The Mine Planning Problem: from the Discrete to the Continuous Framework

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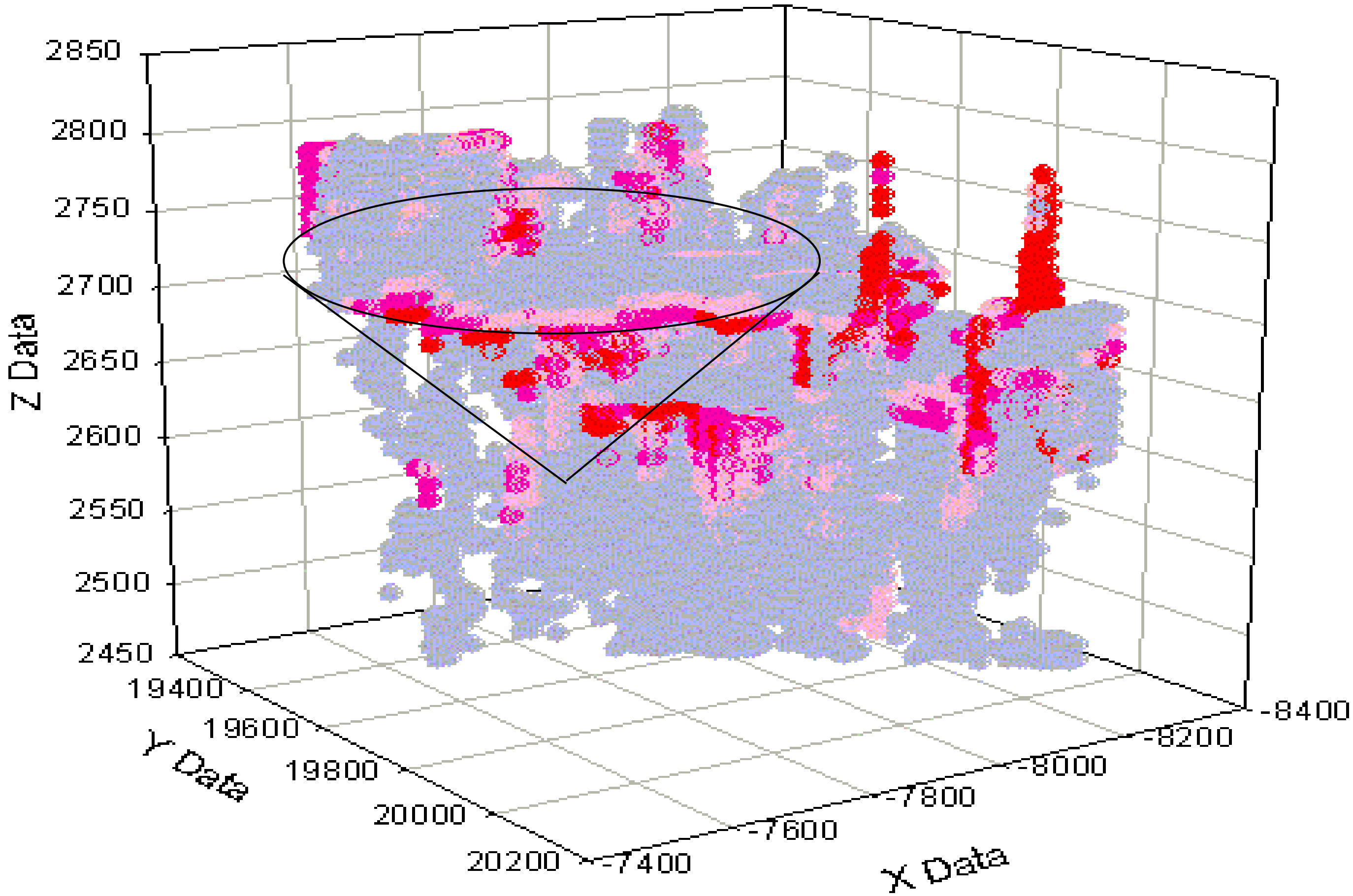
The Chuquicamata open pit is 5 x 3 km large and 1 km deep, and is the largest metal mine of the world.

Copper mine

Marmato (Caldas, Colombia). Gold mine



The information in the data base for our purposes is essentially:

- **coordinates *(x, y, z)***
- **grade at each block (% of Copper/Total mass) and other characteristics**

THE PROBLEM

Given an **estimation** of the value distribution in situ, one needs to schedule the portions of the mine to be extracted at each period, which the aim is to find an economic sequence of extraction.

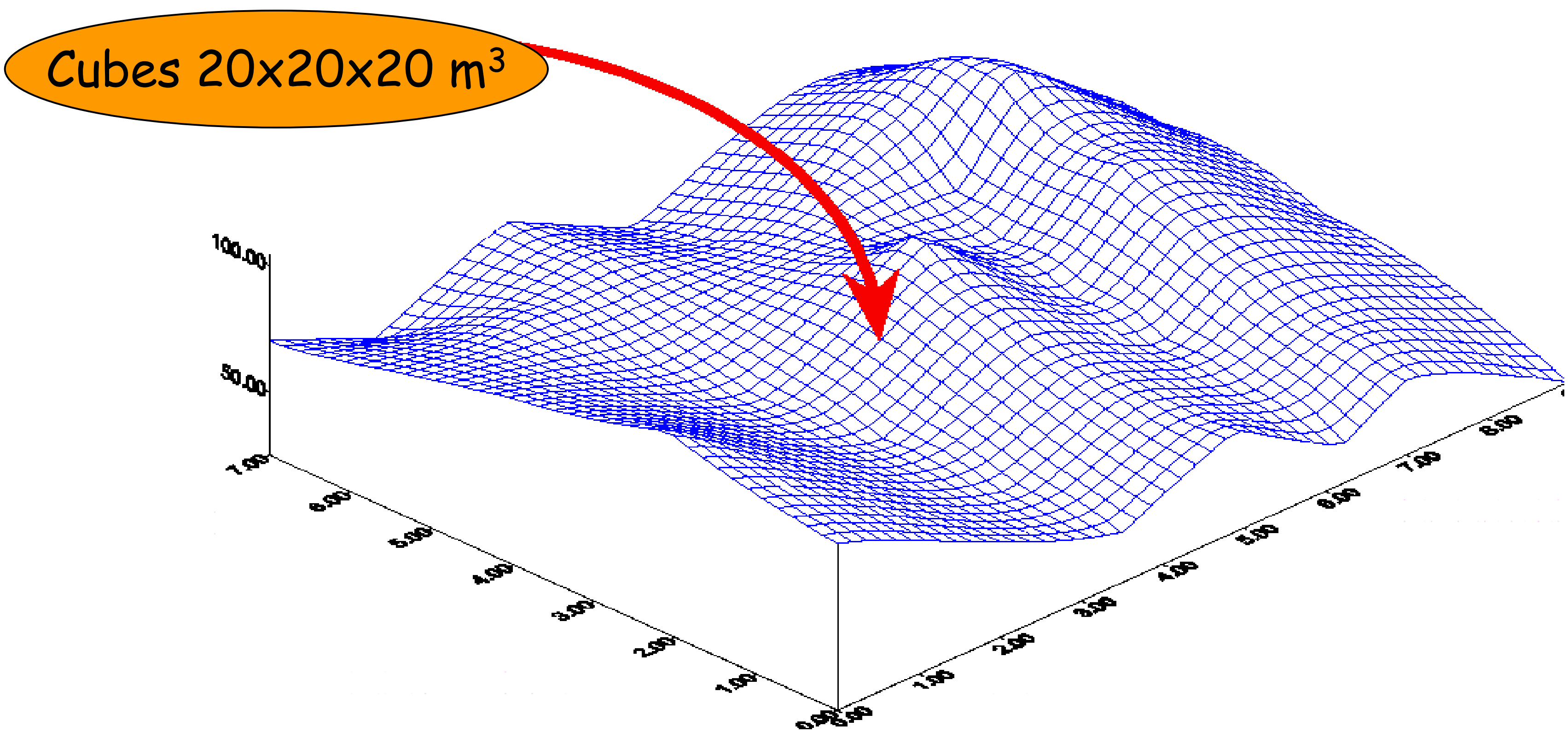
We show two frameworks:

- a discrete optimization model (binary decision variables), and
- a continuous approach (posed in an appropriate functional space).

