A Heuristic Approach for Short-Term Production Planning in Open Pits

Michel GAMACHE
Gabriel L'HEUREUX
François SOUMIS

ÉCOLE POLYTECHNIQUE MONTRÉAL

GERAD Groupe d'études et de recherche en analyse des décisions

COSMO - Stochastic Mine Planning Laboratory
McGill University
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Definition of the problem

Short Term Planning (Tactical planning)

The short-term planning problem in open pit mines consists in scheduling mining activities over a period of 10 to 90 days while taking into account operational constraints (precedence, blending, stripping ratio, equipment capacity, etc).

For each period of the planning horizon, the schedule must indicate

- which block will be drilled, blasted and extracted;
- the location of each shovel (faces).

Objectives

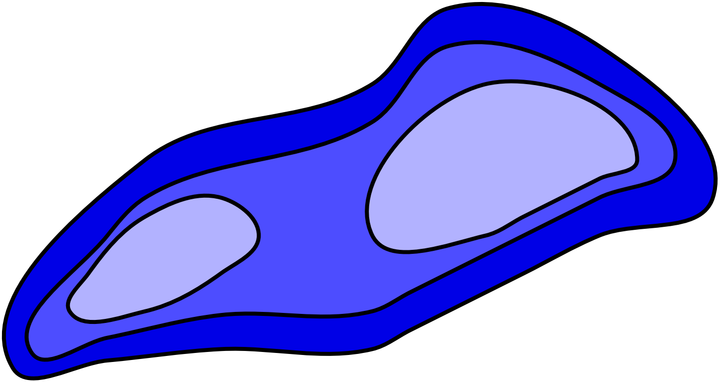
- Develop an optimal approach:
  - What is the maximum size of problems that can be solved to optimality in a reasonable time?
  - What are the elements of the problem that make this problem difficult to solve?
- Identify potential heuristic approaches.

Outline

Outline

1. Overview of the MIP model
   - Definitions
   - Objective Function
   - Constraints
2. Strategies
3. Heuristic approaches

Definitions : Areas



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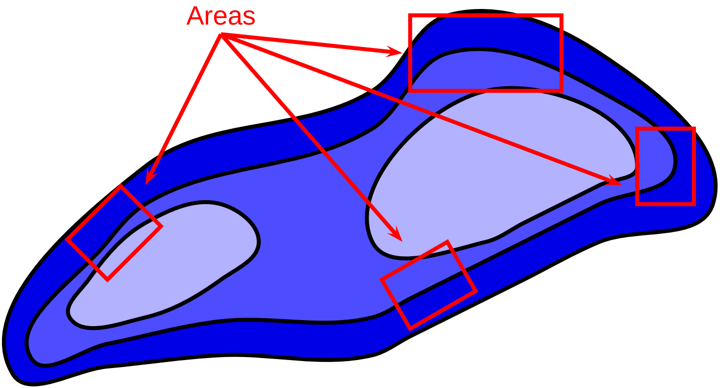

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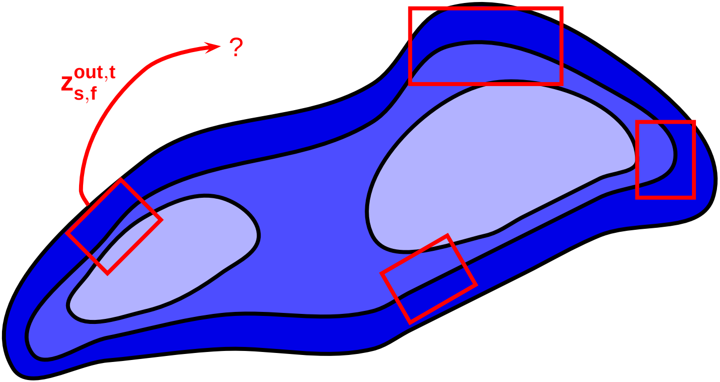

$$z_{s,f}^{out,t} = \begin{cases} 1 & \text{if shovel } s \text{ moves from face } f \text{ to another area at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

