situ, one needs to schedule the portions of the mine to be extracted at each period, which the aim is to find an optimal sequence of extraction.

There are two frameworks for this problem:

I. a continuous approach (in a functional space).
II. a discrete optimization model

I. CONTINUOUS APPROACH

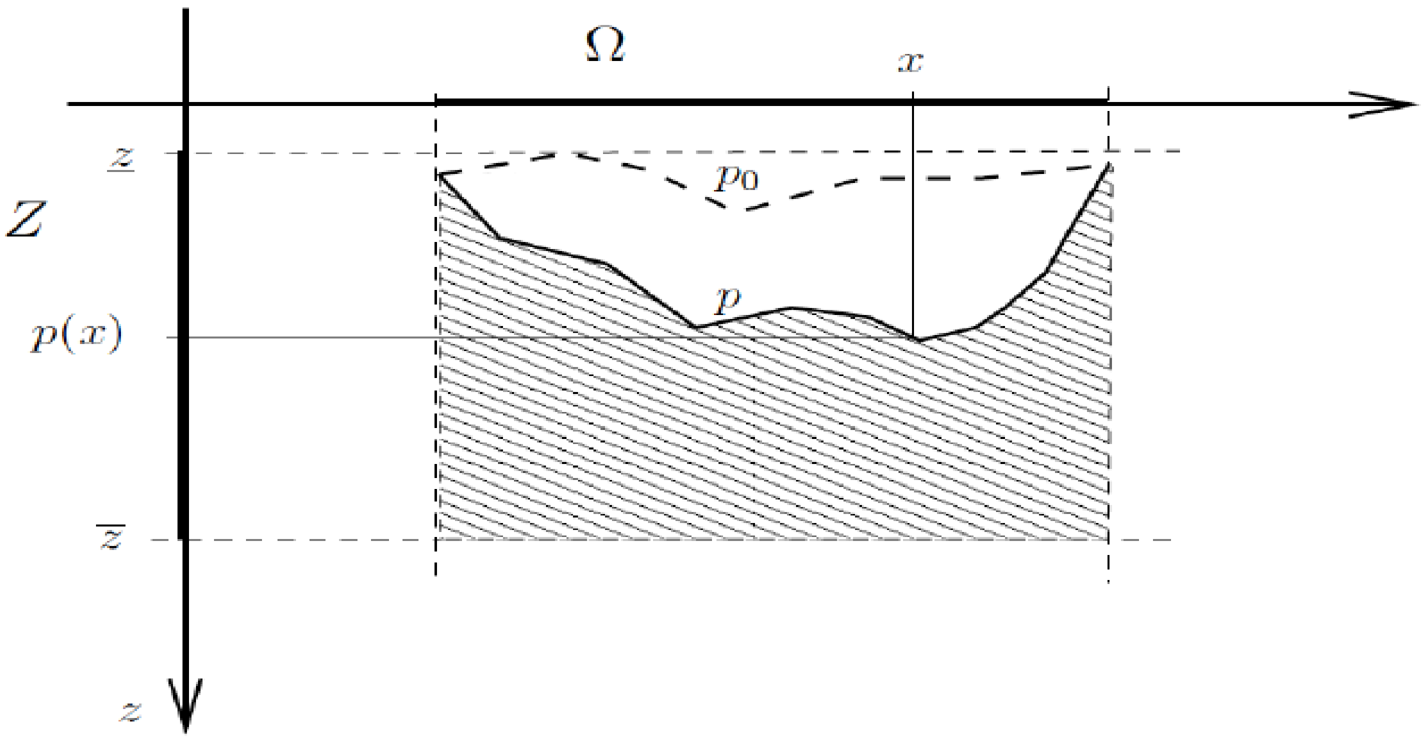


Figure: Sketch of a vertical section for a feasible profile

Alvarez-Amaya-Griewank-Strogies. **A continuous framework for open pit mine planning.** Mathematical Methods for Operations Research, Vol 73, pp 29-54, 2011.

Amaya-Hermosilla-Molina. **Characterizing the optimal profile of the open pit problem in the continuous framework.** Manuscript, in progress, 2017.

Math Meth Oper Res (2011) 73:29–54
DOI 10.1007/s00186-010-0332-3
ORIGINAL ARTICLE

A continuous framework for open pit mine planning

Felipe Alvarez · Jorge Amaya · Andreas Griewank · Nikolai Strogies
Received: 2 February 2010 / Accepted: 29 September 2010 / Published online: 17 October 2010

Keywords Mine planning · Continuous optimization · Calculus of variations · Functional analysis

To be submitted:

Characterizing the optimal profile of the open pit problem in the continuous framework

