ESTIMATION OF GAS CONTENT AND DIFFUSION PARAMETERS OF COAL BED BASED ON INVERSE PROBLEM SOLUTION BY “CANISTER TEST” DATA

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CONTENTS

Publications

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2. C.Bertard, B.Bruyet, J.Gunther. Determination of desorbable gas concentration of coal (Direct Method). Int. J. of Rock Mech. & Mining Science, 1970, 7(1), pp. 43-65.

3. D.M.Smith. Methane diffusion and desorption in coal. PhD Thesis, University of New Mexico, 1982, 234 p.

4. W.P.Diamond, S.J.Schatzel. Measuring the gas content of coal: A review. Int. J. of Coal Geology, 1998, V. 35, Issues 1-4, pp. 311-331.

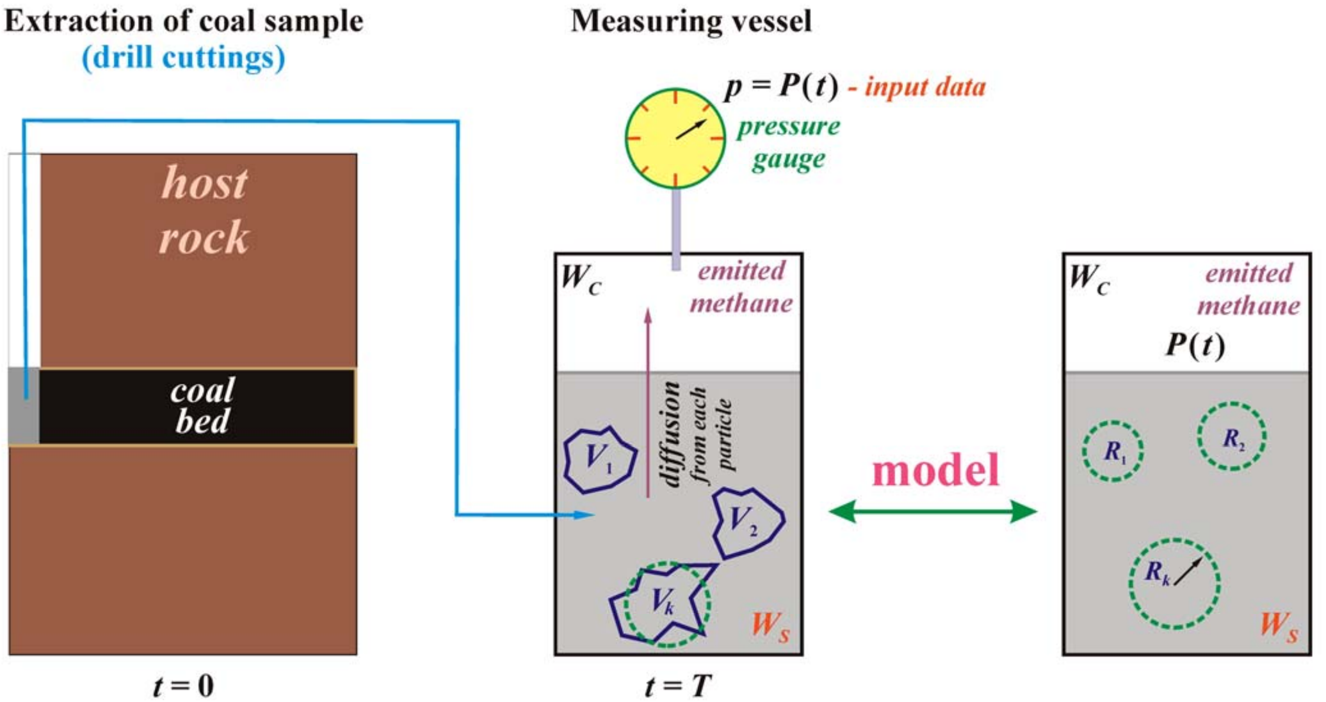
5. Australian Standard TM AS 3980-1999. Guide to the determination of gas content of coal. Direct desorption method. Standards Association of Australia, 1999 , 36 p.

Resume.

All approaches:

- do not take into account grain-size composition of coal sample;
- can not “extract” total information contained in experimental data because
- use simple models (permitting analytical solution) for processing.

Scheme of " *canister test* " experiment



W_S - *volume of coal sample*

W_C - *volume of measuring vessel*

grain-size composition of sample

$$F_i = \{R_k, s_k\}_i$$

$$s_1 + s_2 + \ldots + s_K = 1$$

particle of coal sample fraction R_k

equivalent ball fraction R_k

$$R_k = \sqrt[3]{3V_k / 4\pi}$$

number of particle (fraction R_k*) in sample of volume* W_S

$$N_k = \frac{3 s_k W_S}{4\pi R_k^3}$$

It is the scheme of experiment for determination of gas content in coal bed – "canister test".

We extract sample from coal bed, place the coal in measuring vessel, seal the vessel and record variation of pressure – input data.

Such approach is known but existing methods of input data interpretation "use" very simple models so we lose information about coal substance.

System of equation, initial and boundary conditions

Mass conservation equation

$$\frac{\partial C}{\partial t} + \nabla \cdot \vec{q} = 0$$

Fick's law

$$\vec{q} = -D \nabla C$$

Initial conditions
for each particle

$$C(R,0) = S$$

Boundary conditions
for each particle
of equivalent radius R_k

$$at \quad t < T_0$$
$$C(R_k,t) = S(1 - t/T_0)$$

$$at \quad t \geq T_0$$
$$D\frac{\partial C(R_k,t)}{\partial R} = \begin{cases} \beta[B(t) - C(R_k,t)] & if \;\; C(R_k,t) > B(t) \\ 0 & if \;\; C(R_k,t) \leq B(t) \end{cases}$$

NOMENCLATURE

- $\vec{q}$ - flux
- D - diffusion coefficient
- β - mass transfer coefficient
- C - concentration of free gas in particle
- S - initial gas content in coal bed
- B - concentration of gas in sealed vessel
- P_0 - atmospheric pressure
- T_0 - time moment of vessel sealing

This is statement of direct boundary problem:
evolution of free C gas content in set of spherical coal particles
extracted from coal bed at time moment $t = 0$
and placed in the vessel at time moment $t = T_0$.
It has to be remarked that *boundary condition* is *non-linear*.