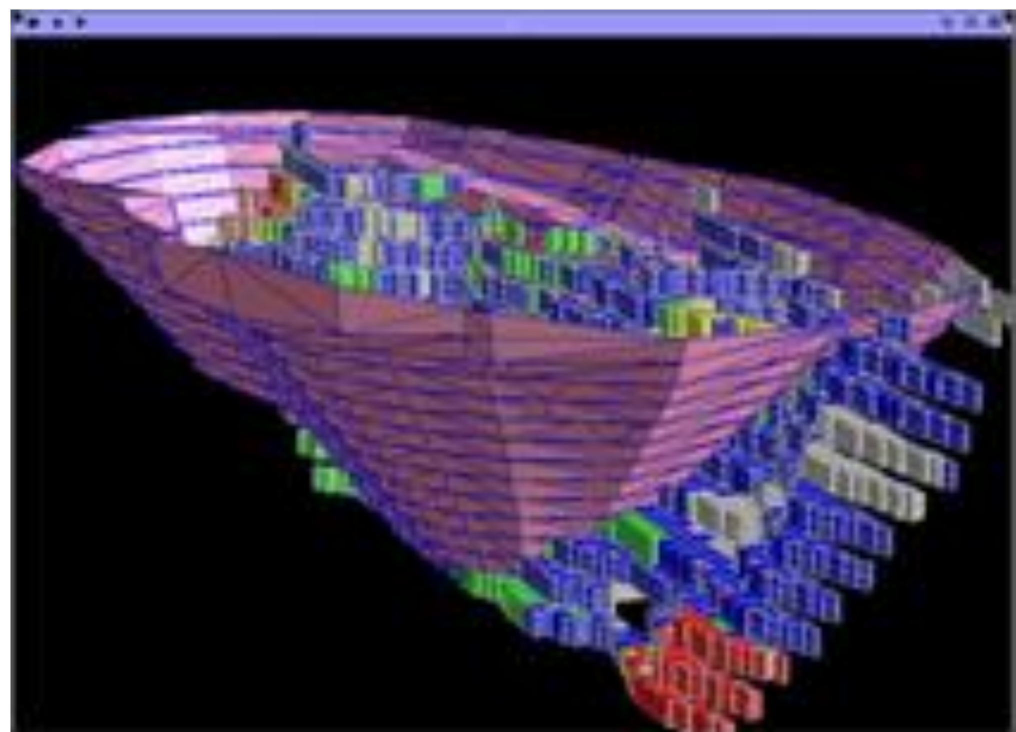
I. DISCRETE APPROACH

The objective of the planning is to determine an optimal sequence of extraction, satisfying production capacity at each period and geotechnical constraints.

The blocks represents physical units of extraction.



Block model
Idealized
image



Objective: mine planning in open pit mines

Given an estimation of the value (grade), the decision-maker needs to decide the economic sequence of blocks, satisfying

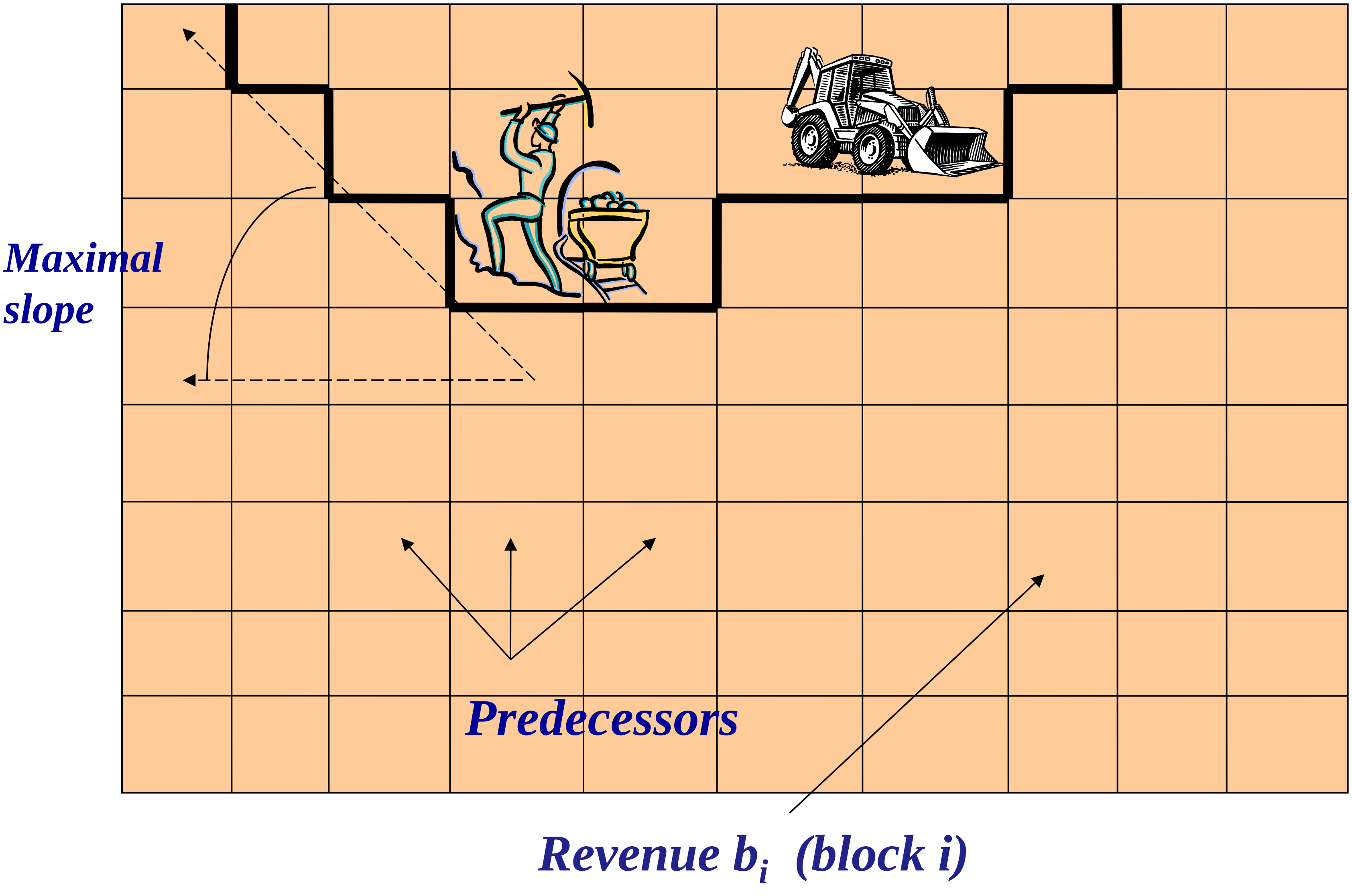
- Capacities (extraction, transport, process…)
- Wall slope of the pit (stability)

A more general model can include the conditions

- Waste/ore rate
- Destinations of the blocks (plant, stock, process…)

This gives rise to **(very) large (linear) binary Optimization problems**.