Jorge Amaya, Cristopher Hermosilla, Emilio Molina

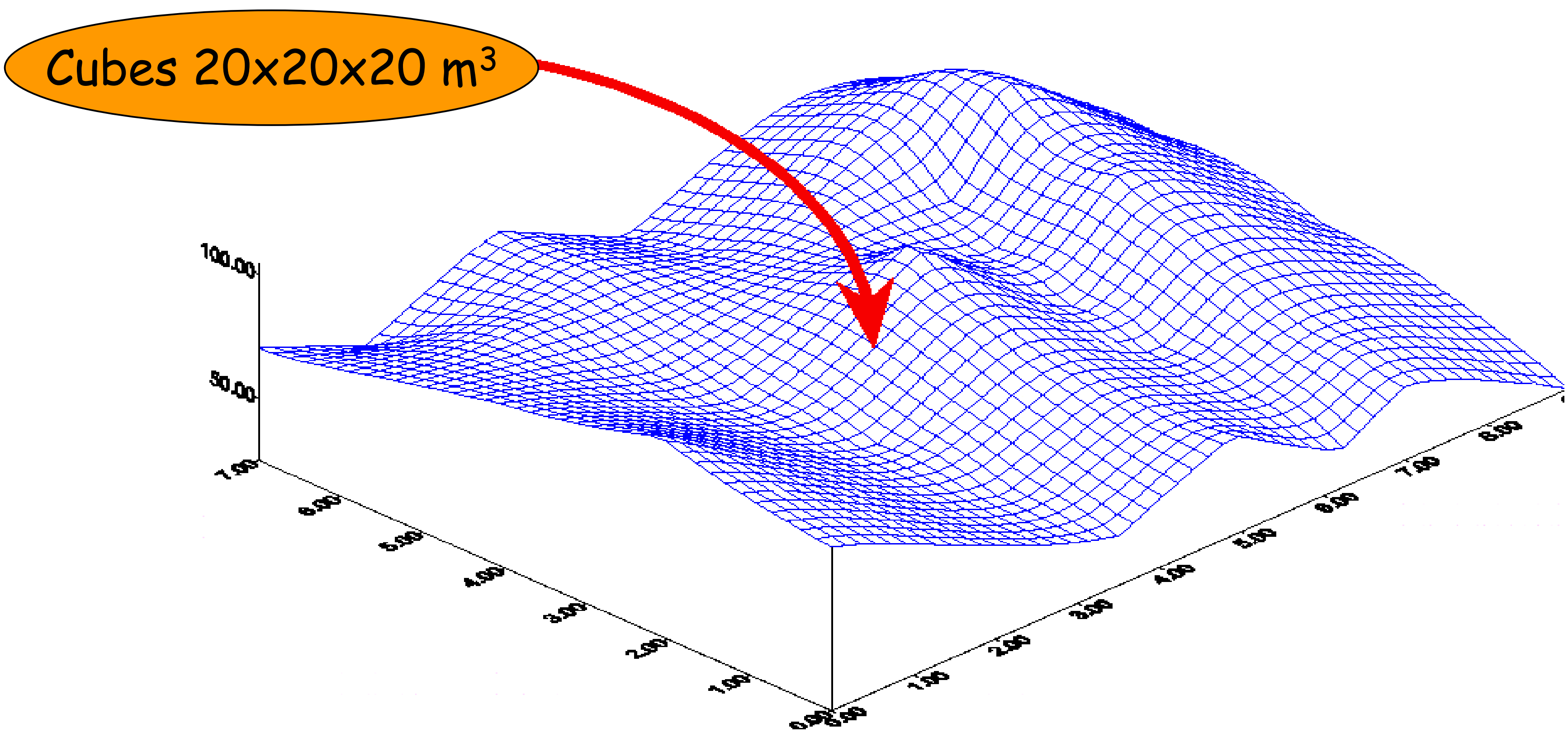
October 2, 2017

Keys words: Open pit modeling; Continuous approach; Optimality characterization.

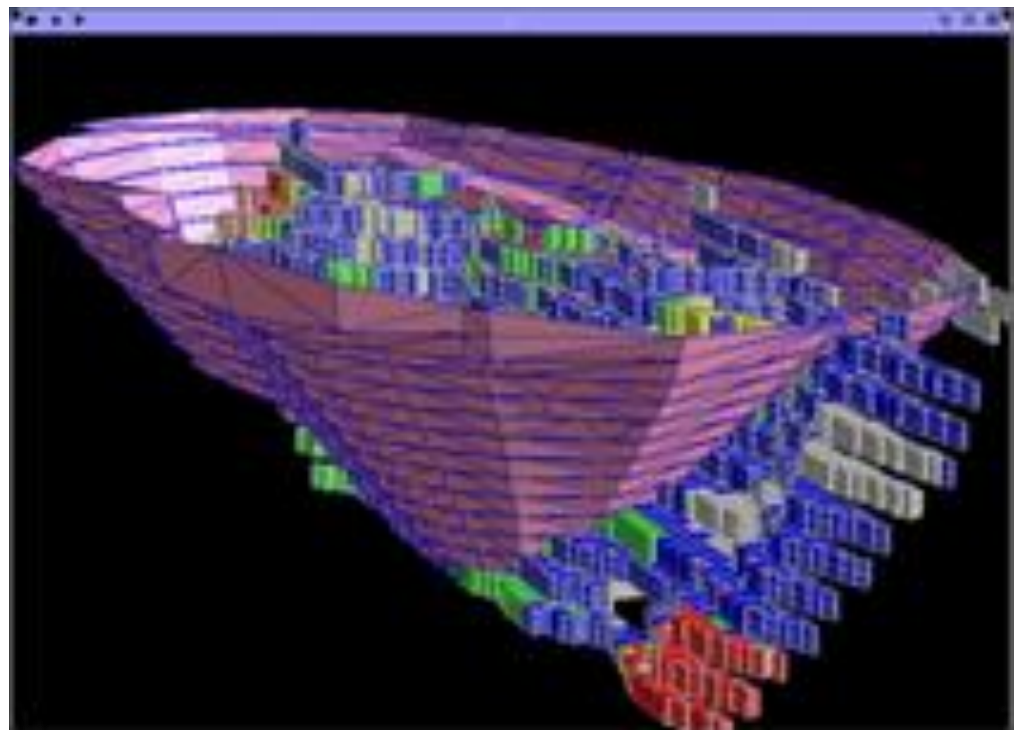
II. DISCRETE APPROACH

The objective of the planning is to determine an optimal sequence of extraction, satisfying production capacity at each period and geotechnical constraints.

The blocks represents physical units of extraction.



Block model
Idealized
image



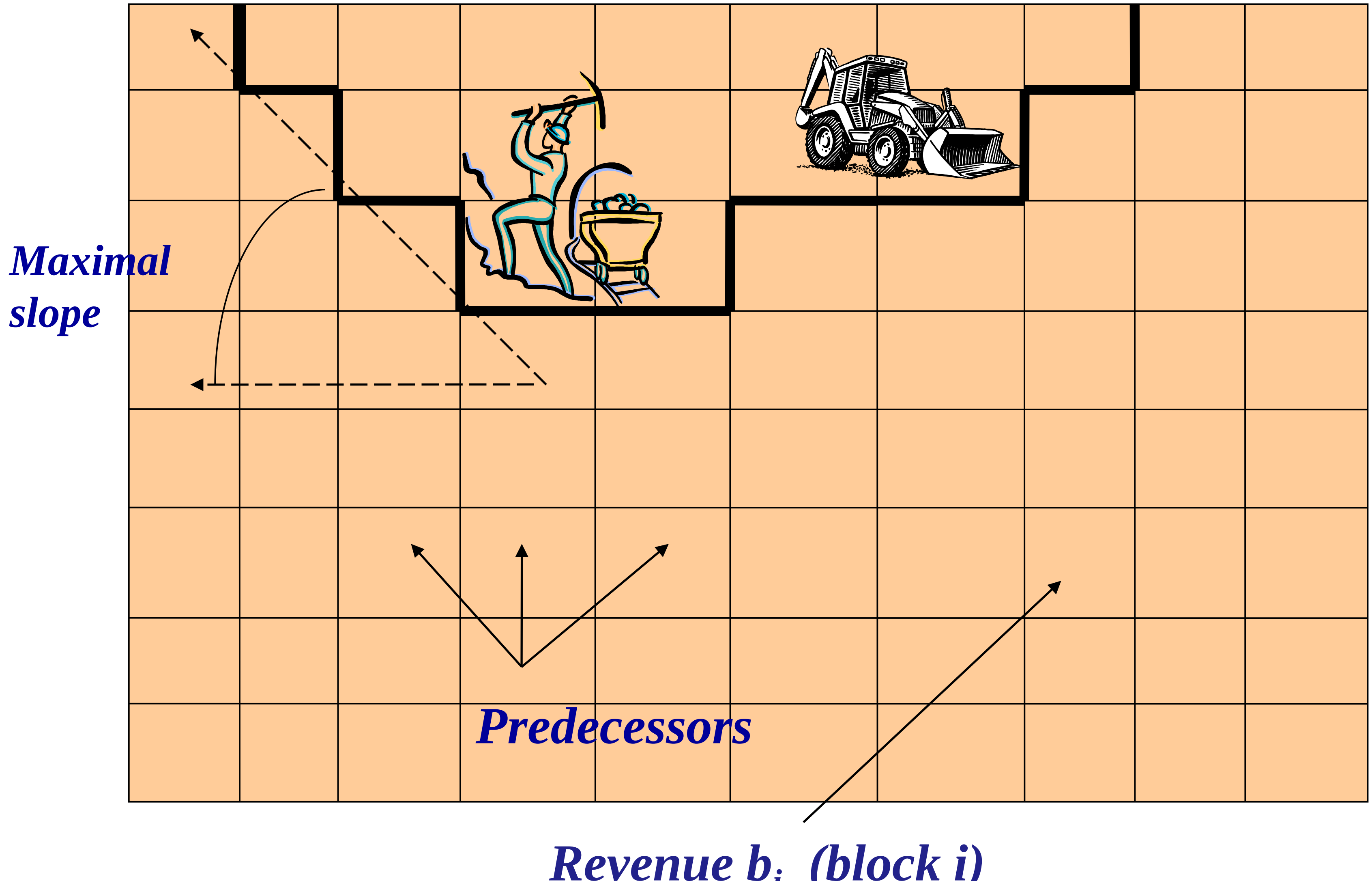
Objective:

Given an estimation of the value (grade), the decision-maker needs to decide the economic sequence of blocks, satisfying

- Capacities (extraction, transport, process…)
- Wall slope of the pit (stability)
- Waste/ore rate
- Destinations of the blocks (plant, stock, waste, …)
- **Exposed ore**

This gives rise to **(very) large (linear) binary Optimization problems**.

Blocks are represented by **nodes**
and precedence relations are represented by **arcs**



Predecessors

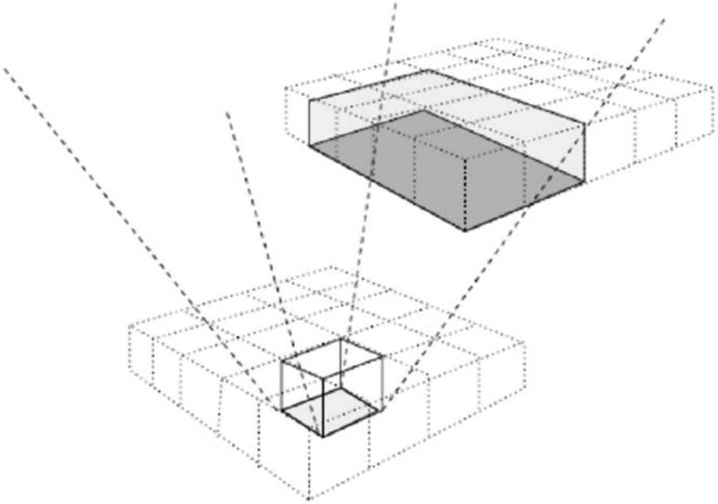


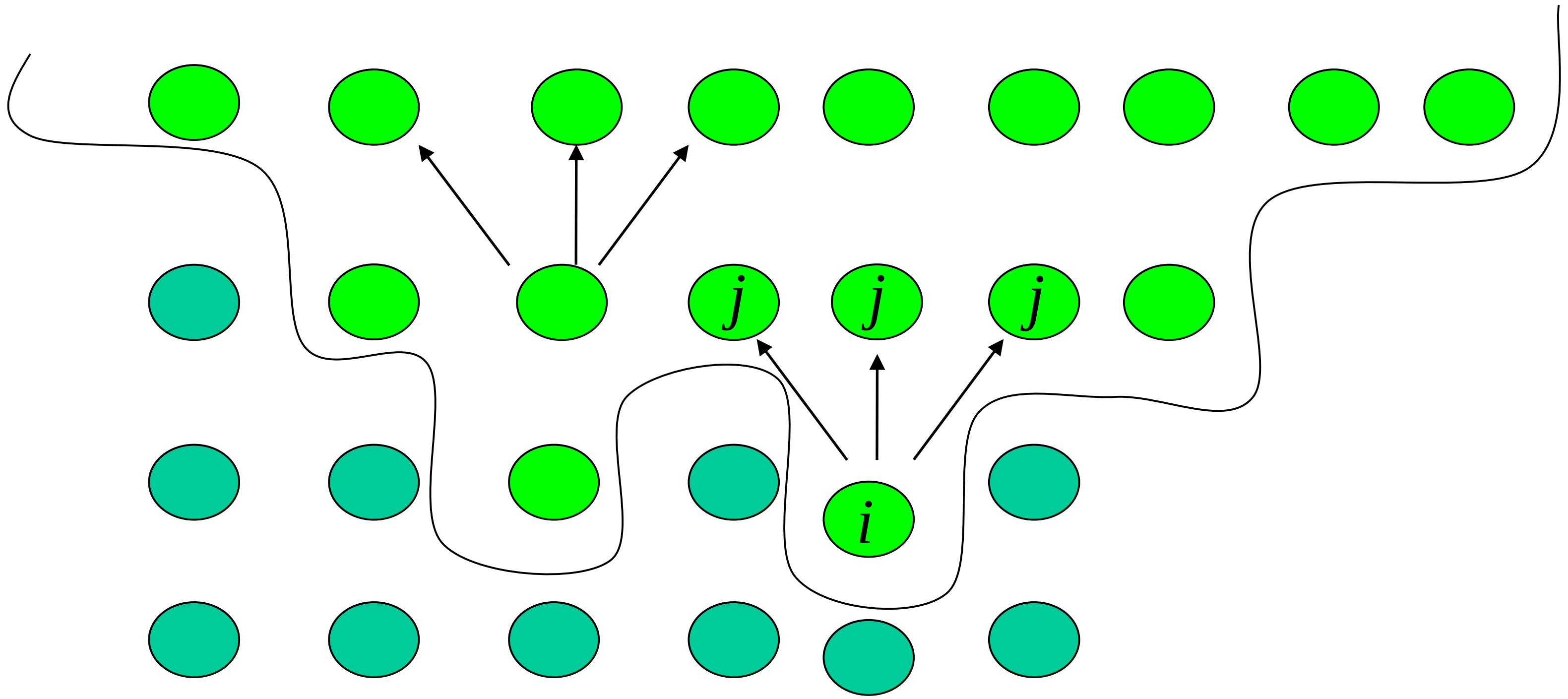
Figure: Precedence relationship between blocks

Definition:

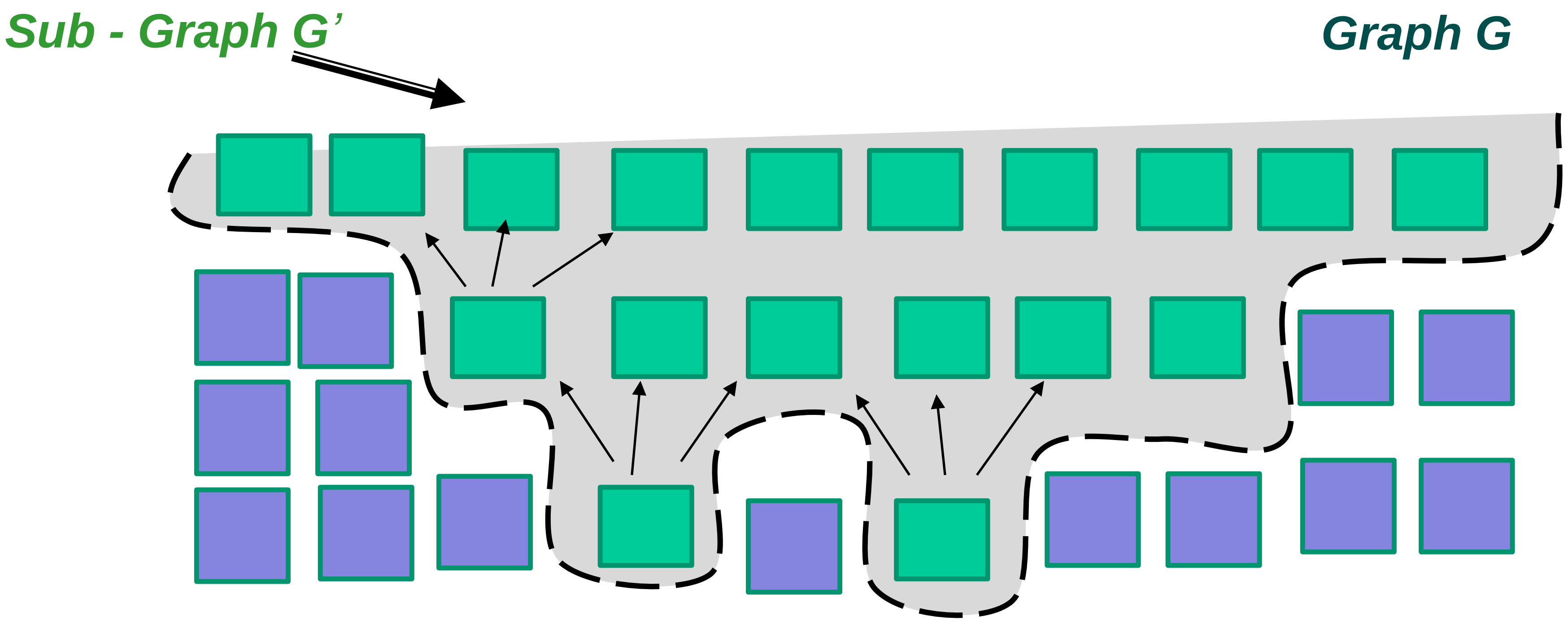
PIT is a set of nodes, closed with respect to the precedence arcs.

$$\text{If} \quad P_i = \{ j \,/\, (i, j) \in A \}, \quad \text{then} \quad i \in G' \Rightarrow j \in G' \quad \forall j \in P_i$$

PIT



FIRST PROBLEM: to find the volume of interest



$$\text{If} \quad P_i = \left\{ j \,/\, (i, j) \in A \right\}, \quad \text{then} \quad i \in G' \Rightarrow j \in G' \quad \forall j \in P_i$$

FINAL PIT ➔ MAXIMAL CLOSURE

To find a sub-graph satisfying the precedence relations and having maximal benefice.

The static problem (maximal closure)

$$x_i = \begin{cases} 1 & \text{block } i \text{ belongs to the chosen set} \\ 0 & \text{block } i \text{ doest not belong to the chosen set} \end{cases}$$

(FOP)

$$\text{Max} \quad \sum_{i \in N} b_i x_i$$

$$x_j - x_i \geq 0 \qquad (i, j) \in A$$

$$x_i = 0 \text{ or } 1 \qquad i \in N.$$

The total capacity constraint could be added to the model:

The capacitated static problem

(CFOP) $$\text{Max} \quad \sum_{i \in N} b_i x_i$$

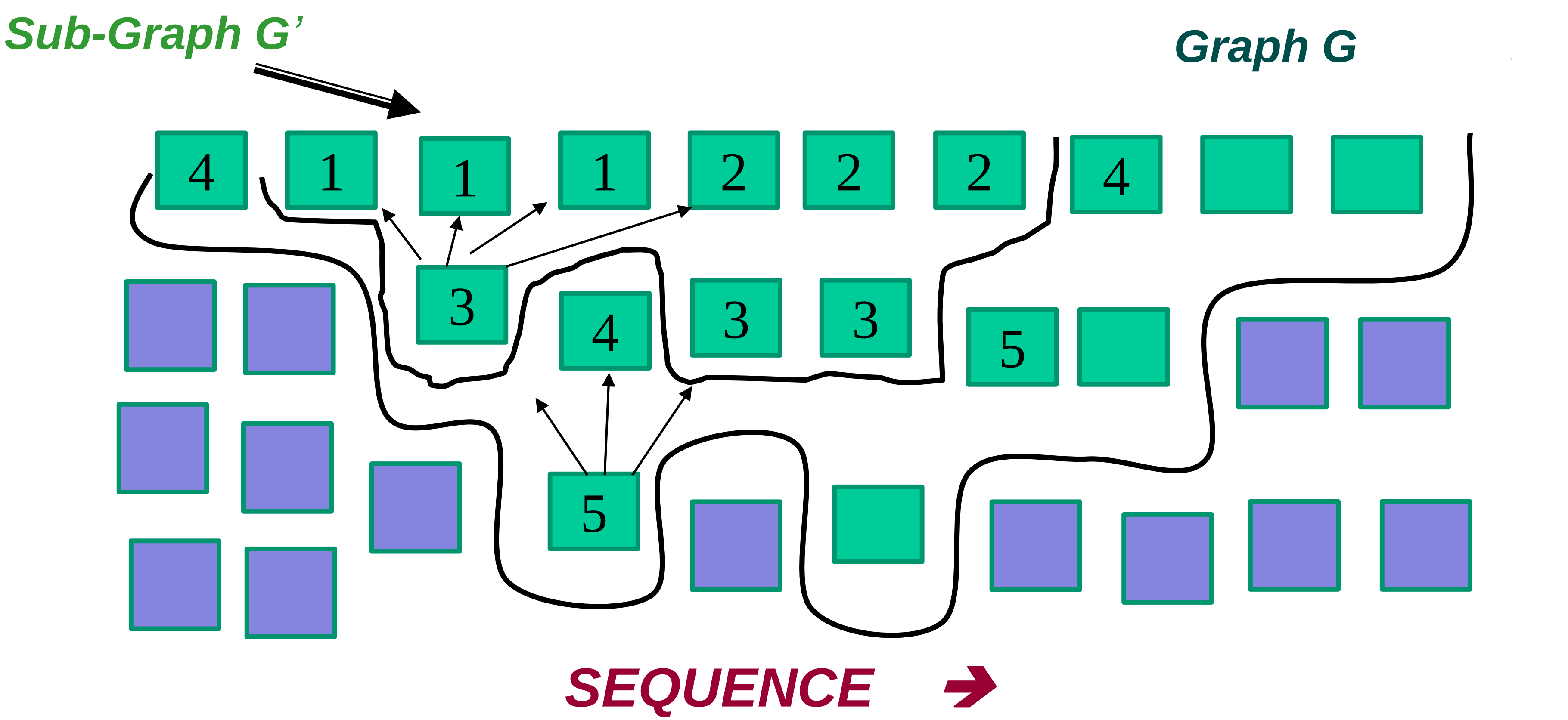
$$x_j - x_i \geq 0 \qquad (i, j) \in A$$

$$\sum_{i \in N} x_i \leq C$$

$$x_i = 0 \text{ or } 1 \qquad i \in N.$$

SECOND PROBLEM:

dynamic version (more realistic)



To find a feasible sequence of blocks having maximal discounted value.