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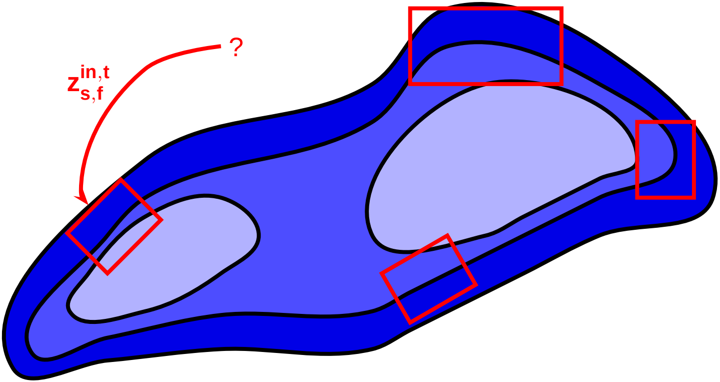
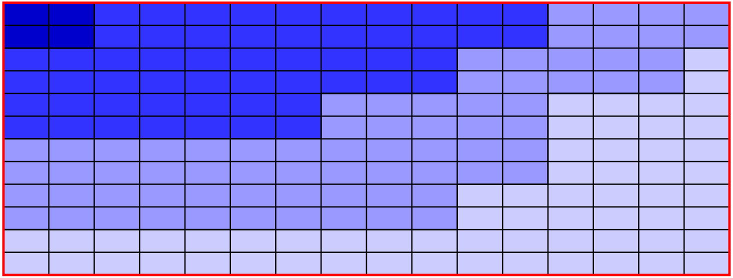


$$z_{s,f}^{in,t} = \begin{cases} 1 & \text{if shovel } s \text{ arrives at face } f \text{ from another area at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Definitions : Blocks and Faces



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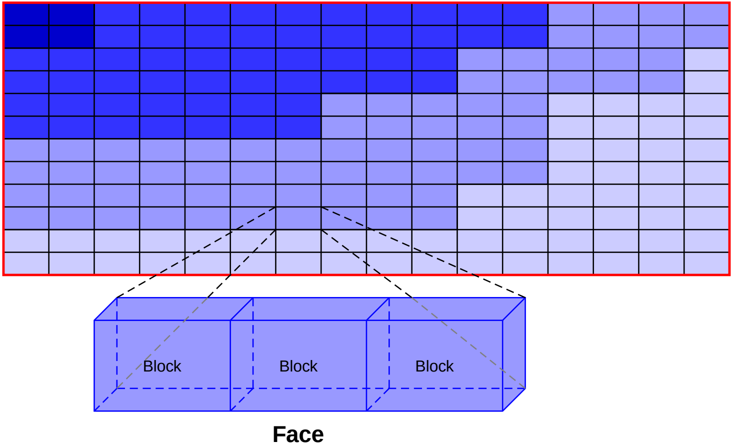

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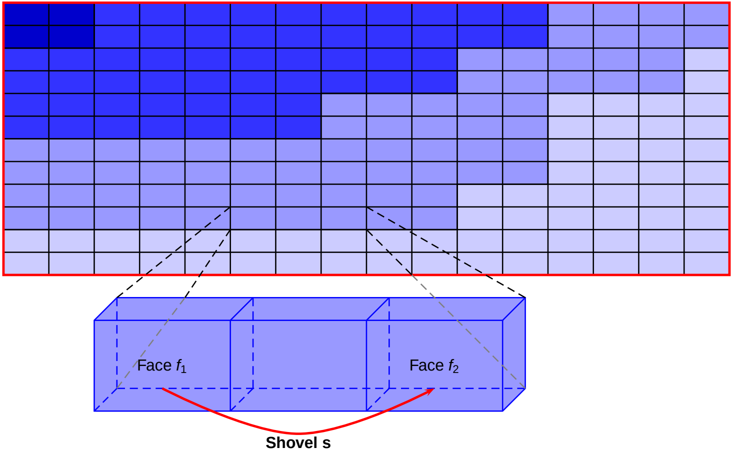
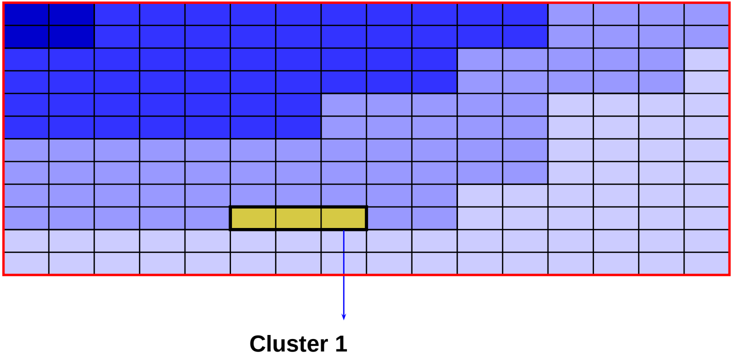


$$z^t_{s,f_1,f_2} = \begin{cases} 1 & \text{If shovel } s \text{ arrives at face } f_2 \text{ from face } f_1 \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

Clusters



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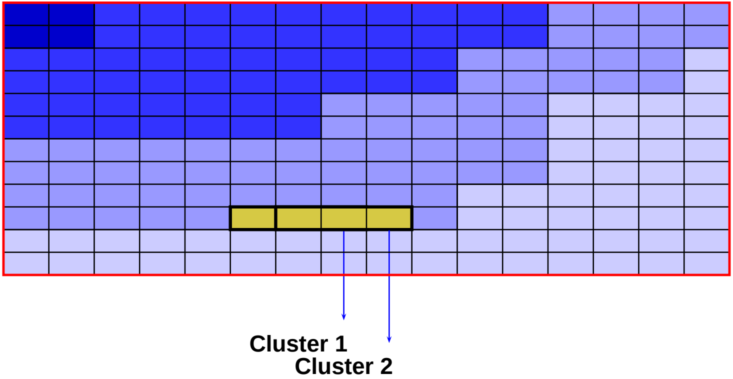