Calculation of pressure *P*(*t*) in measuring vessel

Mass of gas emitted by each particle of R_k size

$$M_k(t) = \int_{T_0}^{t} \iint_{L_k} q_R(R_k, t)\, dL\, dt$$

L_k - particle surface

$$q_R = -D \frac{\partial C}{\partial R}$$

Boyle -Mariotte law

$$P(t)W = P_0\left(W + M(t)/\rho_0\right)$$

ρ_0 - gas density at atmospheric pressure

Total mass of gas in measuring vessel

$$M(t) = \sum_{k=1}^{K} N_k M_k(t)$$

Gas concentration in measuring vessel

$$B(t) = M(t)/W$$

$W = W_C - W_S$ - volume of measuring vessel free part

$$B(t) = \frac{3W_S}{W_C - W_S} D \sum_{k=1}^{K} \frac{s_k}{R_k} \int_{T}^{t} \frac{\partial C(R_k, t)}{\partial R} dt$$

$$B(t) = S\overline{B}(t)$$

Gas concentration *C* in each particle as well as in vessel are in proportion to initial gas content *S*

Pressure in measuring vessel

$$P(t)/P_0 = 1 + S\overline{B}(t)/\rho_0$$

This slide demonstrate method of pressure *P*(*t*) evaluation in sealed vessel – just that information which is recorded in experiment - input data.

Parametric analysis of forward problem

Model parameters

Free parameters

(have to be determined)

D - diffusion coefficient

β - mass transfer coefficient

S - initial gas content in coal bed

Governing parameters

T_0 - time moment of vessel sealing

W_S - volume of sample

Aposteriory determined

$\{R_k, s_k\}$ - grain-size composition of sample

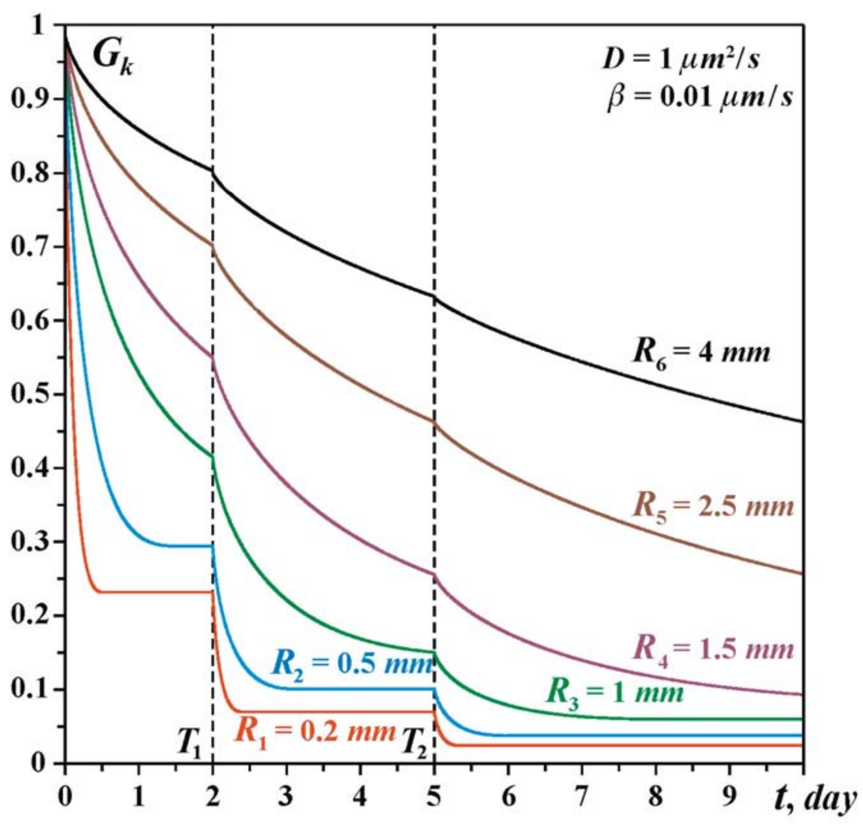
Grain-size composition of sample for the theoretical analysis

number of fraction, k	1	2	3	4	5	6
radius of particle, R_k, mm	0.2	0.5	1.0	1.5	2.5	4.0
relative content, s_k	0.25	0.15	0.15	0.25	0.10	0.10

s_k - relative content of *k*-th fraction in sample

We selected realistic grain-size composition of sample and solved series of direct problems varying model parameters. The next two slides demonstrate the examples of such consideration.

Reduction of gas content in time in various fractions



Relative gas content in particle of *k*-th fraction

$$G_k(t) = \frac{4\pi \int_0^{R_k} C(R,t) R^2 dR}{SV_k}$$

Volume of particle of *k*-th fraction

$$V_k = 4\pi R_k^3 / 3$$

T_1, T_2

time moments of pressure drop

We proposed specific mode of experiment "pressure drop":