For this new model (considering sequence),

we define decision variables:

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted at time } t \\ 0 & \text{if block } i \text{ is not extracted at time } t \end{cases}$$

The dynamic problem (sequencing problem)

(CDOP)

$$\text{Max} \sum_{t=1}^{T} \sum_{i \in N} \frac{b_i}{(1+\alpha)^{t-1}} x_i^t$$

$$\sum_{t=1}^{T} x_i^t \le 1 \qquad i \in N$$

$$\sum_{l=1}^{t} x_j^l - x_i^t \ge 0 \quad (i,j) \in A,\ t = 1, \cdots, T$$

$$\sum_{i \in N} p_i x_i^t \le C_t \qquad t = 1, \cdots, T$$

$$x_i^t = 0 \text{ or } 1 \qquad i \in N,\ t = 1, \cdots, T.$$

1. The first one is the **Final Open Pit (FOP)** problem, which aims to find the region of maximal economic value for exploitation under some geotechnical stability constraints.

2. The second one is the **Capacitated Final Open Pit (CFOP)** which considers an additional constraint on the total capacity for the previous formulation.

3. The multi-period version, which we call here the **Capacitated Dynamic Open Pit (CDOP)** problem, with the goal of finding an optimal sequence of extracted volumes in a certain finite time horizon for bounded capacities at each period.

Let S_{FOP}, S_{CFOP} and S_{CDOP} be the sets of blocks contained in the solution of these 3 previous problems, respectively.

$S_{FOP}:$	Final Open Pit (FOP)
$S_{CFOP}:$	Capacitated Final Open Pit (CFOP)
$S_{CDOP}:$	Capacitated Dynamic Open Pit (CDOP)

Property:

$$S_{CFOP} \subseteq S_{FOP}$$

$$S_{CDOP} \subseteq S_{FOP}$$

Comments:

- Combinatorial nature of the precedence relationship: mathematical formulation based on block models uses graph theory and integer programming

- Pioneer works: Lerchs and Grossman algorithm (1965, graph approach) and Picard’s network flow approach, 1976.

- Real world mines can produce large scale instances having several millions of variables and constraints.

- In practice greedy-type strategies are not optimal and heuristics (combined with B&B and LP relaxations) must be applied to tackle those huge problems.

OPEN PIT SCHEDULING WITH EXPOSED ORE RESERVE

Motivation:

In open pit extraction, usually one needs to remove material with poor economic value (*waste*) to give access to more economically profitable material.

An unmined block is said to be **exposed** at the beginning of a given period if its precedent blocks have been all extracted.

The problem that this work addresses is the design of a block schedule, with the additional constraint of leaving enough **exposed** ore reserve that is readily available at the start of the period.

Our approach is particularly useful in mines having disseminated or irregular ore distribution.

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Optimizing open-pit block scheduling with exposed ore reserve

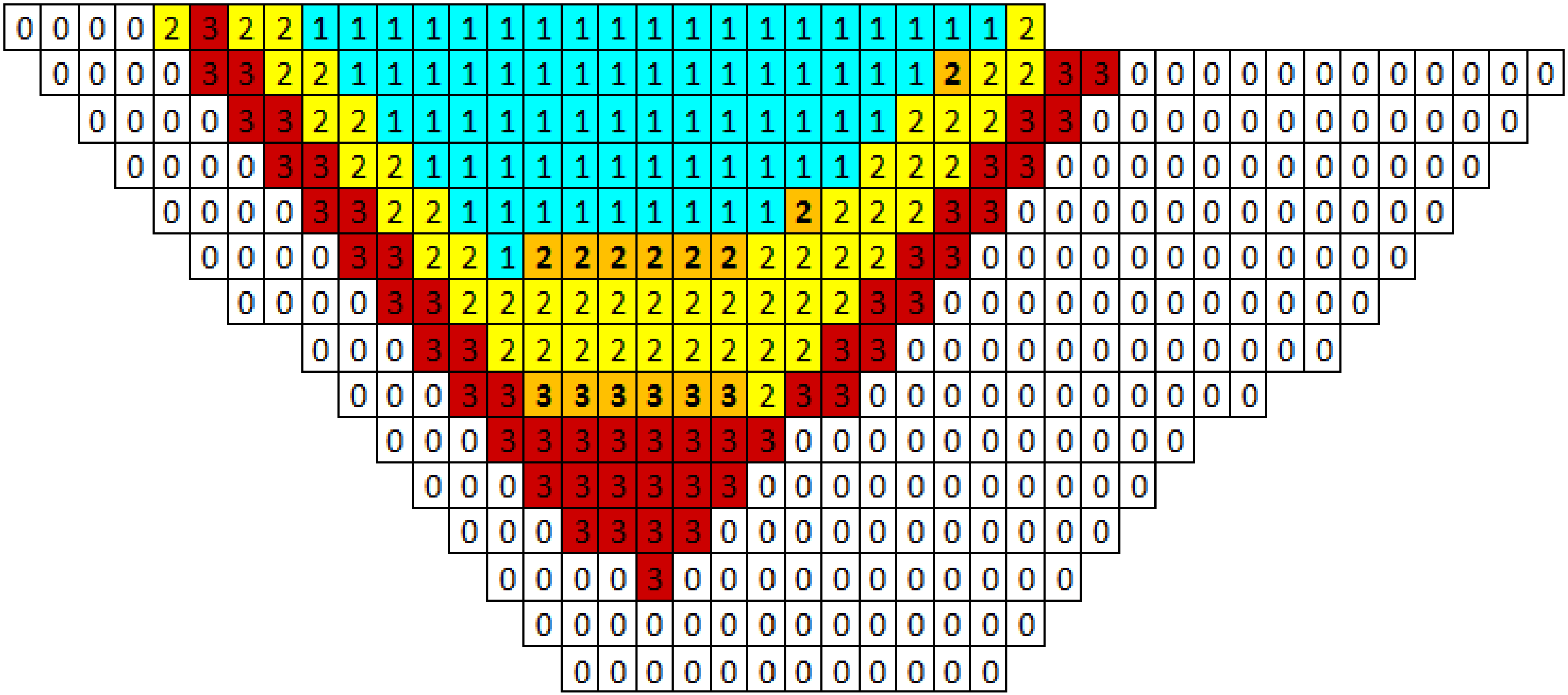
by J. Saavedra-Rosas*, E. Jélvez†, J. Amaya‡, and N. Morales†

Synopsis

A crucial problem in the open pit mining industry is to determine the *optimal block scheduling*, defining how the orebody will be sequenced for exploitation. An orebody is often comprised of several thousand or million blocks and the scheduling models for this structure are very complex, giving rise to very large combinatorial linear problems. Operational mine plans are usually produced on a yearly basis and further scheduling is attempted to provide monthly, weekly, and daily schedules. A portion of the ore reserve is said to be exposed if it is readily available for extraction at the start of the period. In this paper, an integer programming (IP) model is presented to generate pit designs under exposed ore reserve requirements, as an extension of the classical optimization models for mine planning. For this purpose, we introduce a set of new binary variables, representing which blocks can be declared as exposed ore reserve, in addition to the extraction and processing decisions. The model has been coded and tested in a set of standard instances, showing very encouraging results in the generation of operational block schedules.