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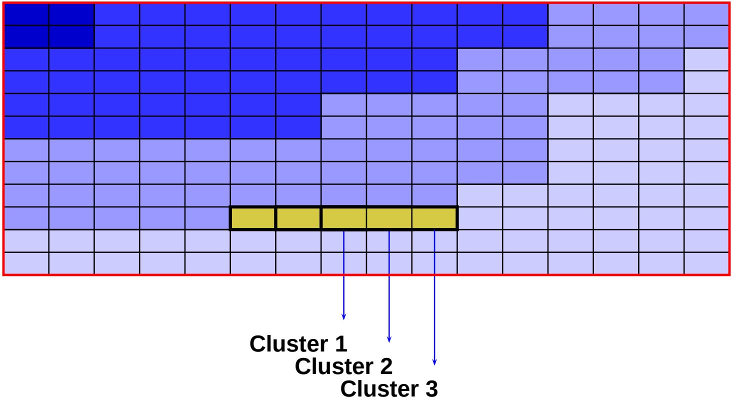
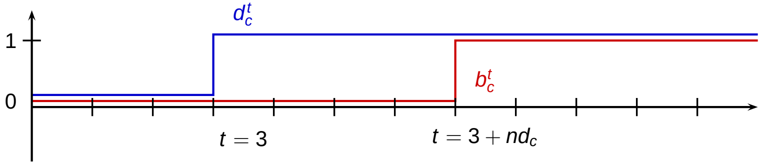


Clusters

$$d_c^t = \begin{cases} 1 & \text{if cluster } c \text{ has been drilled at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$b_c^t = \begin{cases} 1 & \text{if cluster } c \text{ has been blasted at time } t, \\ 0 & \text{otherwise.} \end{cases}$$



Objective function

$$\min \sum_{s} \sum_{t} \sum_{\forall f_1 \in F} TCU(z^{in,t}_{s,f_1} + z^{out,t}_{s,f_1}) \quad \textit{(Long-distance movements)}$$

$$+\sum_{s} \sum_{t} \sum_{\forall f_1 \in F} \sum_{\forall f_2 \in F | f_2 \neq f_1} TC_{f_1,f_2} * z^{t}_{s,f_1,f_2} \quad \textit{(Movements within the same area)}$$

$$+\sum_{c} (DC^1_c d^1_c + \sum_{t=2}^{T} DC^t_c (d^t_c - d^{t-1}_c)) \quad \textit{(Drilling)}$$

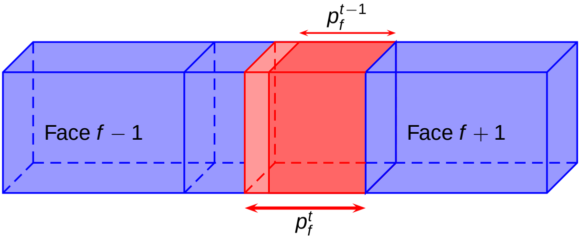
$$+\sum_{c} (BC^1_c b^1_c + \sum_{t=2}^{T} BC^t_c (b^t_c - b^{t-1}_c)) \quad \textit{(Blasting)}$$

Constraints

Blending and Stripping Ratio

$$\sum_{f=1}^{F} m_f \alpha_f q_f^e (p_f^t - p_f^{t-1}) \geq L_e^t \sum_{f=1}^{F} m_f \alpha_f (p_f^t - p_f^{t-1}) \quad t = 2, \ldots, T \; \forall e \quad (2)$$

$$\sum_{f=1}^{F} m_f \alpha_f q_f^e (p_f^t - p_f^{t-1}) \leq U_e^t \sum_{f=1}^{F} m_f \alpha_f (p_f^t - p_f^{t-1}) \quad t = 2, \ldots, T \; \forall e \quad (3)$$



Constraints

Shovels' location and production capacity

$$w_{s,f}^{t} = \begin{cases} 1 & \text{if shovel } s \text{ is assigned to face } f \text{ during period } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{f} w_{s,f}^{t} \leq 1 \qquad \forall t;\ \forall s \qquad (4)$$

$$\sum_{s} w_{s,f}^{t} \leq 1 \qquad \forall t;\ \forall f \qquad (5)$$

$$\sum_{f} m_f (p_f^t - p_f^{t-1}) \geq LS \sum_{s=1}^{S} Us_s \qquad t = 2, ..., T \qquad (6)$$

$$m_f (p_f^t - p_f^{t-1}) \leq \sum_{s} Us_s w_{s,f}^{t} \qquad t = 2, ..., T,\ \forall f \qquad (7)$$