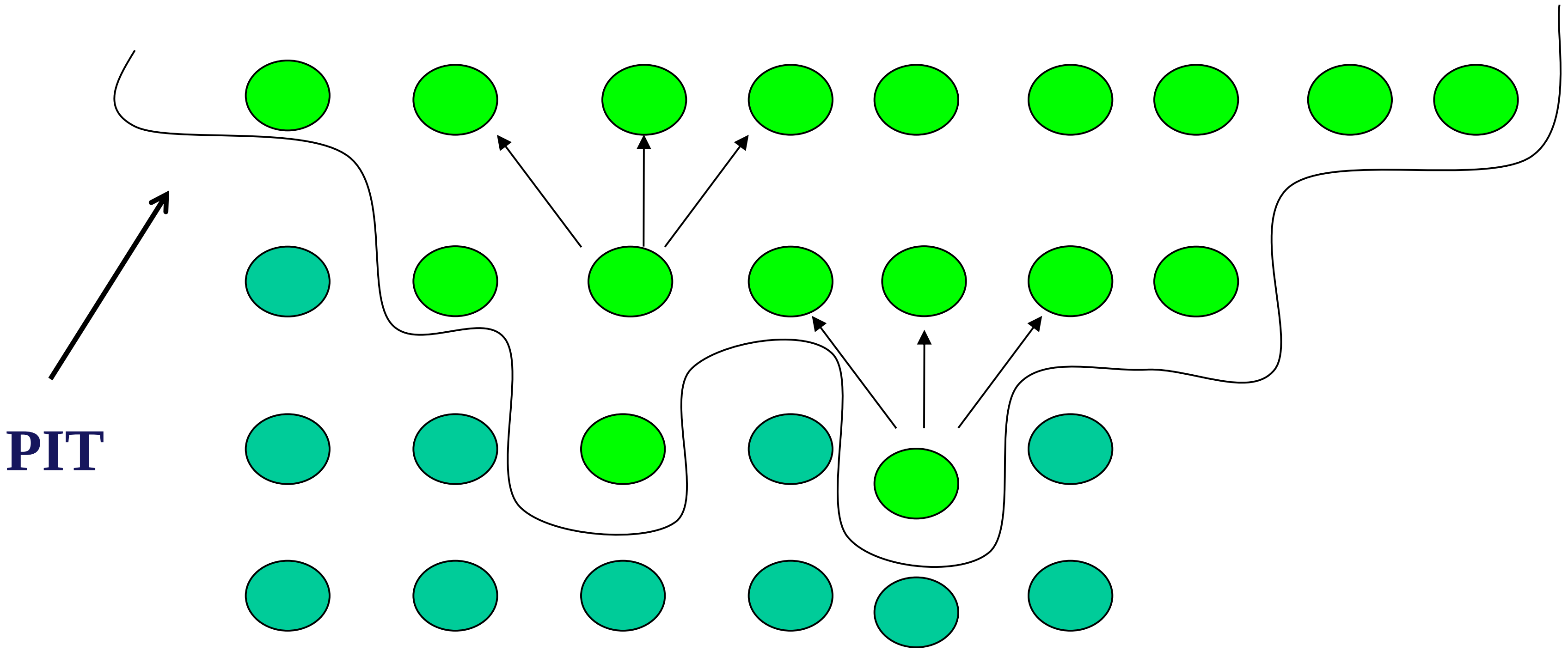
Blocks are represented by **nodes**
and precedence relations are represented by **arcs**



<u>Definition</u>:

PIT is a set of nodes, closed with respect to the precedence arcs.

$$\text{If } \; P_i = \{ j \,/\, (i, j) \in A \}, \;\; \text{then} \quad i \in G' \Rightarrow j \in G' \quad \forall j \in P_i$$



Consider a set $\mathcal{B}$ of blocks. Block j being a predecessor of block i (because of the slope constraints) is denoted as

$$j \prec i$$

A set $P \subset \mathcal{B}$ is said to be a *pit* whenever

$$i \in P \quad \wedge \quad j \prec i \quad \Rightarrow \quad j \in P$$

For each $i \in \mathcal{B}$ an economic benefit/value $v_i \in \mathbb{R}$ and a tonnage (or weight) $w_i > 0$.

The value of subset $S \subset \mathcal{B}$ is denoted as

$$v(S) = \sum_{i \in S} v_i$$

Similarly, the weight of S is

$$w(S) = \sum_{i \in S} w_i$$

Closure

If P and P' are pits, then so are $P \cup P'$ and $P \cap P'$.

Union: if $b \in P \cup P'$ and b' is another block such that $b' \prec b$, then either $b \in P \Rightarrow b' \in P \Rightarrow b' \in P \cup P'$, or similarly if $b \in P'$ then $b' \in P \cup P'$.

A *maximum value pit* is a pit P such that $v(P)$ is maximum over all possible pits.

Ultimate Pit

The *ultimate pit or final pit* is a maximum value pit P^* such that if P is a maximum value pit then $P \subset P^*$.

The existence and unicity of P^* follows from the closure property (union).

Dynamic Problem: Pit Sequences

Nested sequence

A sequence of pits $(P_t)_{t\geq 1}$ is said *nested* if for any $t > 1$ we have that

$$P_{t-1} \subset P_t$$

Phases

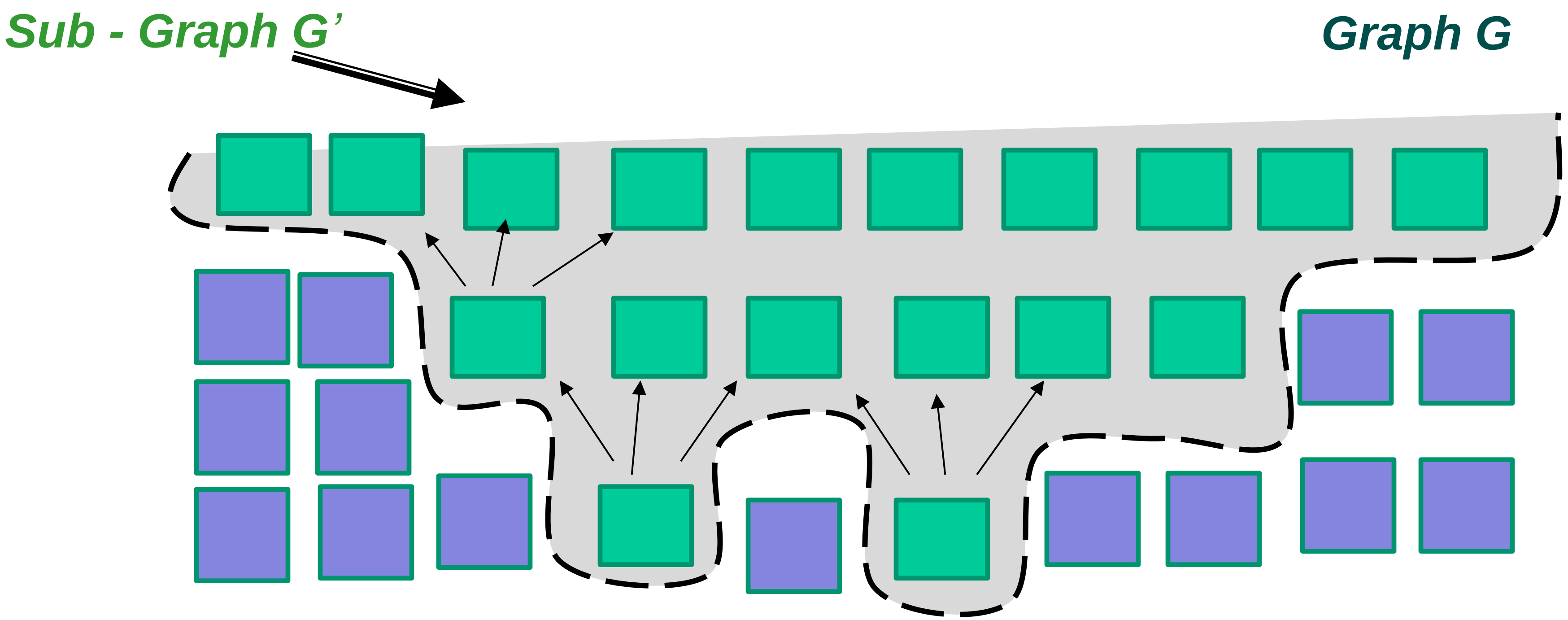
If $\mathcal{S} = (P_t)_{t\geq 1}$ is a nested sequence of pits, we define the associated *phases* as the sequence $\mathcal{F}(\mathcal{S}) = (F_t)_{t\geq 1}$ given by

$$F_t := P_t \setminus P_{t-1}$$

where $P_0 = \emptyset$ so that $F_1 = P_1$.