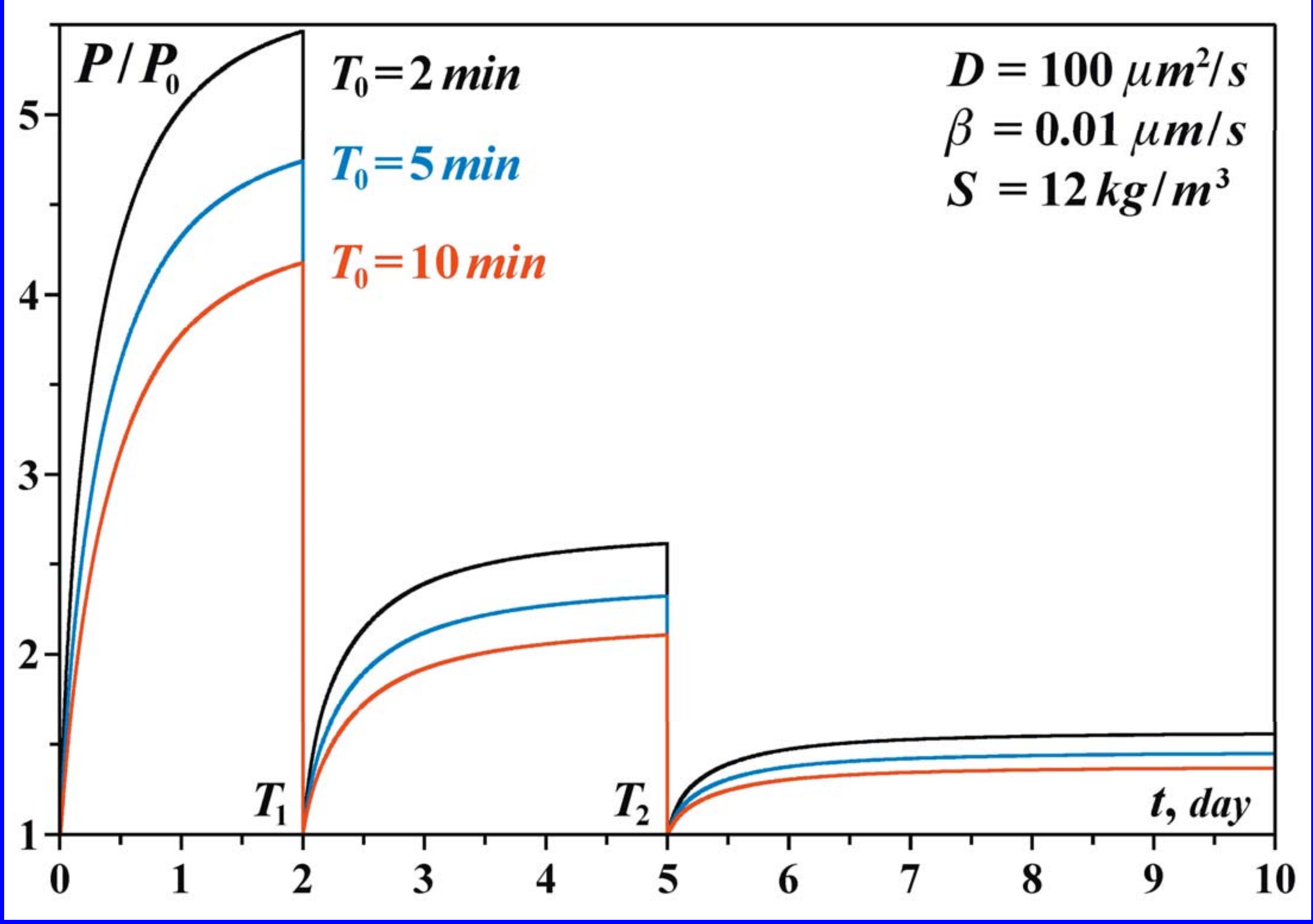
1) at different time moments the vessel is opened and pressure fall down to the value of the atmospheric pressure.

2) then the container was sealed again.

Numerical simulation has shown that small fractions "lose" majority of gas at the first stage of the experiment: at $T_0 < t < T_1$.

Later on the pressure in vessel grows due to the gas emission from the large fractions.

Variation of pressure in the vessel at different T_0



Numerical experiments revealed that the greater time T_0 (placing sample in the vessel) the lesser pressure *P*.
It can be explained by gas loss in the time interval [0, T_0] between extraction of the sample and placing it in the vessel.

THEORETICAL INVESTIGATION OF INVERSE PROBLEM RESOLVABILITY BASED ON SYNTHETIC INPUT DATA

Inverse problem

to find:

S - gas content

D - diffusion coefficient

β - mass transfer coefficient

by input data $P_*(t)$

for known grain-size composition of coal sample

Input data $P_*(t)$ synthesis

$$P_*(t) = [1 + \gamma\, \psi(t)] P(t, S_*, D_*, \beta_*)$$

$P(t, S_*, D_*, \beta_*)$ - exact solution of direct problem

at $S = S_*$ $D = D_*$ $\beta = \beta_*$

$\psi(t) \in [-1, 1]$ - random function

γ - noise level

We investigate resolvability of inverse problem using synthetic data:
impose multiplicative noise (relative level γ) on exact solution.