Constraints

Resource usage at time t

Number of drilling

$$\sum_c (d_c^t - d_c^{t-nd_c}) \geq Ld^t \quad t = 2, \ldots, T \quad (8)$$

$$\sum_c (d_c^t - d_c^{t-nd_c}) \leq Ud^t \quad t = 2, \ldots, T \quad (9)$$

Number of blasting

$$\sum_c (b_c^t - b_c^{t-1}) \geq Ld^t \quad t = 2, \ldots, T \quad (10)$$

$$\sum_c (b_c^t - b_c^{t-1}) \leq Ud^t \quad t = 2, \ldots, T \quad (11)$$

Constraints

The number of faces in operation at time t

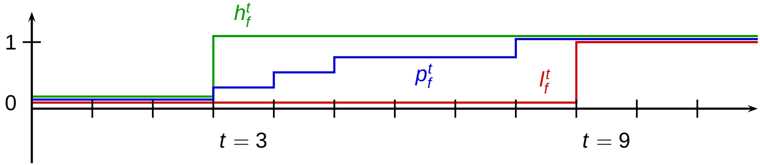
$$h_f^t = \begin{cases} 1 & \text{if the extraction of the material at face } f \text{ has begun at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$l_f^t = \begin{cases} 1 & \text{if all the material at face } f \text{ has been extracted at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_f (h_f^t - l_f^t) \leq nf \quad t = 2, \ldots, T \qquad (12)$$

$$l_f^t \leq p_f^t \quad \forall t;\ \forall f \qquad (13)$$

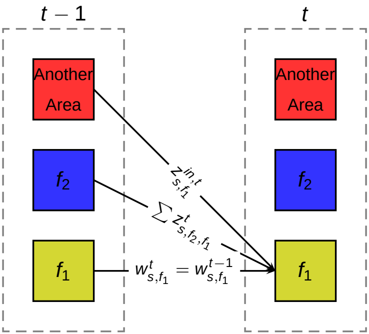
$$p_f^t \leq h_f^t \quad \forall t;\ \forall f \qquad (14)$$



Constraints

Flow conservation

$$w^t_{s,f_1} = w^{t-1}_{s,f_1} + \left(z^{in,t}_{s,f_1} + \sum_{f_2 \in ARCS_{f_1}} z^t_{s,f_2,f_1}\right) - \left(z^{out,t}_{s,f_1} + \sum_{f_2 \in ARCS_{f_1}} z^t_{s,f_1,f_2}\right) \quad t = 2, \dots, T;\ \forall s\ \forall f_1 \in F \qquad (15)$$



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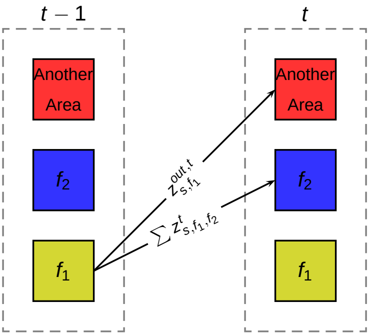

Constraints

Precedence constraints

- Precedence between operations

Drilling → Blasting → Excavation

$$
\begin{aligned}
d_c^{t-nd_c} &\geq b_c^t \quad t = 2, \ldots, T;\ \forall c \qquad (16)\\
\sum_{c \in C_f} b_c^{t-nb_c} &\geq h_f^t \quad t = 2, \ldots, T;\ \forall f \qquad (17)\\
l_f^{t-1} &\geq b_c^t \quad t = 2, \ldots, T;\ \forall c;\ \forall f \in F_c \qquad (18)
\end{aligned}
$$

- Temporal link

$$
\begin{aligned}
d_c^{t-1} &\leq d_c^t \quad t = 2, \ldots, T;\ \forall c \qquad (19)\\
b_c^{t-1} &\leq b_c^t \quad t = 2, \ldots, T;\ \forall c \qquad (20)\\
h_f^{t-1} &\leq h_f^t \quad t = 2, \ldots, T;\ \forall f \qquad (21)\\
l_f^{t-1} &\leq l_f^t \quad t = 2, \ldots, T;\ \forall f \qquad (22)\\
p_f^{t-1} &\leq p_f^t \quad t = 2, \ldots, T;\ \forall f \qquad (23)
\end{aligned}
$$

Constraints
Clusters

$$y_c = \begin{cases} 1 & \text{if cluster } c \text{ is chosen,} \\ 0 & \text{otherwise.} \end{cases}$$

$$\sum_{c \in K_k} y_c \leq 1 \quad \forall k \quad (24)$$

$$d_c^T \leq y_c \quad \forall c; \quad (25)$$

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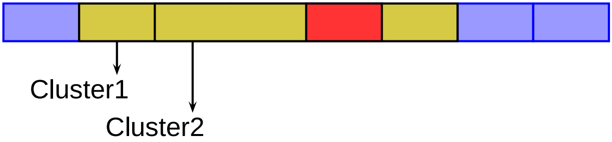