Remark: $P_t = P_{t-1} \cup F_t$.

FIRST PROBLEM



$$\text{If} \quad P_i = \left\{ j \,/\, (i, j) \in A \right\}, \quad \text{then} \quad i \in G' \Rightarrow j \in G' \quad \forall j \in P_i$$

FINAL PIT ➔ MAXIMAL CLOSURE

To find a sub-graph satisfying the precedence relations and maximal benefece.

First model: The static problem (maximal closure)

$$x_i = \begin{cases} 1 & \text{block } i \text{ belongs to the chosen set} \\ 0 & \text{block } i \text{ doest not belong to the chosen set} \end{cases}$$

(FOP) $$\text{Max} \quad \sum_{i \in N} b_i x_i$$

$$x_j - x_i \geq 0 \qquad (i, j) \in A$$

$$x_i = 0 \text{ or } 1 \qquad i \in N.$$

The total capacity constraint could be added to the model:

The capacitated static problem

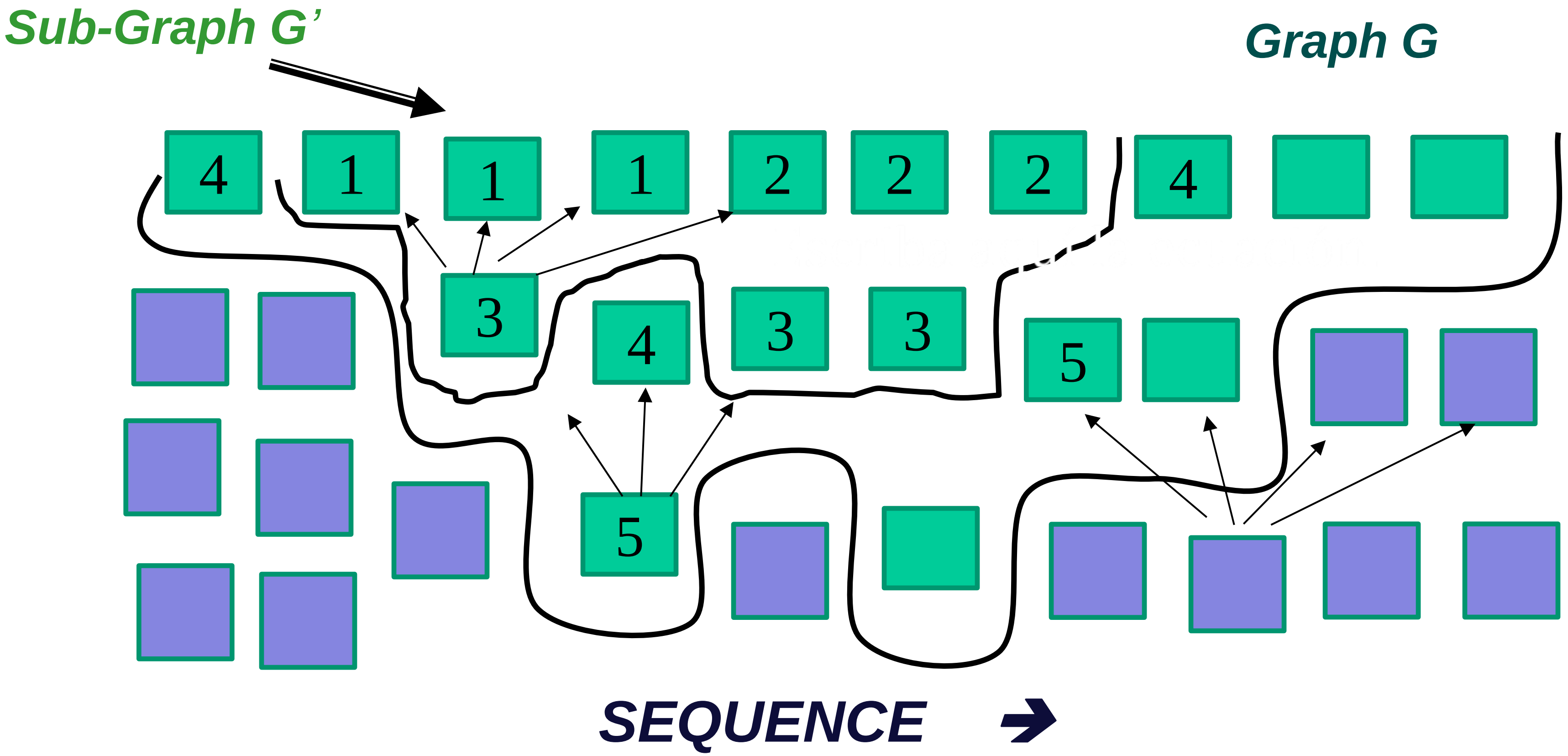
(CFOP) $$\text{Max} \quad \sum_{i \in N} b_i x_i$$

$$x_j - x_i \geq 0 \qquad (i, j) \in A$$

$$\sum_{i \in N} x_i \leq C$$

$$x_i = 0 \text{ or } 1 \qquad i \in N.$$

SECOND (more realistic) PROBLEM



To find a feasible sequence of blocks having maximal discounted value.

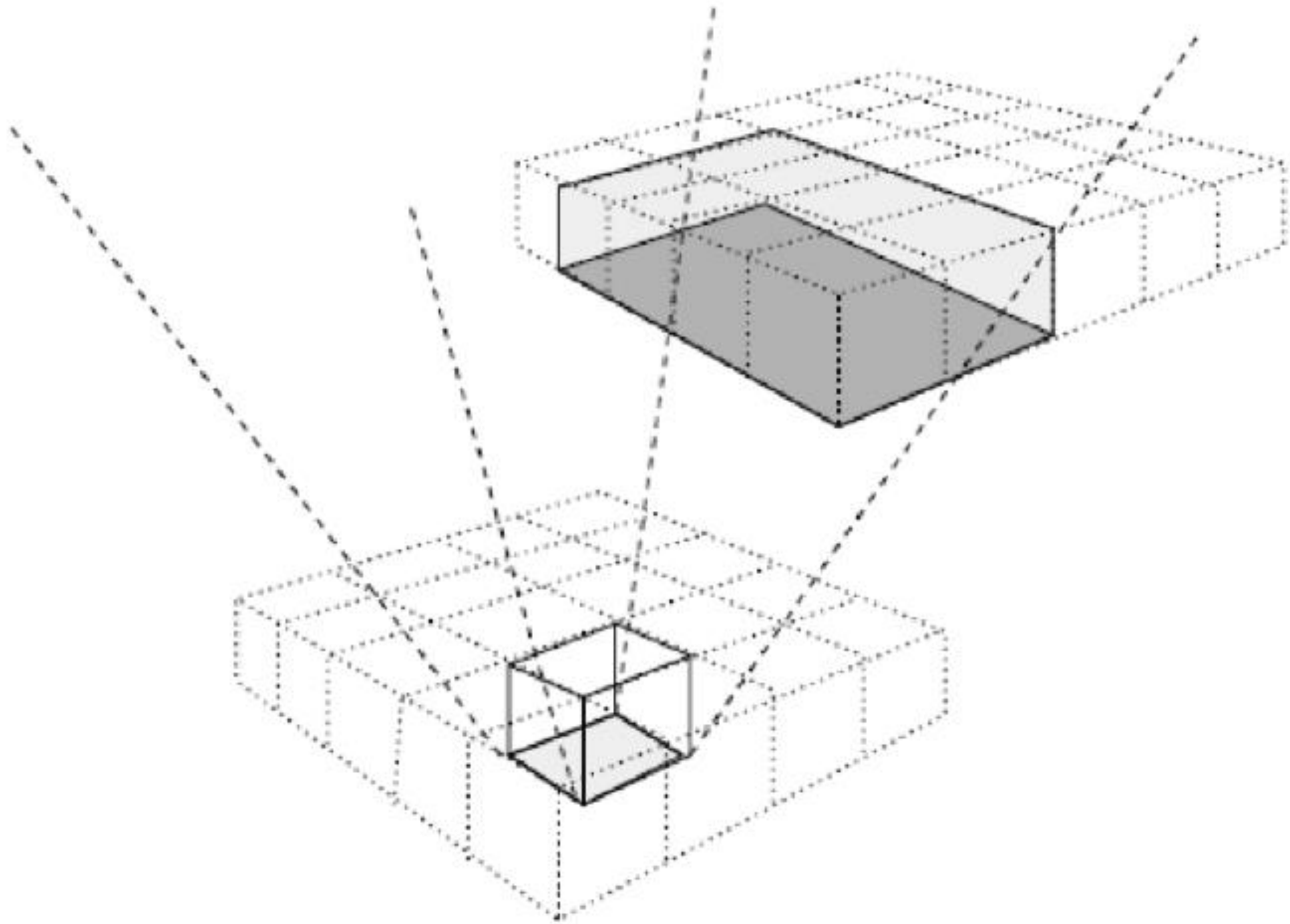


Figure: Precedence relationship between blocks

New model (considering sequence):