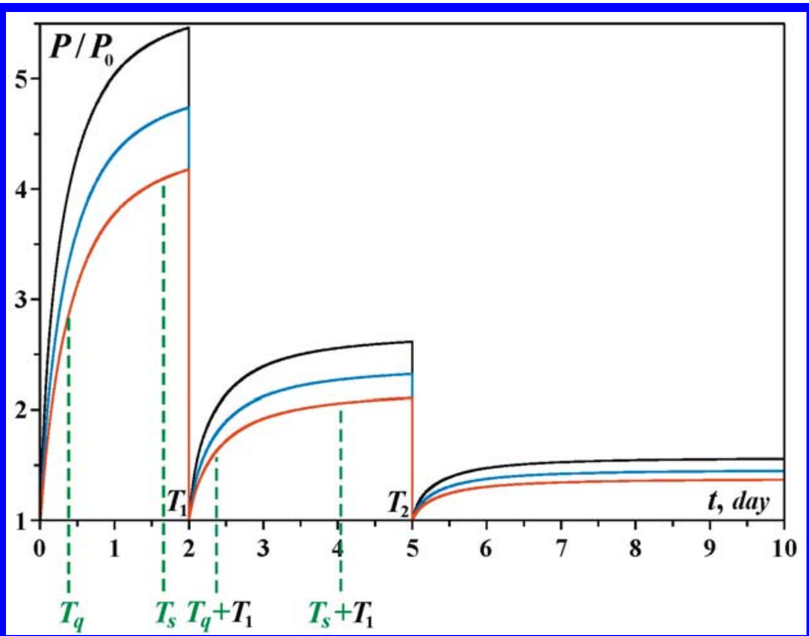
Inverse problem: resolvability analysis

Consider the function

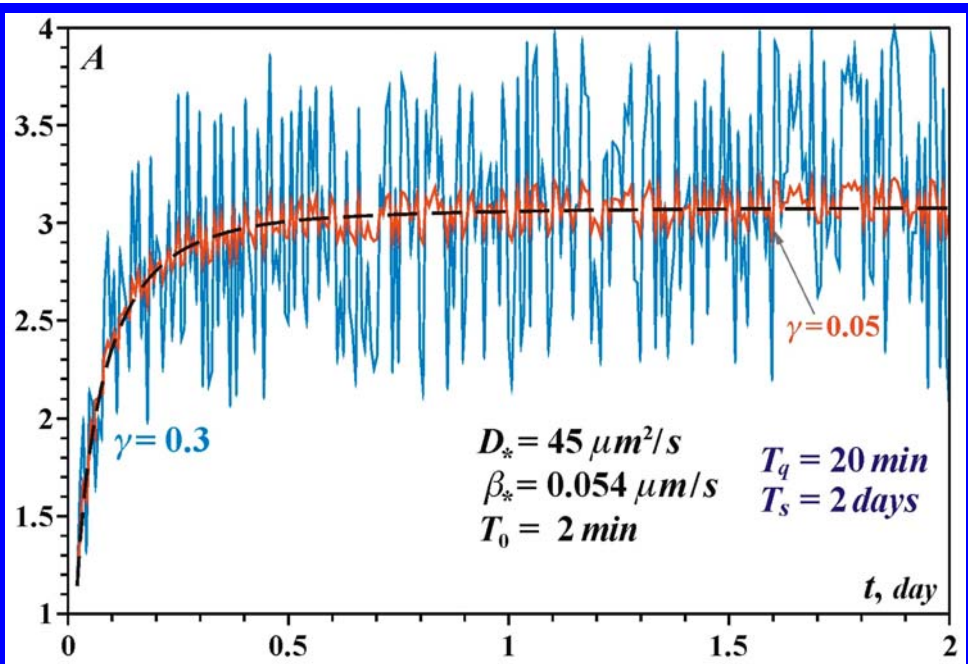
$$A(t,D,\beta)=\frac{P(t,S,D,\beta)-P_0}{P(t+T_1,S,D,\beta)-P_0}$$

$$T_q < t < T_s = \min\,(T_1\,,\,T_2 - T_1)$$

Choice of time limits for data interpretation: illustration



Function $A_*(t)$ calculated by synthesized input data $P_*(t)$



$$A_*(t)=\frac{P_*(t)-P_0}{P_*(t+T_1)-P_0}$$

Function A doesn't depend on gas content S in sample. So we may temporarily eliminate S from analysis.

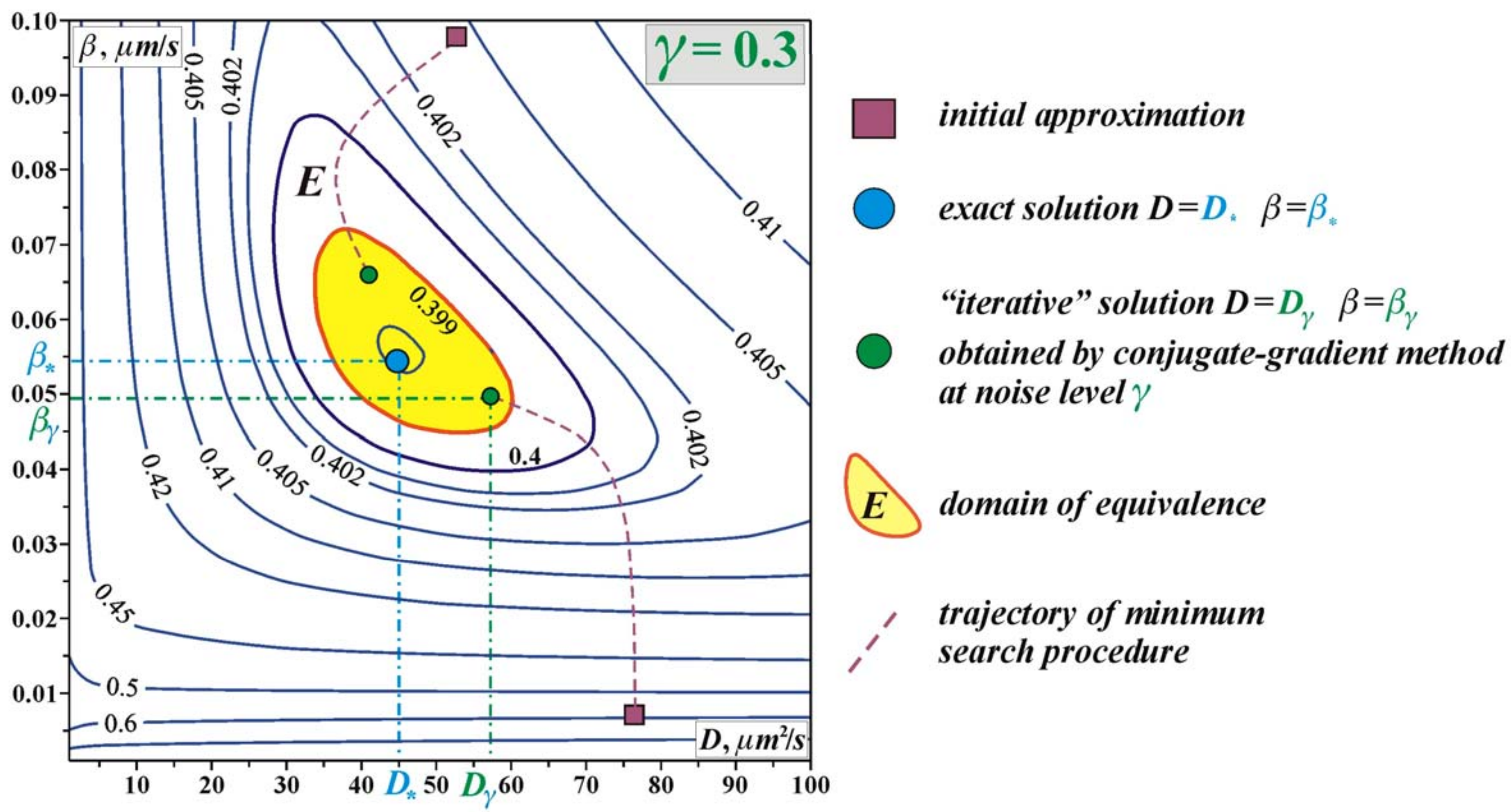
Upper picture illustrates time moments that were used for data interpretation.