Keywords

block scheduling, surface mining, open pit planning, optimization model, exposed ore reserve.



Extraction Period

0	Unmined blocks
1	Extracted blocks at period 1
2	Extracted blocks at period 2
3	Extracted blocks at period 3
2	Exposed ore reserve at period 1 and extracted at period 2
3	Exposed ore reserve at period 2 and extracted at period 3

Symbol	Description
B	Set of blocks
A	Set of precedence arcs
T	Time horizon (number of periods)
b_i^t	Profit resulting from the mining of block i at period t
p_i^t	Cost of mining and processing block i at period t
m_i^t	Cost of mining block i at period t
M^t	Maximum mining capacity for period t
P^t	Maximum processing capacity for period t
F^t	Minimum exposed ore reserve required at period t (as metal)
τ_i	Tonnage of block i
λ_i	Ore grade of block i
λ_{cg}	Cut-off grade to define minimum exposed ore reserve requirement

Three types of variables are used in the model, all of them are binary. The first type is the variable associated to the extraction for processing purposes for each block

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and processed at time } t \\ 0 & \text{otherwise} \end{cases}$$

The second variable type describes the decision relating to the disposal of a block by sending it to the waste dump

$$w_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and sent to waste dump at time } t \\ 0 & \text{otherwise} \end{cases}$$

The third variable type is used to identify exposed blocks; throughout the paper it will indistinctively be called "visibility" or "exposure" variable

$$y_i^t = \begin{cases} 1 & \text{if block } i \text{ is exposed at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{\lambda}_i = \begin{cases} \lambda_i & if\ \lambda_i \geq \lambda_{cg} \\ 0 & otherwise. \end{cases}$$

$$\textbf{(OPBS} - \textbf{EO)} \quad max \sum_{i \in B} \sum_{t=1}^{T} [(b_i^t - p_i^t) x_i^t - m_i^t w_i^t]$$

$$\sum_{t=1}^{T} (x_i^t + w_i^t) \leq 1 \qquad \forall\, i \in B \qquad (1)$$

$$\sum_{i \in B} \tau_i (x_i^t + w_i^t) \leq M^t \qquad \forall\, t \in \{1, \dots, T\} \qquad (2)$$

$$\sum_{i \in B} \tau_i x_i^t \leq P^t \qquad \forall\, t \in \{1, \dots, T\} \qquad (3)$$

$$y_i^t + \sum_{s=1}^{t} (x_i^s + w_i^s) \leq \sum_{s=1}^{t} \left(x_j^s + w_j^s\right) \qquad \forall\, (i, j) \in A, \;\; t \in \{1, \dots, T\} \qquad (4)$$

$$y_i^t \leq x_i^{t+1} \qquad \forall\, i \in B, \;\; t \in \{1, \dots, T-1\} \qquad (5)$$

$$\sum_{i \in B} \tau_i \bar{\lambda}_i \, y_i^t \geq F^t \qquad \forall\, t \in \{1, \dots, T-1\} \qquad (6)$$

$$x_i^t, w_i^t, y_i^t \in \{0,1\} \qquad \forall\, i \in B, \;\; t \in \{1, \dots, T\} \qquad (7)$$

Marvin case study

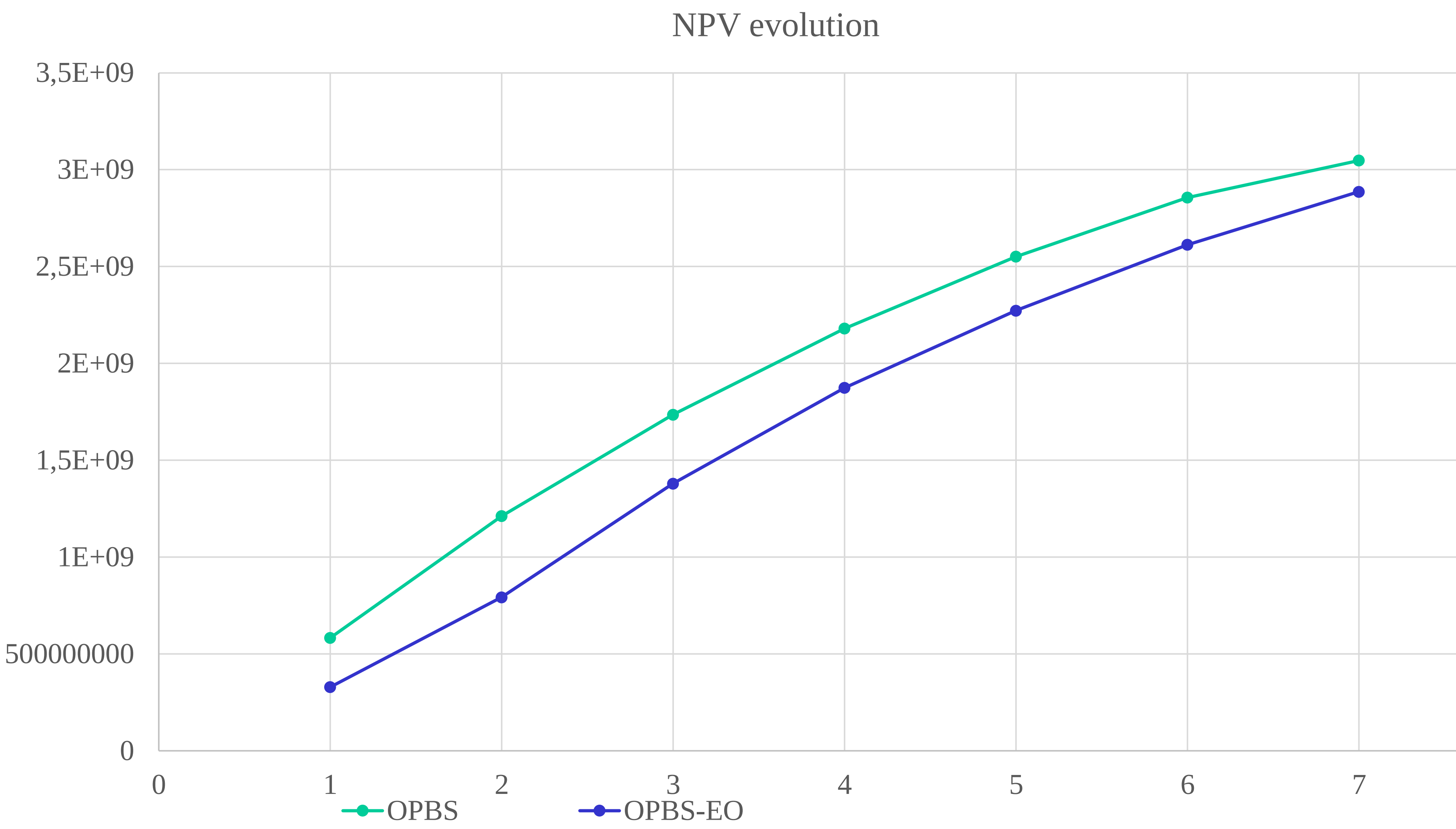
- This block model contains 53,271 blocks of $30 \times 30 \times 30$ meters.
- We consider T=7 periods
- The wall slope requirements are given by a 45° slope angle

RESULTS: Comparison of production plans for the Marvin case study.