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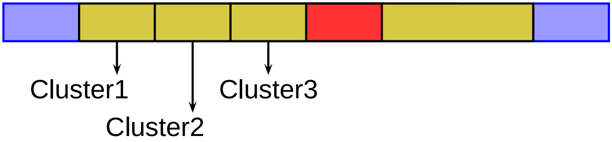
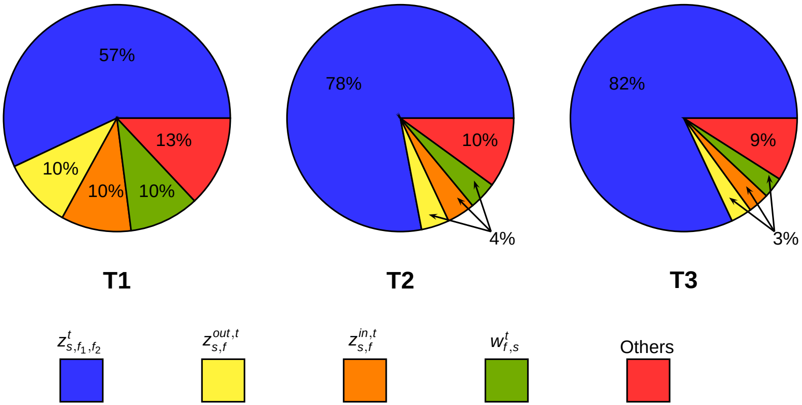


Outline

Instances

	T1	T2	T3
Periods	10	30	90
Faces	30	90	192
Areas	5	5	8
Shovels	5	5	5
Clusters	60	315	864
Variables	15 660	310 815	2 541 024
Constraints	5 912	83 729	627 178

Distribution of variables in the models



Results : M_0 (basic model)

	Number Constraints	Number Variables	Number Nodes	Integrality Gap (%)	Times Z_{IP}^{opt} (sec)	Times $Z_{IP}^{5\%}$ (sec)	Times Z_{IP}^{Best} (sec)
M_0	4060	12020	1457910	N/A	36000	N/A	N/A

Results : M_1

1. Set the value of some variables based on the earliest start of each task (considering precedence constraints only);
2. Prioritize the branching on the variables $w_{s,f}^{t}$.

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M_0	4060	12020	1457910	N/A	36000	N/A	N/A
M_1	2680	10735	736204	31%	6464	40	40

- At least 5 times faster;
- The number of nodes was reduced by half;
- The best solution found in less than 40 sec.

Results : M_2

1. Symmetric solutions;
2. Adding discounts in the objective function based on the ore grade of each face : $-\sum_t \sum_f r_f^t p_f^t$;

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M_1	2680	10735	736204	31%	6464	40	40
M_2	2423	8875	24682	33%	200	5	17

- At least 30 (180) times faster than M1 (M0);
- 10% reduction on the number of constraints and variables;
- 97% reduction on the branching nodes;
- Integrality gap ???

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- 97% reduction on the branching nodes;
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Results : M_3

1. Fractional variables ($w^t_{f,s}$) $\Rightarrow$ this eliminates almost all the movements of shovels which greatly reduces costs;
2. Solution $\Rightarrow$ Adding a new constraint that forces a minimum number of movements for each shovel.

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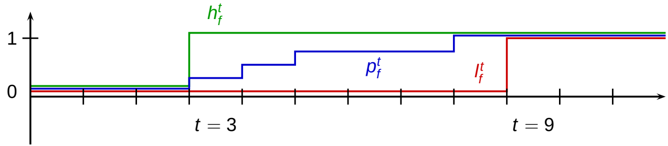
M_1	2680	10735	736204	31%	6464	40	40
M_2	2423	8875	24682	33%	200	5	17
M_3	2664	10557	38687	0.8%	400	13	310

- It reduces the gap;
- The solution time increases.