Lower picture demonstrate function $A_*(t)$ calculated by synthesized input data at different noise $\gamma = 0.05$ and 0.3 .

First objective function

$$\Phi_1(D,\beta) = \int_{T_q}^{T_s} [A(t,D,\beta) - A_*(t)]^2\, dt$$

Level lines of objective function Φ_1



Numerical experiments have shown:

for arbitrary initial approximation iterative process of minimum search converges to definite limit point (D_γ, β_γ) which belongs to domain E . This domain is called domain of equivalence.

Inverse problem: resolvability analysis

Second objective function

$$\Phi_2(S) = \int_{T_q}^{T_s} [P(t, S, D_\gamma, \beta_\gamma) - P_*(t)]^2 dt \to \min$$

$$P(t, S, D_*, \beta_*) = [1 + S\overline{B}(t, D_*, \beta_*)/\rho_0] P_0$$

$\Longrightarrow$

$$S = \rho_0 \frac{\int_{T_q}^{T_s} [P_*(t)/P_0 - 1]\overline{B}(t)dt}{\int_{T_q}^{T_s} \overline{B}^2(t)dt}$$

Results of gas content *S* calculation

by known coefficients of diffusion D_γ and mass transfer β_γ

determined at noise level $\gamma = 0.4$

	D_γ, μm^2/s	β_γ, μm/s	S_γ, kg/m^3	Error, %
exact solution	45	0.054	12	0
points D_γ and β_γ belong to domain of equivalence *E*	70	0.045	12.74	6.12
	35	0.065	11.62	3.20
	65	0.050	12.55	4.61
	36	0.048	11.78	1.69

In spite of possible moderate (or sometimes - great) error in evaluation of coefficients of diffusion and mass transfer the gas content *S* estimation is sharp.

INTERPRETATION OF IN SITU MEASUREMENTS DATA

Site Information

where sampling was carried out

Location: Coal mine «Berezovskaya», Kuznetsk Coal Basin, Russia

Depth: 320 m

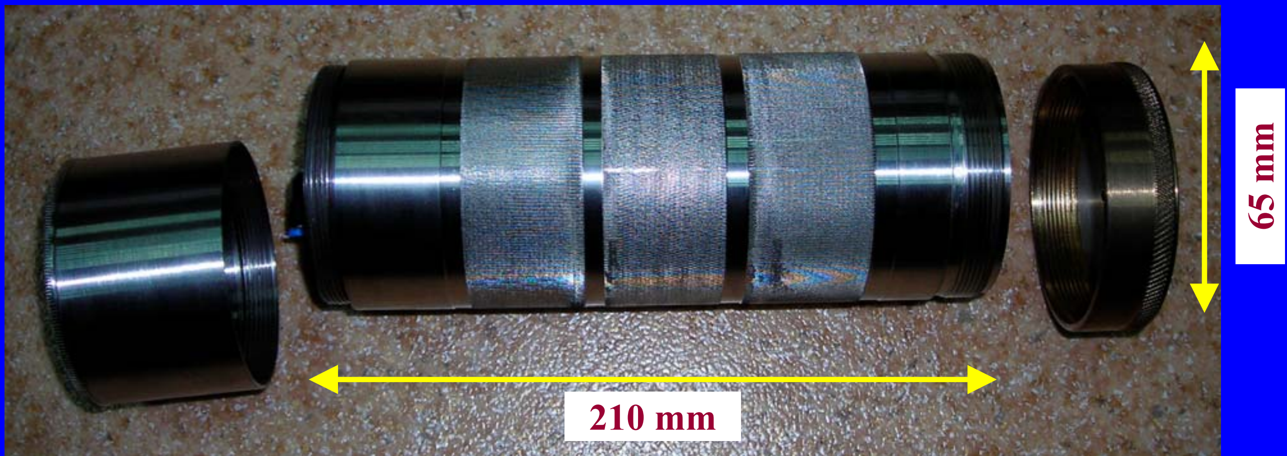
Boring Method: Auger Drilling

Borehole Length: 5 m

Borehole Diameter 50 mm

Geological (natural) gas content: 10 m^3/t (9 kg/m^3)

Measuring vessel



Vessel specification

Measurement pitch 5 s

Recoding time 15 days

Volume 673 cm^3

Absolute accuracy:

- pressure 10 Pa
- temperature 0.01°C

It is measuring vessels for "canister test" and its performance specification.
We manufactured ten vessels.