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted at time } t \\ 0 & \text{if block } i \text{ is not extracted at time } t \end{cases}$$

The dynamic problem (sequencing problem)

(CDOP)

$$\text{Max} \sum_{t=1}^{T} \sum_{i \in N} \frac{b_i}{(1+\alpha)^{t-1}} x_i^t$$

$$\sum_{t=1}^{T} x_i^t \le 1 \qquad i \in N$$

$$\sum_{l=1}^{t} x_j^l - x_i^t \ge 0 \quad (i,j) \in A,\ t = 1, \cdots, T$$

$$\sum_{\mathrm{i} \in N} p_i x_i^t \le C_t \qquad t = 1, \cdots, T$$

$$x_{\mathrm{i}}^{\mathrm{t}} = 0 \text{ or } 1 \qquad i \in N,\ t = 1, \cdots, T.$$

Generally speaking, three different problems are usually considered by mining engineers for the economic valuation, design and planning of open pit mines.

1. The first one is the **Final Open Pit (FOP)** problem, which aims to find the region of maximal economic value for exploitation under some geotechnical stability constraints.

2. The second one is the **Capacitated Final Open Pit (CFOP)** which considers an additional constraint on the total capacity for the previous formulation.

3. The multi-period version, which we call here the **Capacitated Dynamic Open Pit (CDOP)** problem, with the goal of finding an optimal sequence of extracted volumes in a certain finite time horizon for bounded capacities at each period.

Let S_{FOP}, S_{CFOP} and S_{CDOP} be the sets of blocks contained in the solution of these 3 previous problems, respectively.

S_{FOP} : Final Open Pit (FOP)

S_{CFOP} : Capacitated Final Open Pit (CFOP)

S_{CDOP} : Capacitated Dynamic Open Pit (CDOP)

Property: $S_{CFOP} \subseteq S_{FOP}$

$$S_{CDOP} \subseteq S_{FOP}$$

- Several problems:
  - Final Open Pit: one period and unbounded capacity.
  - Capacitated Final Open Pit: one period but bounded total capacity.
  - Capacitated Dynamic Open Pit: multiperiod scheduling problem.
- Combinatorial nature of the precedence relationship: Mathematical formulation based on block models uses Graph Theory and Integer Programming. Pioneering works: the Lerchs-Grossmann graph theoretic algorithm (1965) and Picard's network flow method (1976).
- Real-world mines produce large scale instances with hundreds of thousands of variables and constraints.
- In practice greedy-type strategies are not optimal. Suboptimal solutions are obtained by means of MLIP techniques (LP relaxations and Branch-and-Bound algorithms), also genetic-type algorithms.

Mathematical model: resolution

a. Linear programming
b. Branch and Bound
c. Heuristics: Greedy, Local search, Relaxation and Expected period of extraction
d. Pre-processing: eliminating variables and redundancies, via final pit or first period of possible extraction

Main possible (and relevant) extensions

a. Robust optimization (large number of instances due to sampling process of random data)
b. Multiple destinations of blocks (very large number of variables and constraints)
c. Multi-mine (several mines, sharing plant facilities, some capacities and demands)
d. Optimal scheduling with reserve

Open Pit Block Scheduling with Exposed Ore Reserve

$$(OPBSEM) \quad max \sum_{i \in B} \sum_{t=1}^{T} [(b_i^t - p_i^t) x_i^t - m_i^t w_i^t] \tag{1}$$

$$\sum_{t=1}^{T} (x_i^t + w_i^t) \leq 1 \qquad \forall\, i \in B \tag{2}$$

$$\sum_{i \in B} \tau_i (x_i^t + w_i^t) \leq M^t \qquad \forall\, t \in \{1, \dots, T\} \tag{3}$$

$$\sum_{i \in B} \tau_i x_i^t \leq P^t \qquad \forall\, t \in \{1, \dots, T\} \tag{4}$$

$$y_j^t + \sum_{s=1}^{t} \left(x_j^s + w_j^s\right) \leq \sum_{s=1}^{t} (x_i^s + w_i^s) \qquad \forall\, (i, j) \in A, \;\; t \in \{1, \dots, T\} \tag{5}$$

$$y_i^t \leq x_i^{t+1} \qquad \forall\, i \in B, \;\; t \in \{1, \dots, T-1\} \tag{6}$$

$$\sum_{i \in B} \tau_i \lambda_i (\Lambda_{cg})\, y_i^t \geq F^t \qquad \forall\, t \in \{1, \dots, T-1\} \tag{7}$$

$$x_i^t, w_i^t, y_i^t \in \{0,1\} \qquad \forall\, i \in B, \;\; t \in \{1, \dots, T\} \tag{8}$$

Three types of variables are used in the model, all of them are binary. The first type is the variable associated to the extraction for processing purposes for each block

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and processed at time } t \\ 0 & \text{otherwise} \end{cases}$$

The second variable type describes the decision relating to the disposal of a block by sending it to the waste dump

$$w_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and sent to waste dump at time } t \\ 0 & \text{otherwise} \end{cases}$$

The third variable type is used to identify exposed blocks; throughout the paper it will indistinctively be called "visibility" or "exposure" variable

$$y_i^t = \begin{cases} 1 & \text{if block } i \text{ is exposed at time } t \\ 0 & \text{otherwise} \end{cases}$$

Generating an initial feasible solution

1. Stage:

Find the final pit (LP problem), and then delete the blocks not included in it.

From this stage, we work with the residual graph.

2. Pre-processing:

For each block *i*, define the first period in which the block could be mined. Let t_i that period. Then we can fix:

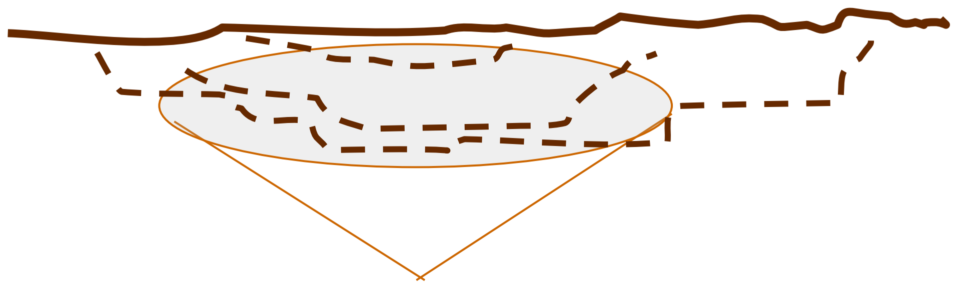
$$x_i^s = 0 \quad \text{for all} \quad s < t_i$$

To calculate t_i we define $\mathcal{N}_i^* = \{j \in \mathcal{N}_{FP} \; / \text{ there is a path from } i \text{ to } j\}$. This represent the cone over block i. Then t_i is given by:

$$t_i = \min\{t \; / \sum_{j \in \mathcal{N}_i^*} p_j \leq \sum_{k=0}^{t} C_k\}$$

3. Pre-processing: Redefine benefits:

$$b^*{}_i = \rho^{t_i} b_i$$



Other definitions of b_i^* could be envisaged.
Example: The total benefit contained in the cone-above.