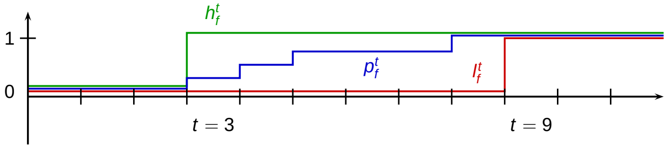
Results : M_4

Solution of the MILP :

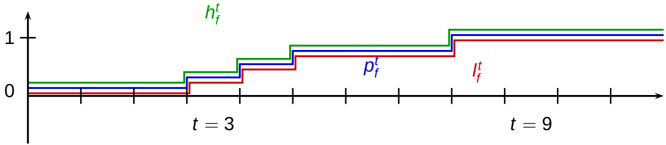


Results : M_4

Solution of the MILP :



Solution of the LP relaxation



Results : M_4

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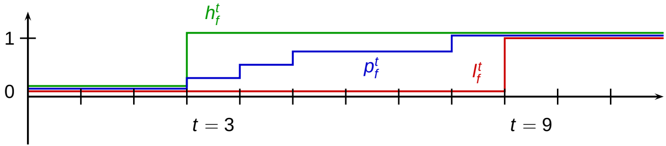

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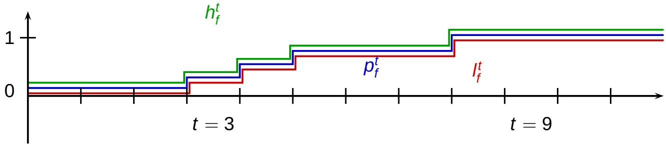

$$1 - (h_f^t - h_f^{t-1}) \geq l_f^{t+1} \quad t = 2, \ldots, T-1; \forall f \quad (41)$$
$$1 - (h_f^t - h_f^{t-1}) \geq l_f^{t+2} \quad t = 2, \ldots, T-2; \forall f \quad (42)$$

Results : M_4

1. Based on M2
2. Stronger link between variables :

$$
\begin{array}{rcll}
w^t_{s,f_1} & \geq & \displaystyle\sum_{\forall f_2 \in ARCS_{f_1}} z^t_{s,f_2,f_1} & \forall t;\ \forall f_1 \in F;\ \forall s \quad (39)\\
w^t_{s,f_1} & \geq & \displaystyle\sum_{\forall f_3 \in ARCS_{f_1}} z^t_{s,f_1,f_3} & t = 1,\dots,T-1;\ \forall f_1 \in F;\ \forall s \quad (40)\\
1-(h^t_f - h^{t-1}_f) & \geq & l^{t+1}_f & t = 2,\dots,T-1; \forall f \quad (41)\\
1-(h^t_f - h^{t-1}_f) & \geq & l^{t+2}_f & t = 2,\dots,T-2; \forall f \quad (42)\\
d^t_c & = & b^{t+3}_c & t = 1,\dots,T-3; \forall c \quad (43)\\
h^t_f & = & l^{t+5}_f & t = 1,\dots,T-5; \forall c \quad (44)\\
y_c & = & d^T_c & \forall c \quad (45)\\
y_c & = & b^T_c & \forall c \quad (46)
\end{array}
$$

Results : M_4

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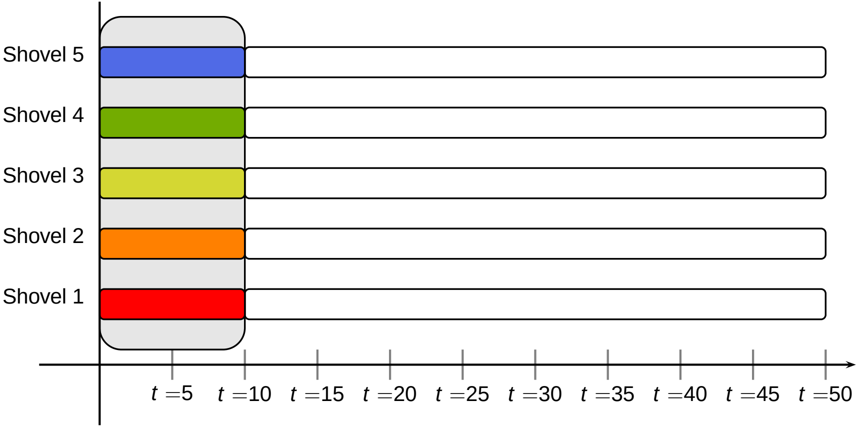
M_2	2423	8875	24682	33%	200	5	17
M_4	1718	830	22145	33%	31	0	9

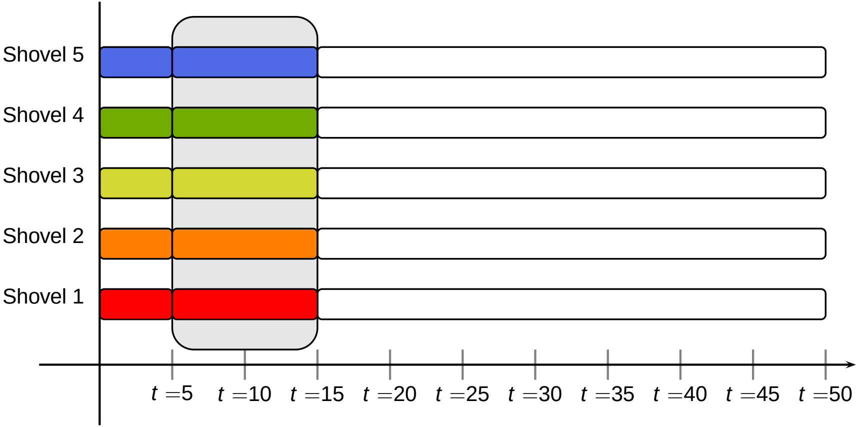
- 6 (1500) times faster than M2 (M0);
- 90% less variables than M2;
- 50% less constraints than M0;
- 98% less nodes in the B&B than M0.