OPBS: **standard model**

OPBS-EO: **our model, with exposed ore**

	Discounted value	
Period	**OPBS**	**OPBS-EO**
1	582,605,720	329,142,653
2	629,312,551	463,833,658
3	523,828,567	586,467,752
4	444,162,523	494,561,419
5	371,571,924	398,567,122
6	304,876,581	339,961,783
7	191,140,416	273,394,271
Total NPV	3,047,498,282	2,885,928,658


NPV evolution
3,5E+09
3E+09
2,5E+09
2E+09
1,5E+09
1E+09
500000000
0
0
1
2
3
4
5
6
7
OPBS
OPBS-EO

	OPBS				OPBS-EO			
Period	**Grade %**	**Ore Ton**	**Total Ton**	**Exposed Ton**	**Grade %**	**Ore Ton**	**Total Ton**	**Exposed Ton**
1	1.03	19,994,670	59,989,965	**0**	0.72	19,999,638	59,891,451	**0**
2	1.14	19,971,010	49,490,568	**0**	0.92	19,966,980	45,939,352	**100,134**
3	1.05	19,980,880	40,052,602	**359**	1.14	19,985,370	39,798,429	**100,055**
4	1.02	19,985,340	44,985,249	**1,462**	1.09	19,987,840	42,930,865	**100,026**
5	0.97	19,999,920	49,166,274	**4,719**	1.14	19,990,580	52,232,416	**100,099**
6	0.92	19,999,290	52,191,380	**7,569**	1.14	19,992,410	46,784,718	**100,362**
7	0.88	15,535,850	46,060,989	**6,254**	1.47	15,516,740	16,587,549	**100,159**
Total		135,466,960	341,937,027	**20,363**		135,439,558	304,164,781	**600,835**

Two stage resolution approach

- **First initial solution (Ferland)**

- **Solving the optimization problem starting from this warm solution**

Monolithic approach

Solve the problem directly without apply any heuristic method

Hard to be solved.

Two stage approach

First solve the problem using a greedy strategy (period by period) to find a feasible solution.

Then put this solution in optimization solver to start the B&B from there.

Comparison in terms of NPV, computation time and optimality gap is interesting…

To generate an initial solution, solve a sequence of sub problems associated with each period $t = 1, \ldots, T$ in increasing order of t

$$(\mathbf{OPBS-EO}) \quad max \sum_{i \in B} \sum_{t=1}^{T} [(b_i^t - p_i^t) x_i^t - m_i^t w_i^t]$$

$$\sum_{t=1}^{T} (x_i^t + w_i^t) \leq 1 \qquad \forall\, i \in B \qquad (1)$$

$$\sum_{i \in B} \tau_i (x_i^t + w_i^t) \leq M^t \qquad \forall\, t \in \{1, \ldots, T\} \qquad (2)$$

$$\sum_{i \in B} \tau_i x_i^t \leq P^t \qquad \forall\, t \in \{1, \ldots, T\} \qquad (3)$$

$$y_i^t + \sum_{s=1}^{t} (x_i^s + w_i^s) \leq \sum_{s=1}^{t} (x_j^s + w_j^s) \qquad \forall\, (i,j) \in A,\ \ t \in \{1, \ldots, T\} \qquad (4)$$

$$y_i^t \leq x_i^{t+1} \qquad \forall\, i \in B,\ \ t \in \{1, \ldots, T-1\} \qquad (5)$$

$$\sum_{i \in B} \tau_i \bar{\lambda}_i y_i^t \geq F^t \qquad \forall\, t \in \{1, \ldots, T-1\} \qquad (6)$$

$$x_i^t, w_i^t, y_i^t \in \{0,1\} \qquad \forall\, i \in B,\ \ t \in \{1, \ldots, T\} \qquad (7)$$

B^t blocks not extracted yet

$$\max \quad \sum_{i \in B^t} \left[\left(b_i^t - p_i^t \right) \bar{x}_i - m_i^t \bar{w}_i \right]$$

S. t.

$$\sum_{i \in B^t} \tau_i \left(\bar{x}_i + \bar{w}_i \right) \leq M^t - \bar{M}^{t-1}$$

$$\sum_{i \in B^t} \tau_i \bar{x}_i \leq P^t - \bar{M}^{t-1}$$

$$y_j^t + \bar{x}_j + \bar{w}_j \leq \bar{x}_i + \bar{w}_i \qquad \left(i, j\right) \in A, i \in B^t$$

$$\sum_{i \in B^t} \tau_i \bar{\lambda}_i y_i^t \geq F^t$$

$$\bar{x}_i, \bar{w}_i, y_i^t \in \left\{0,1\right\} \qquad i \in B^t$$

To generate an initial solution, solve a sequence of sub problems associated with each period $t = 1, \ldots, T$ in increasing order of t

$$
\begin{aligned}
\max \quad & \sum_{i \in B^t} \left[\left(b_i^t - p_i^t \right) \bar{x}_i - m_i^t \bar{w}_i \right] \\
\text{S. t.} \quad & \\
& \sum_{i \in B^t} \tau_i \left(\bar{x}_i + \bar{w}_i \right) \le M^t - \bar{M}^{t-1} \\
& \sum_{i \in B^t} \tau_i \bar{x}_i \le P^t - \bar{M}^{t-1} \\
& y_j^t + \bar{x}_j + \bar{w}_j \le \bar{x}_i + \bar{w}_i && (i, j) \in A, i \in B^t \\
& \sum_{i \in B^t} \tau_i \bar{\lambda}_i y_i^t \ge F^t \\
& \bar{x}_i, \bar{w}_i, y_i^t \in \{0, 1\} && i \in B^t
\end{aligned}
$$

After solving the sub problem t, we obtain the values of the variables for period t as follows:

$$
\begin{aligned}
& \bar{x}_i = 1 \quad \Rightarrow \quad x_i^t = 1 \text{ and } w_i^t = 0 \\
& \bar{w}_i = 1 \quad \Rightarrow \quad w_i^t = 1 \text{ and } x_i^t = 0 \\
& y_i^t = 1 \quad \Rightarrow \quad x_i^{t+1} = 1 \text{ and } w_i^{t+1} = 0 \\
& B^{t+1} = B^t - \left\{ i \in B^t : x_i^t = 1 \text{ or } w_i^t = 1 \text{ or } y_i^t = 1 \right\}
\end{aligned}
$$

Also, denote the total weight of the exposed blocks determined in period t

$$\boxed{\bar{M}^t = \sum_{i \in B^t} \tau_i y_i^t}$$

Results for Marvin 2D case

Monolithic approach

NPV : 133,982,359

Time (s): 166

Gap (%): 0.999

Two stage approach

First feasible solution (Ferland)