Grain-size composition of coal samples

obtained upon completion of experiments “canister test”

Vessel number	Grain size composition { R_k , s_k }						Sample volume W_s , cm^3	W_s / W_c
	average diameter of particle R_k , mm							
	0.13	0.40	0.78	1.5	2.5	4.0		
1	0.30	0.16	0.18	0.26	0.08	0.03	205	0.306
2	0.34	0.18	0.19	0.23	0.04	0.02	243	0.362
3	0.38	0.18	0.19	0.20	0.03	0.02	211	0.314
4	0.26	0.14	0.16	0.28	0.11	0.05	257	0.383
5	0.28	0.12	0.14	0.23	0.13	0.10	252	0.375

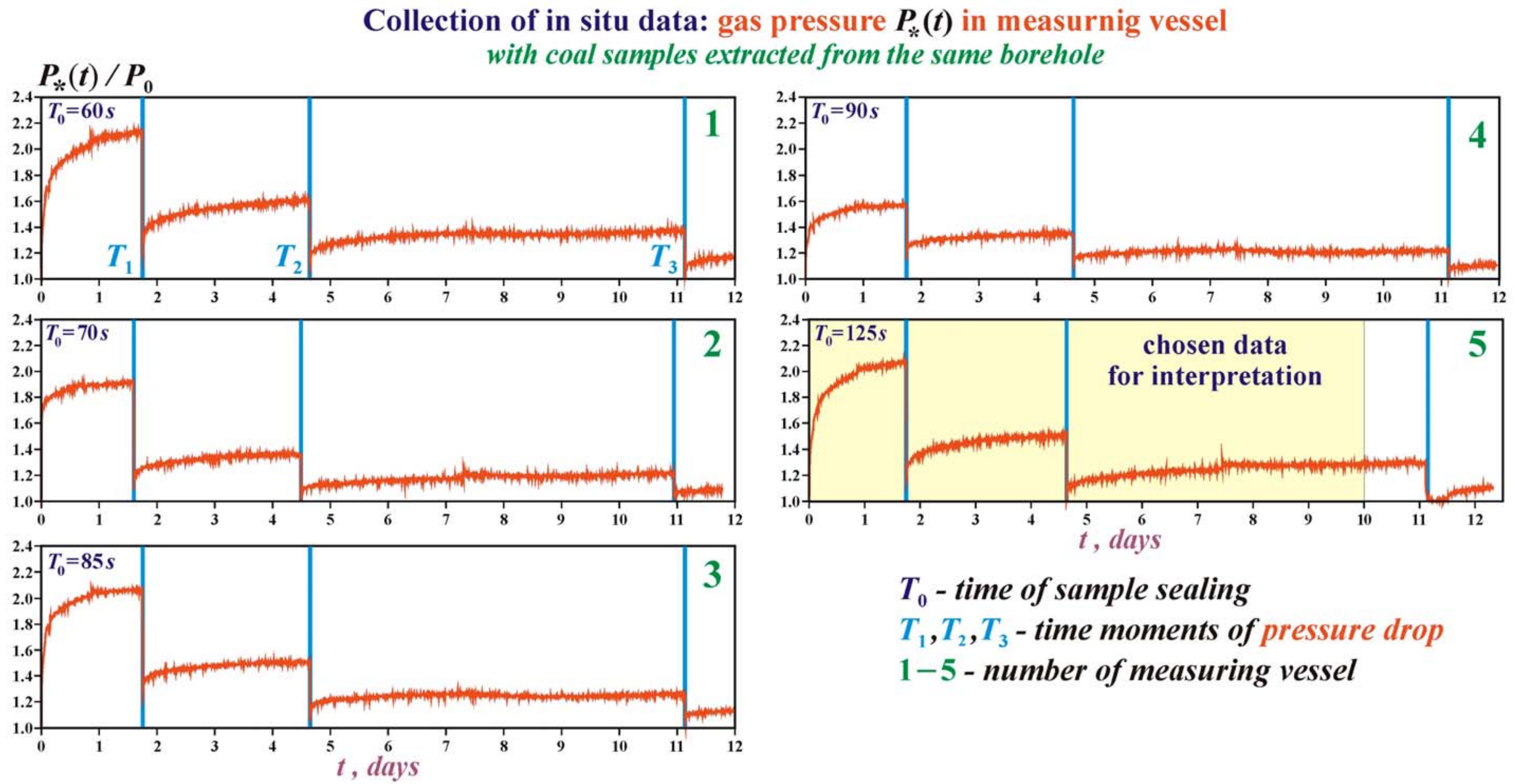
s_k - relative content of k-th fraction in sample

W_c - volume of vessel (W_c=673 cm^3)

We carried out fractional analysis upon completion of measurements.

Results of analysis is presented in Table.

We used the data in line 5 for interpretation.



We filled five vessels with coal chip and sealed them at different time moments T_0.

Measurements last about 12 days.

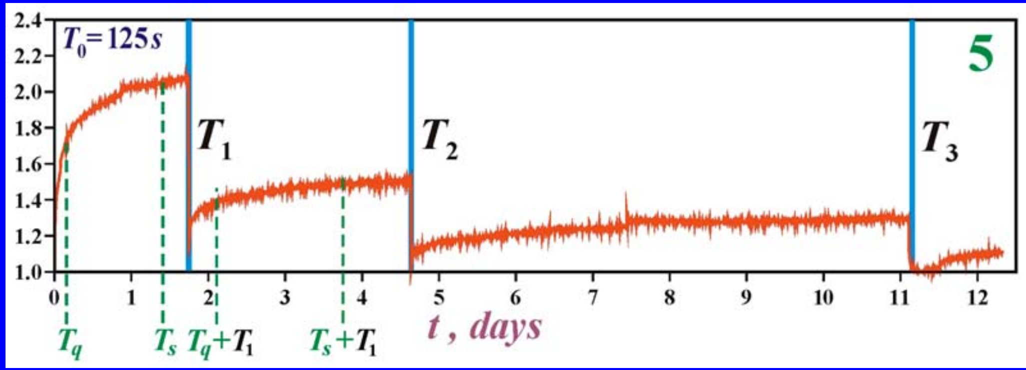
We realized "pressure drop" regime.