Outline

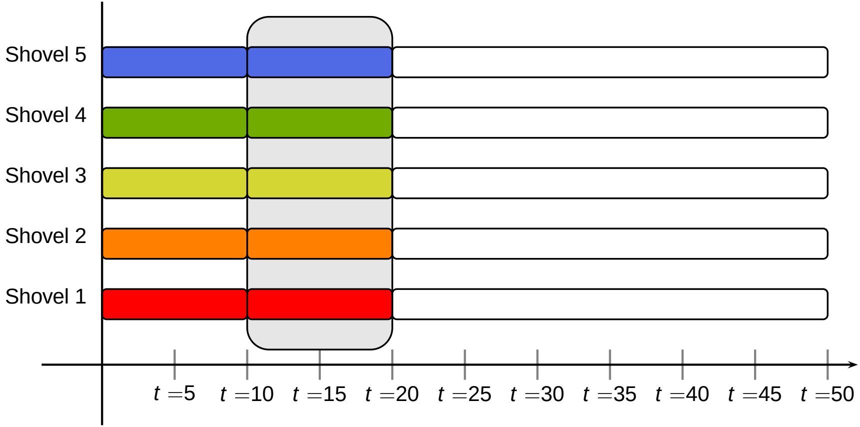
Sliding Time Horizon



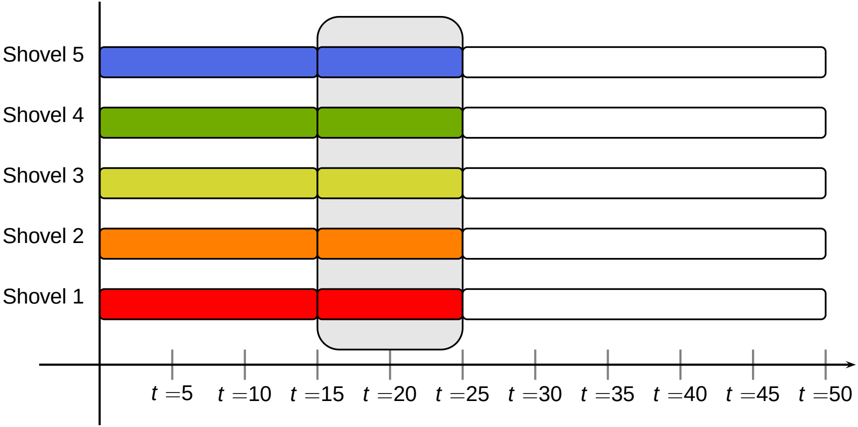
Sliding Time Horizon



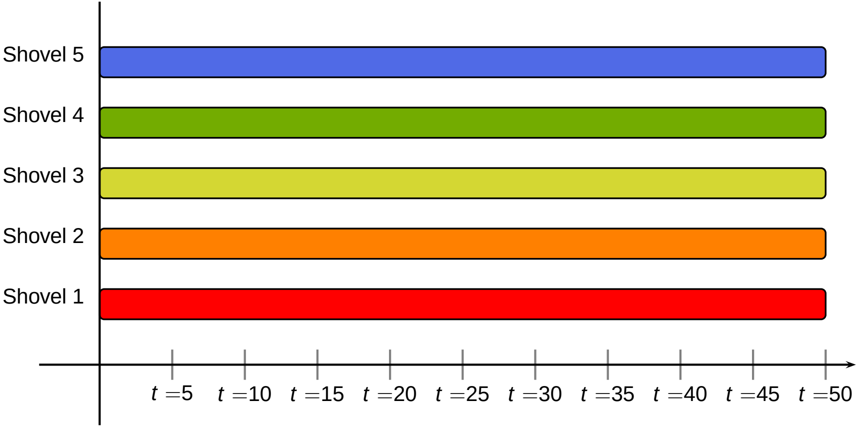
Sliding Time Horizon



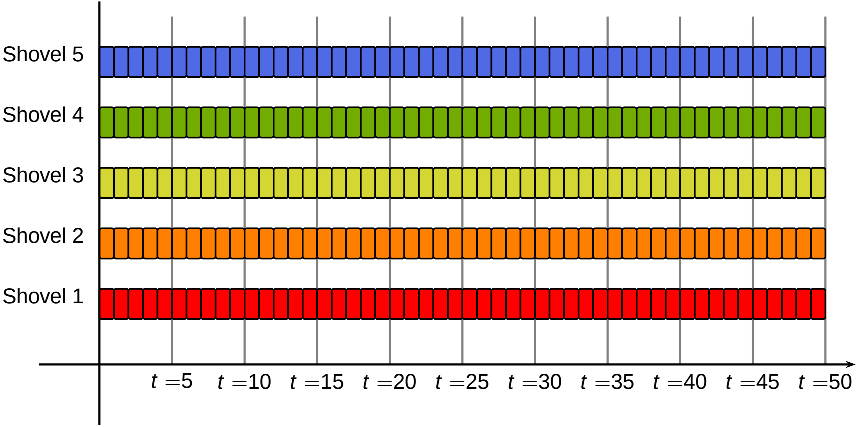
Sliding Time Horizon



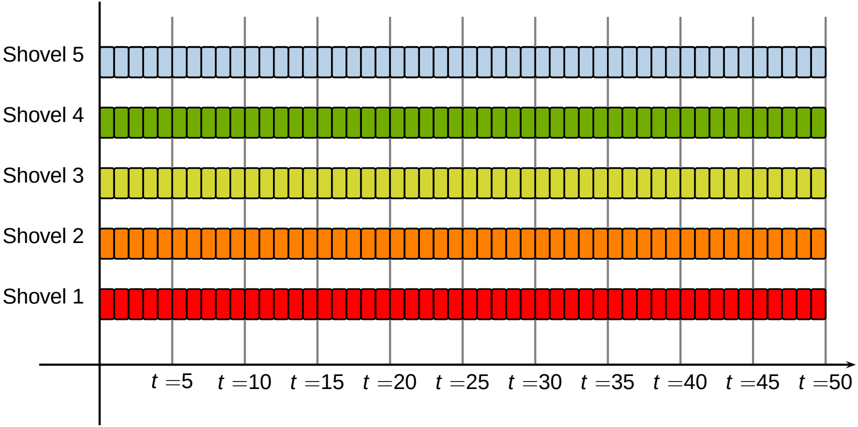