4. Apply Greedy algorithm with these new benefits.

(**Ferland et al, in:** ***Studies in Computational Intelligence*****, Springer Verlag, 2007)**

A scalable approach to optimal block scheduling

J. Amaya, D. Espinoza, M. Goycoolea, E. Moreno, Th. Prévost, E. Rubio

Proceedings APCOM2009, *Applications of Computers on Mining Industry*, Vancouver, Canada, 2009.

A critical aspect of long-term open-pit mine planning consists in computing a production schedule based upon a block sequencing strategy.

Here we describe a scalable IP-based methodology for solving very large (millions of blocks) instances of this problem.

We show that embedding standard IP technologies in a local-search based algorithm we are able to obtain near-optimal solutions to large problems in reasonable time. This methodology has been tested in several mine wide block models.

Notation: $$x_i^t = \begin{cases} 1 & \text{if block } i \text{ has been extracted by time } t \\ 0 & \text{otherwise.} \end{cases}$$

Resource-Constraint Pit Optimization Problem (RC-PIT)

$$\max \sum_{t=1}^{T} \sum_{i \in N} b_i^t (x_i^t - x_i^{t-1})$$

$$x_i^t \le x_i^{t+1} \quad \text{consistency of the variable definition}$$

$$x_i^t \le x_j^t \quad \text{wall - slope (or precedence) condition}$$

$$\sum_{i \in N} q_r^i (x_i^t - x_i^{t-1}) \le c_r^t \quad \text{resource } r \text{ available at time } t$$

Improving feasible solutions: a local search heuristic

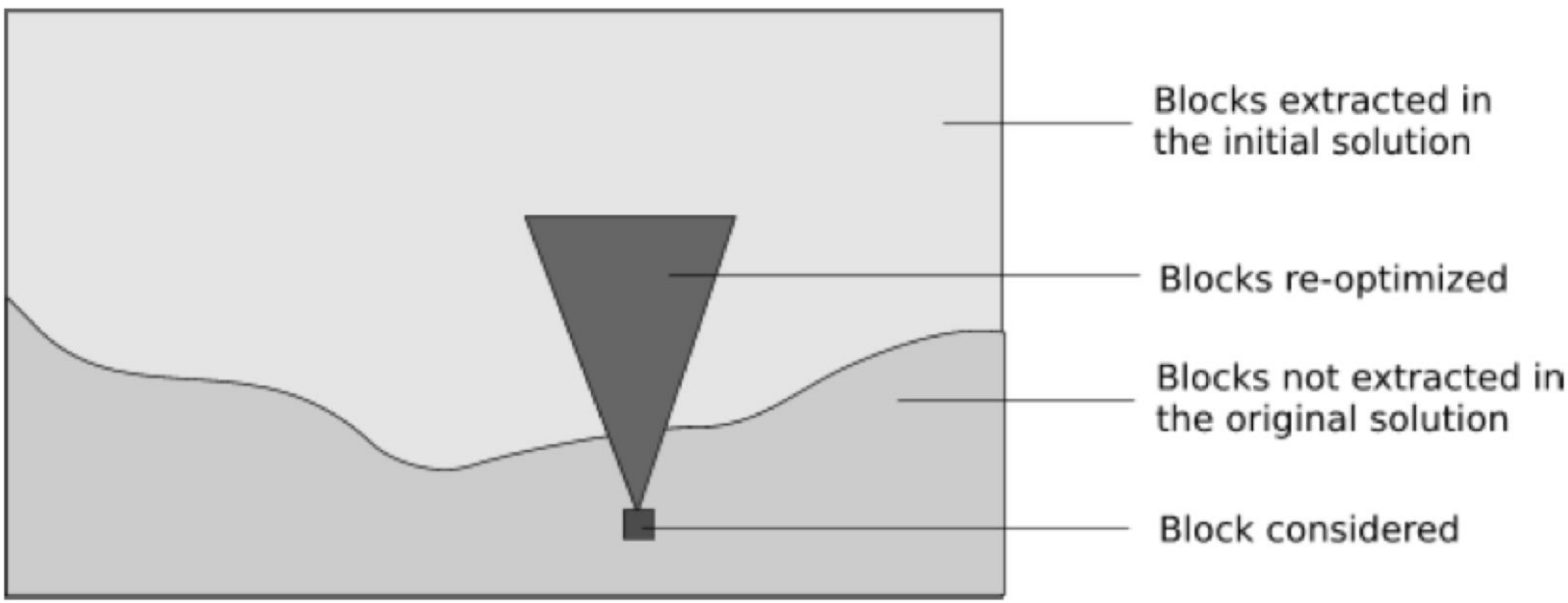
Given a current feasible solution (the first one could be proposed by the Greedy Algorithm) we define a neighborhood of a given block and then we re-optimize over this subset.

This can be accomplished using the formulation described before and adding additional constraints to ensure that blocks outside the chosen neighborhood remain at their original values.

How to choose the neighborhood for pertubation?

Cone-above strategy: Consider a block *i*, and define *P(i)* as the cone of all blocks which are predecessors of *i*.

In order to find a local improvement to a solution, we randomly select a block *i* and find the best solution in the *P(i)*-neighborhood of as indicated above.



Periods strategy: Consider time periods *t* and *t'*, and a solution vector.

In order to find a local improvement of solution we randomly select a pair of time periods (the distance between them not too large) and re-optimize to find the best solution in that neighborhood.

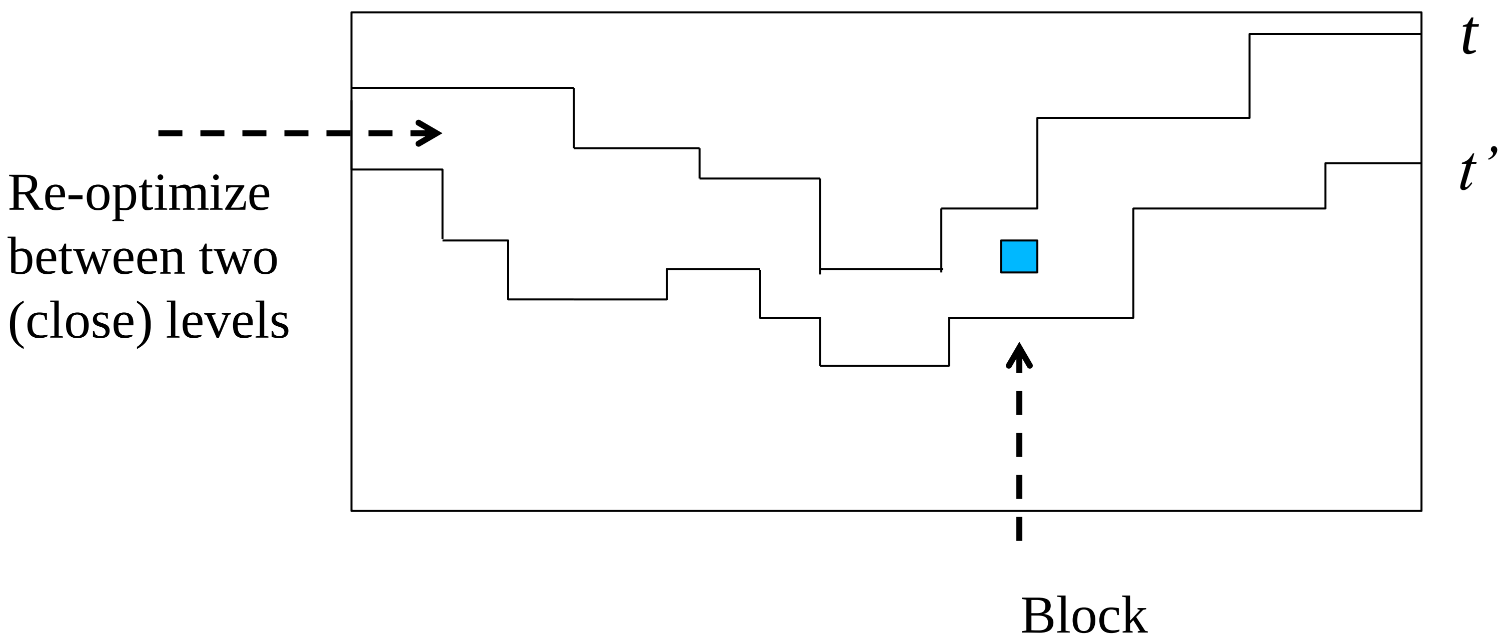


Table 1. Description of the ore bodies used for the study.