In situ data interpretation

$$A(t,D,\beta) = \frac{P(t,S,D,\beta) - P_0}{P(t+T_1,S,D,\beta) - P_0}$$

$$T_q < t < T_s = \min\,(T_1\,,\,T_2 - T_1)$$

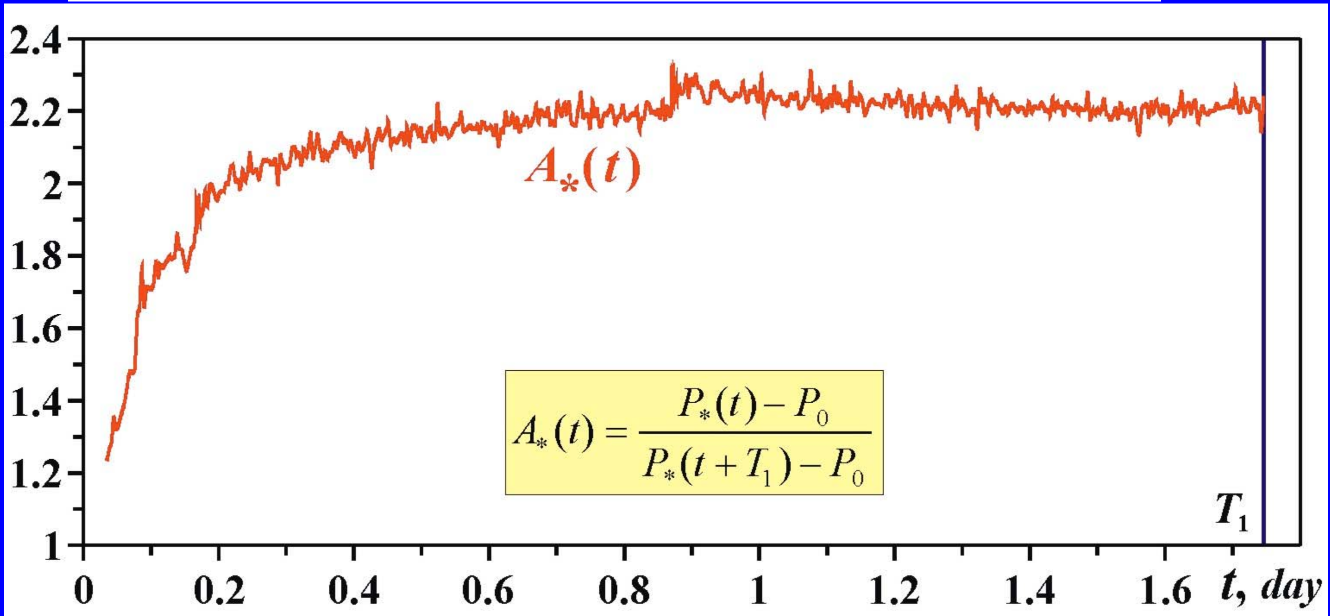
Choice of time limits for data interpretation: illustration



T_q = 20 min

T_s = 1.7 days

Function $A_*(t)$ calculated by real input data $P_*(t)$



We step-by-step carried out the procedure proposed theoretically.
Step I. Calculation of function $A_*(t)$ by recorded pressure $P_*(t)$