NPV : 131,366,038

Time (s): 4

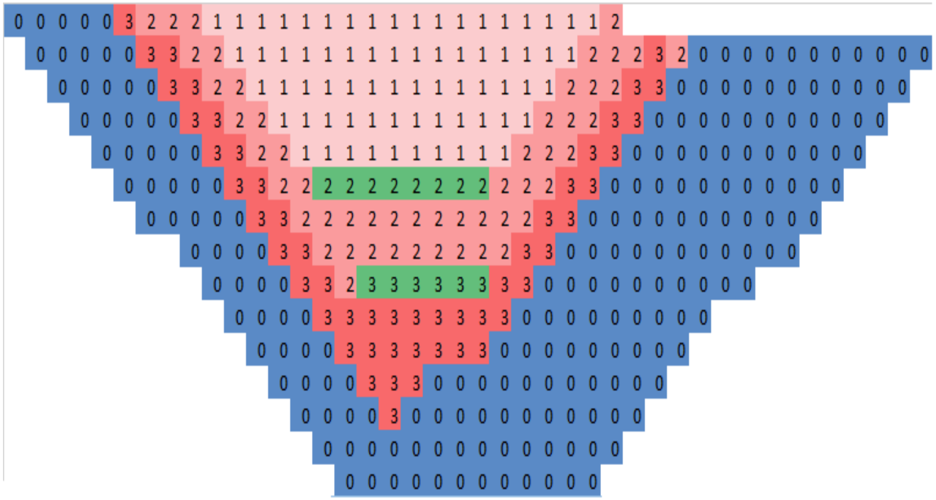
Gap (%): 7.2

After MIPStart solution

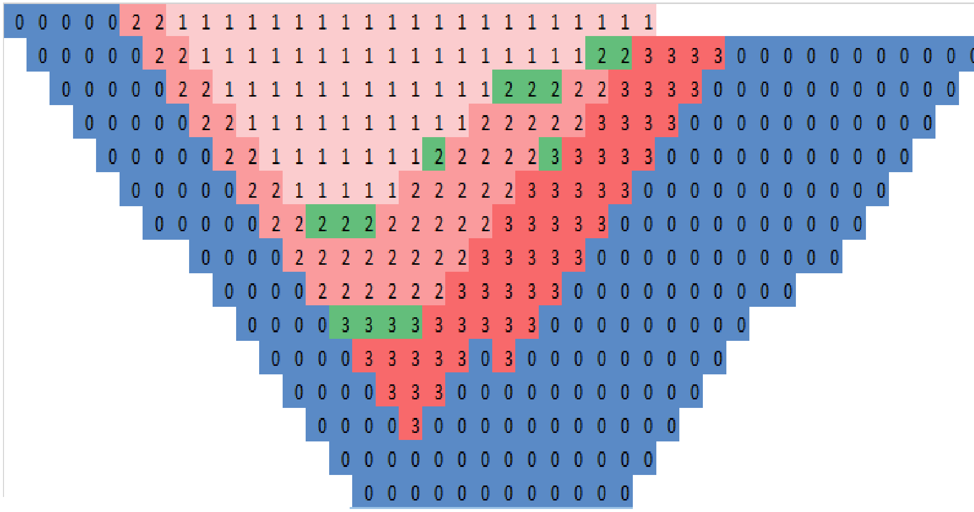
- NPV : 133,865,904
- Time (s): 150
- Gap (%): 0.999

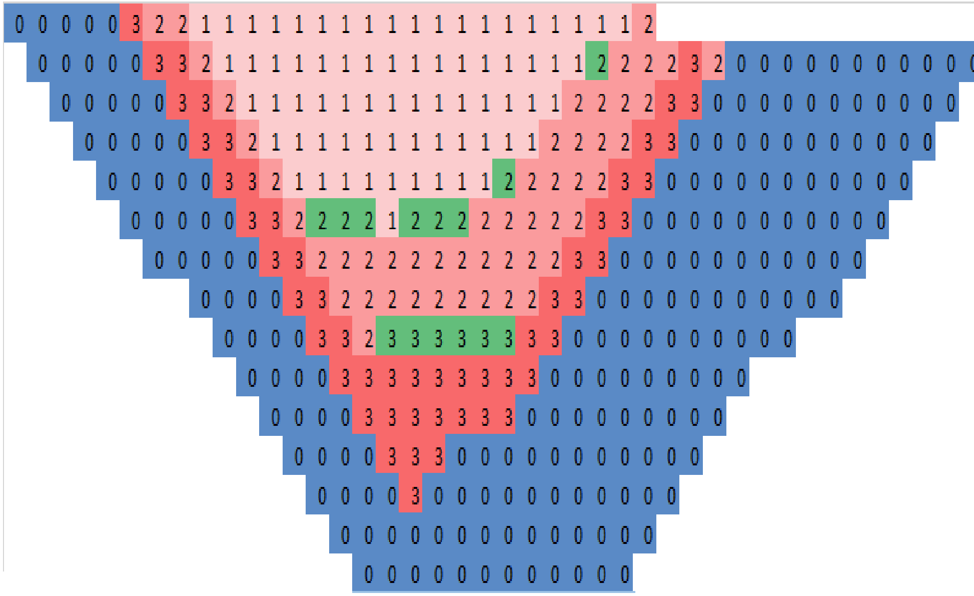
Results for Marvin 2D

Monolithic approach



Two stage approach





Conclusions

- The new model is very close the classical one in terms of discounted value.

- The tons of mineral are also very similar

- The model with reserve generates a more regular production planning

- The model with reserve also permits a more efficient use of the processing plant, avoiding idle times at the beginning of each period

THANK YOU

Generating an initial feasible solution

1. Stage:
Find the final pit (LP problem), and then delete the blocks not included in it.
From this stage, we work with the residual graph.

2. Pre-processing:
For each block *i*, define the first period in which the block could be mined. Let t_i that period. Then we can fix:

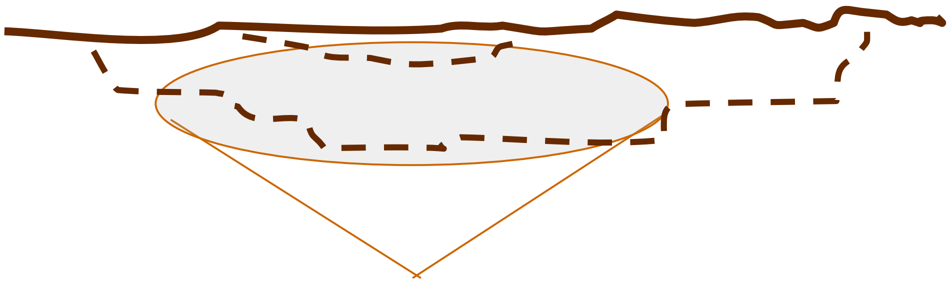
$$x_i^s = 0 \quad \text{for all } s < t_i$$

To calculate t_i we define $\mathcal{N}_i^* = \{j \in \mathcal{N}_{FP} \ / \text{ there is a path from } i \text{ to } j\}$. This represent the cone over block i. Then t_i is given by:

$$t_i = \min\{t \ / \sum_{j \in \mathcal{N}_i^*} p_j \leq \sum_{k=0}^{t} C_k\}$$

3. Pre-processing: Redefine benefits:

$$b^*{}_i = \rho^{t_i} b_i$$



Other definitions of b_i^* could be envisaged.
Example: The total benefit contained in the cone-above.

4. Apply Greedy algorithm with these new benefits.

(Ferland et al, in: *Studies in Computational Intelligence*, Springer Verlag, 2007)