Name	# Blocks	Grade range	Observations
Marvin	61x60x17	0.03-1.46 %Cu 0.1-1.2 ppm Au	fictitious copper gold ore body included in the Whittle 4X mine planning software
AmericaMine	61x42x60	% Cu : 0.08-3.68	hard rock polymetallic mine
AsiaMine	112x230x38	0-1.91 % Cu	Polymetallic ore body with a pipe shape
Andina	184x269x121	0.02-3.64 % Cu 0-0.42 % Mo	Copper molybdenum ore body taken from Andina Sur Sur deposit located at 50 Km north of Santiago. Typical porphyry copper ore body

Table 2. Description of the test set instances used for the study.

	N.Blocks	Real Blocks	P.P. Blocks	N. Periods
Marvin	61x60x17	53668	8553	13
AmericaMine	61x42x60	19320	6445	18
AsiaMine	112x230x38	772800	97900	15
Andina	184x269x121	4320480	3340898	15

Table 3. Summary of Local Search performance after running 4 hours.

	Gershon	Local Search (4 hrs)	LP relaxation	LP time
Marvin	1.0	1.08	1.09	26 min
AmericaMine	1.0	1.15	1.15	19 min
AsiaMine	1.0	1.23	1.24	4h 13 min
Andina	1.0	1.15	Unknown	Unknown

II. A CONTINUOUS APPROACH

(Alvarez, Amaya, Griewank, Strogies, in *Math. Methods for Op. Res.*, 2011)

We propose here a continuous approach which allows for a refined imposition of slope constraints associated with geotechnical stability.

The model introduced here is posed in a suitable functional space, essentially the real-valued functions that are Lipschitz continuous on a given two dimensional bounded region.

We derive existence results and investigate some qualitative properties of the solutions.

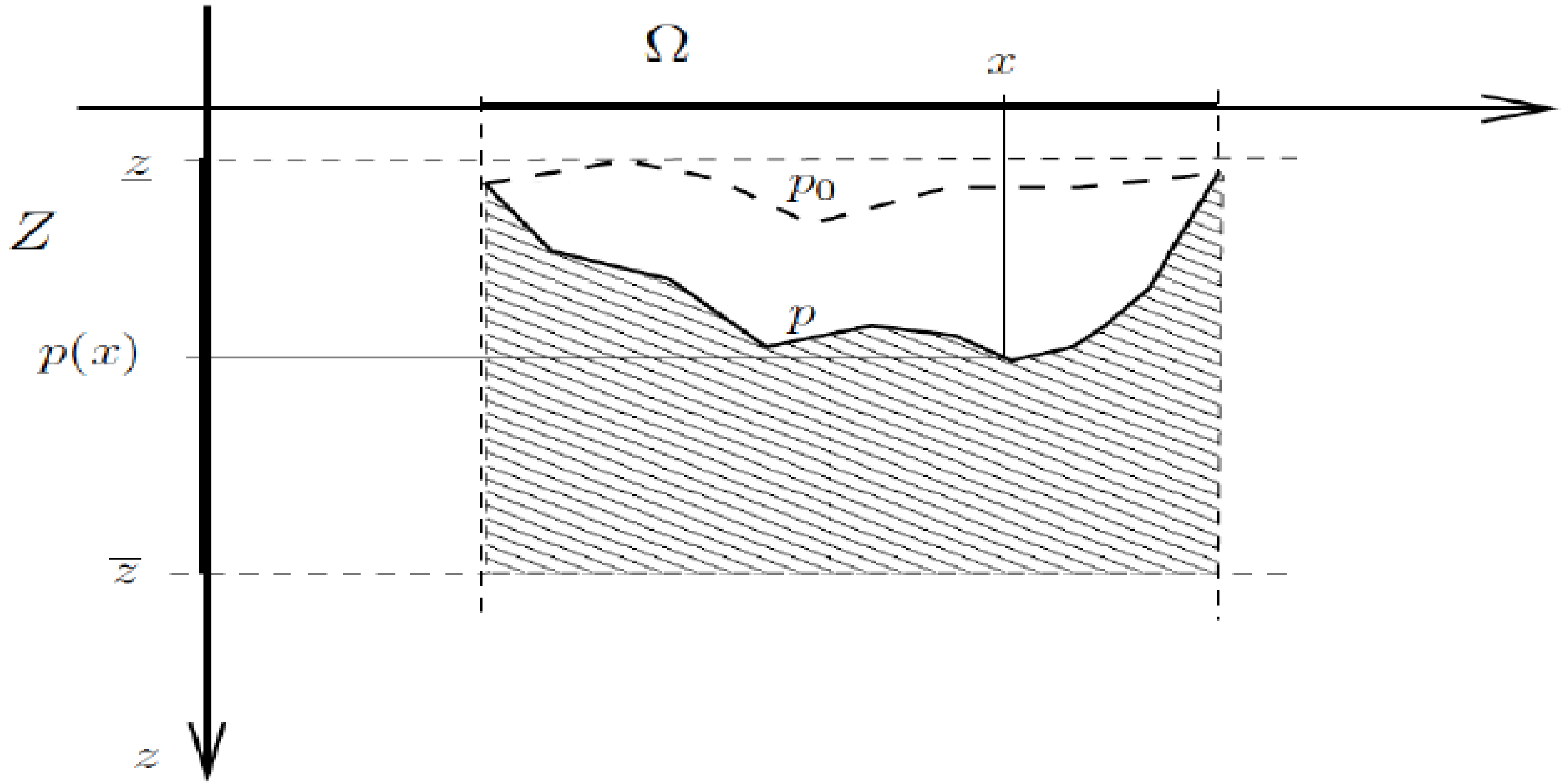


Figure: Sketch of a vertical section for a feasible profile

The bi-dimensional domain Ω is supposed to be bounded.

The profil p belongs to $C(\overline{\Omega})$, the Banach space of continuous real valued functions, equipped with the supremum norm.

Definition of Stable Profile Functions

- Given a bounded, regular and connected domain $\Omega \subset \mathbb{R}^2$.
- Stable Profile: continuous function $p : \overline{\Omega} \to \mathbb{R}$ fulfilling
  - Nonegativity condition:
    $$p(x) - p_0(x) \geq 0 \text{ for } x \in \overline{\Omega}$$
  - Dirichlet boundary condition:
    $$p(x) - p_0(x) = 0 \text{ for } x \in \partial\Omega$$
  - Boundedness:
    $$p(x) \in Z = [\underline{z}, \overline{z}]$$
  - Local stability condition:
    $$\Lambda_p(x) \equiv \limsup_{\tilde{x} \to x \leftarrow \hat{x}} \frac{|p(\tilde{x}) - p(\hat{x})|}{\|\tilde{x} - \hat{x}\|} \leq \omega(x, p(x))$$

    with $\omega : \overline{\Omega} \times Z \to \mathbb{R}_+$ is a given bounded function.

Feasible set

Definition

$\mathcal{P} \equiv \{p \in C(\overline{\Omega}) \mid p \text{ is a stable profile}\}$ is called the set of feasible profiles.

Remark: If $p \in \mathcal{P}$ then $\|\nabla p(x)\| \leq \omega(x, p(x))$ for a.e. $x \in \Omega$.

Lemma 1

If $\omega(x, z)$ is concave w.r.t. z, then $\mathcal{P}$ is convex.

Proposition 1

If ω is u.s.c. then $\mathcal{P}$ is compact in $(C(\overline{\Omega}), \|\cdot\|_\infty)$.