Inverse problem

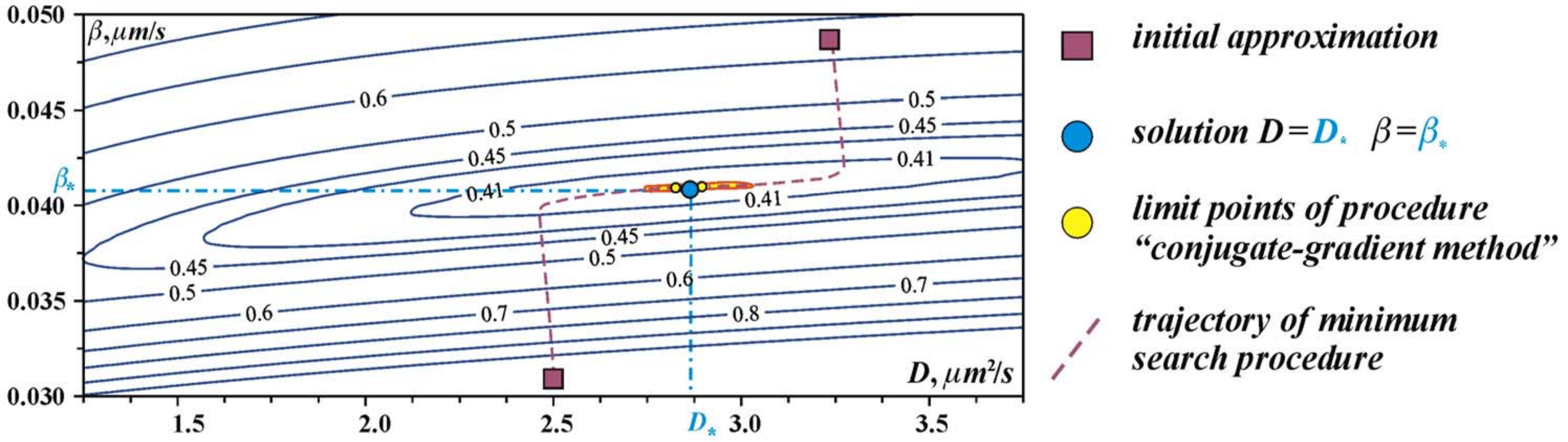
Objective function

$$\Phi_1(D,\beta)=\int_{T_q}^{T_s}[A(t,D,\beta)-A_*(t)]^2\,dt$$

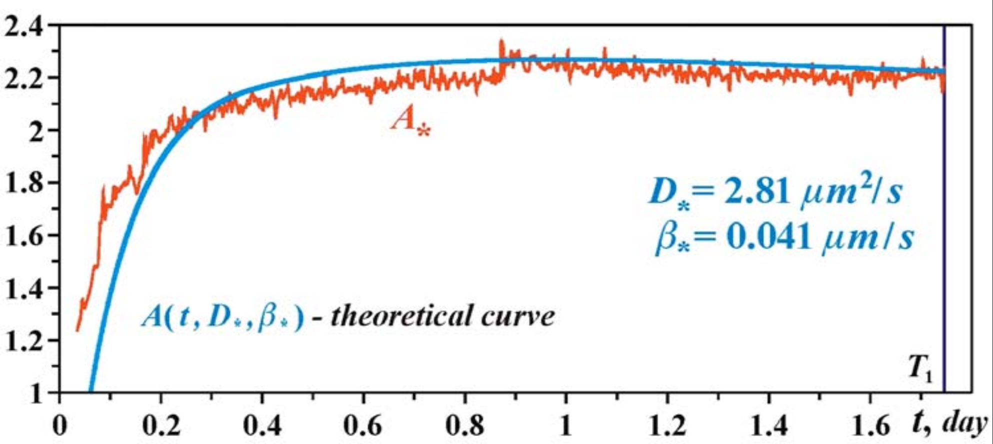
T_q = 20 min

T_s = 1.7 days

Level lines of objective function Φ_1 and illustration of minimum search procedure



Comparison of real $A_*(t)$ and fitted $A(t, D_*, \beta_*)$



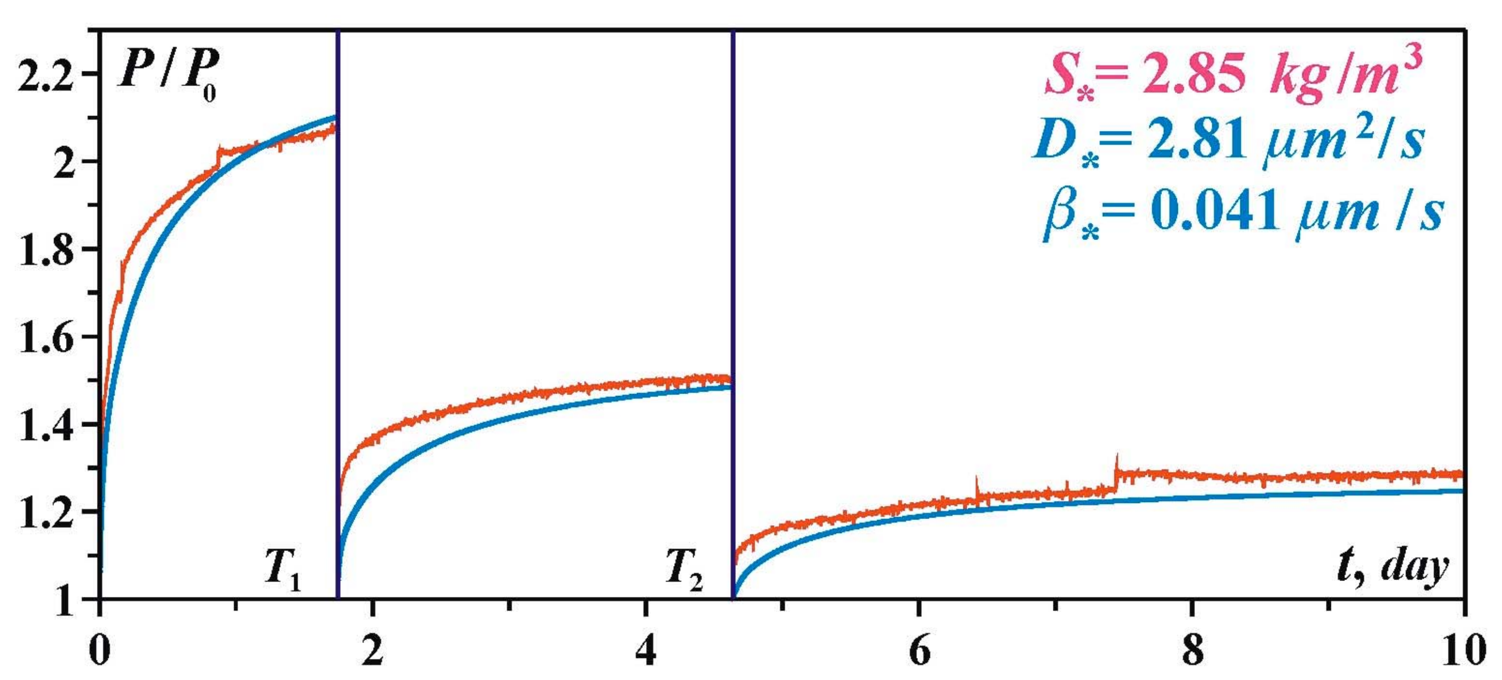
We introduced objective function Φ_1, where $A(t,D,\beta)$ -is "theoretical" function, calculated at certain values of D and β.

Then we found its minimum point (D_*, β_*) using conjugate-gradient method. Thus, we determined coefficients of diffusion and mass transfer.

Comparison of $A_*(t)$ and $A(t, D_*, \beta_*)$ revealed its good fitness: coefficient of variation did not exceed 15%

Inverse problem

$$\Phi_2(S) = \int_{T_q}^{T_s} [P(t, S, D_*, \beta_*) - P_*(t)]^2 \, dt \to \min$$



At last we introduced the second objective function Φ_2 of single variable *S* and found its minimum S_* analytically.
Result: S_* is equal to 2.85 kg per cubic meter.
It is lesser than natural gas content (in intact coal bed, 9 kg per cubic meter).
Explanation: the borehole was very short (5 m) so we extracted coal sample from partly degasified coal mass.

Remark. Value of S_* was calculated using time interval $t < T_1$, but we reached good correspondence theoretical pressure $P(t)$ and real pressure $P_*(t)$ for $t > T_1$ as well .

CONCLUSION

The new method for interpretation of «canister test» data was substantiated theoretically and tested using results of in situ measurements.

The method is based on inverse problem solution and permits to evaluate not only natural gas content S in coal bed, but also gas-kinetic parameters: coefficients of diffusion D and mass transfer β.

The method gives sharp estimation of S

in spite of possible moderate or great error in evaluation of D and β .

THANK YOU FOR ATTENTION!