Sketch of Proof

Obviously $\mathcal{P}$ is bounded and equicontinuous. Closedness of $\mathcal{P}$ can be shown by contradiction. For this, the u.s.c. of ω is necessary.

Stationary Effort and Objective Functions

Gain Function:

$$G([p_1,p_2]) = \int_{\Omega} \int_{p_1(x)}^{p_2(x)} g(x,z)\,dxdz$$

with $g \in L^\infty(\overline{\Omega} \times Z)$

Effort Function:

$$E([p_1,p_2]) = \int_{\Omega} \int_{p_1(x)}^{p_2(x)} e(x,z)\,dxdz$$

with $e \in L^\infty$ and $e(x,z) \geq e_0 > 0$

Final Open Pit Problem - FOP

$$\max\{G(p) := G([p_0,p]) \mid p \in \mathcal{P}\}$$

Capacitated Final Open Pit Problem - CFOP

$$\max\{G(p) \mid p \in \mathcal{P}, E(p) := E([p_0,p]) \leq \overline{E}\}$$

Final Open Pit (FOP)

Discrete case

$$\text{Max} \quad \sum_{i \in N} b_i x_i$$
$$x_j - x_i \ge 0 \qquad (i, j) \in A$$
$$x_i = 0 \text{ or } 1 \qquad i \in N.$$

Continuous case

$$\text{Max} \int_{\Omega} \int_{p_0(x)}^{p(x)} g(x,z)\,dz\,dx$$

Nonegativity condition:

$$p(x) - p_0(x) \ge 0 \text{ for } x \in \overline{\Omega}$$

Dirichlet boundary condition:

$$p(x) - p_0(x) = 0 \text{ for } x \in \partial\Omega$$

Boundedness:

$$p(x) \in Z = [\underline{z}, \overline{z}]$$

Local stability condition:

$$\Lambda_p(x) \equiv \limsup_{\tilde{x} \to x \leftarrow \hat{x}} \frac{|p(\tilde{x}) - p(\hat{x})|}{\|\tilde{x} - \hat{x}\|} \le \omega(x, p(x))$$

with $\omega : \overline{\Omega} \times Z \to \mathbb{R}_+$ is a given bounded function.

Capacitated Final Open Pit (CFOP)

Discrete case: We add the condition

$$\sum_{i \in N} x_i \leq C$$

Continuous case: We add the condition

$$E(p) := \int_{\Omega} \int_{p_0}^{p(x)} e(x,z)\,dz\,dx \leq \bar{E}$$

e is an "effort function", lower bounded by a positive constant.

Continuity, differentiability and convexity

Lemma 2

- G is Lipschitz continuous on $C(\overline{\Omega})$ with constant $\|g\|_\infty |\Omega|$.
- If g is continuous on $\overline{\Omega} \times Z$ then $G(p)$ is everywhere Fréchet differentiable. In particular, for any $\phi \in C(\overline{\Omega})$ we have that

$$dE(p)\,\phi = \int_\Omega e(x, p(x))\phi(x)dx.$$

Similar for E.

Lemma 3

- E is convex if e is monotonically increasing w.r.t. z.
- Respectively G is concave if g is monotonically decreasing w.r.t. z.

Existence Results for Stationary Problems

Theorem 1 (Existence)

There exist solutions for FOP and CFOP.

Remark: Under realistic conditions, problems FOP and CFOP may be nonconvex and nondifferentiable.

Dynamic continuous model

Excavation Paths

- Given the time horizon $T > 0$ for the mine.
- Feasible Excavation Path: continuous function $P : [0, T] \to \mathcal{P}$, so that $p(t, \cdot) := P(t)(\cdot)$ is a feasible profile, fulfilling
  - Monotonicity: $P(t) \geq P(s)$ for $0 \leq s \leq t \leq T$
  - Dynamic capacity constraint:

$$E([P(s), P(t)]) \leq \int_s^t c(\tau)dt$$

for $0 \leq s \leq t \leq T$ and some capacity speed $c \in L^\infty([0, T])$

Definition

$\mathcal{U} = \{P \in C([0, T], \mathcal{P}) |\ P$ is feasible and $P(t) \geq p_0$ for $t \in [0, T]\}$ is called set of feasible excavation paths.

Objective Function and Problem Formulation

Discount Function: $\varphi \in C^1([0,T])$, φ monotonically decreasing, $\varphi(0) = 1$, $\varphi(T) < 1$

Typical choice: $\varphi(t) = e^{-\delta t}$

Dynamic Objective Function :

$$\begin{aligned}\hat{G}(P) &= \int_0^T \varphi(t) \int_\Omega g(x, P(t)(x)) dx dP(t) \\ &= \varphi(T) \int_\Omega \int_{P(0)(x)}^{P(T)(x)} g(x,z) dz dx + \int_\Omega \int_0^T \int_{P(0)(x)}^{P(t)(x)} [-\varphi'(t)] g(x,z) dz dt dx\end{aligned}$$

Capacitated Dynamic Open Pit Problem - CDOP

$$\max\{\hat{G}(P) \mid P \in \mathcal{U}, P(0) = p_0\}$$

The Capacitated Dynamic Open Pit Problem (CDOP)

$$\text{Max} \quad \varphi(T)\int_{\Omega}\int_{P(0)(x)}^{P(T)(x)} g(x,z)\,dz\,dx + \int_{\Omega}\int_{0}^{T}\int_{P(0)(x)}^{P(t)(x)} [-\varphi'(t)]g(x,z)\,dz\,dt\,dx$$

$$P(0) = p_0$$

$$\{P(t)\}\ \textit{is monotone in } t\ (\textit{embeded profiles})$$

$$P(t)\ \textit{is feasible for the static problem,} \qquad \textit{for all } t \in [0,T]:$$

Nonegativity condition:

$$p(x) - p_0(x) \geq 0 \text{ for } x \in \overline{\Omega}$$

Dirichlet boundary condition:

$$p(x) - p_0(x) = 0 \text{ for } x \in \partial\Omega$$

Boundedness:

$$p(x) \in Z = [\underline{z}, \overline{z}]$$

Local stability condition:

$$\Lambda_p(x) \equiv \limsup_{\tilde{x}\to x\leftarrow\hat{x}} \frac{|p(\tilde{x}) - p(\hat{x})|}{\|\tilde{x} - \hat{x}\|} \leq \omega(x, p(x))$$

with $\omega : \overline{\Omega} \times Z \to \mathbb{R}_+$ is a given bounded function.

Structural Properties

Lemma 5

If $\omega(x,z)$ is concave and $e(x,z)$ is constant w.r.t. z, then $\mathcal{U}$ is convex.

Lemma 6

$\hat{G}$ is Lipschitz continuous on $C([0,T] \times \overline{\Omega})$ with constant

$$2\|g\|_\infty |\Omega|.$$

Lemma 7

$\hat{G}$ is concave on the feasible set of CDOP if g is monotonically decreasing w.r.t. z.

Existence Results for the Dynamic Problem

Proposition 4

If ω is u.s.c. then the set $\mathcal{U}$ is compact in $(C([0,T]\times\overline{\Omega}), \|\cdot\|_\infty)$.

Sketch of Proof

- Show
$$\|P(t)-P(s)\|_\infty \leq \left(\frac{\|c\|_\infty}{e_0\pi} + 2\overline{\Omega}\right)(t-s)^{1/3}$$
- Obviously $\mathcal{U}$ bounded and closed.
- AA: $\mathcal{U}$ is compact

Theorem 2

There exists a solution for CDOP.

Main possible and relevant extensions:

- **Limiting connections between the discrete and continuous models**
- **Duality for the open pit continuous model**
- **Numerical resolution by discretization or reduction to a finite-dimensional problem**
- **Properties of the final profile (the value along this profile is equal to 0, a.e.)**

- **Robust optimization (large number of instances due to sampling process of random data)**
- **Multiple destinations of blocks (very large number of variables and constraints)**
- **Multi-mine (several mines, sharing: plant facilities, capacities and demands)**
- **Optimal scheduling with reserve** (at the end of each period, the available blocks provide a certain amount of ore for the next period)

THANK YOU