Sliding Time Horizon



Large Neighborhood Search



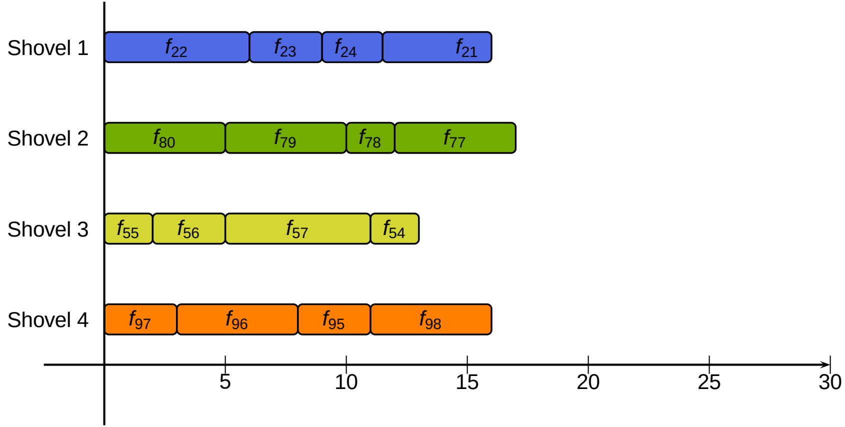
Large Neighborhood Search



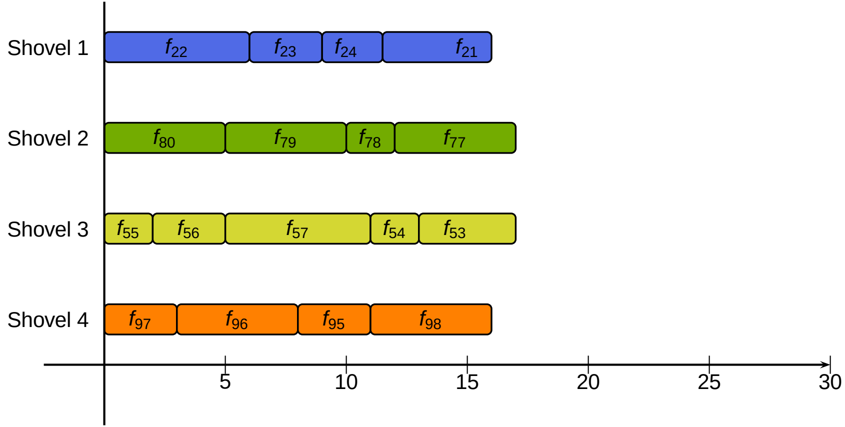
Heuristic Approach

- Construct a simple and intuitive schedule;
- Use the heuristic schedule as a guide when solving model M4

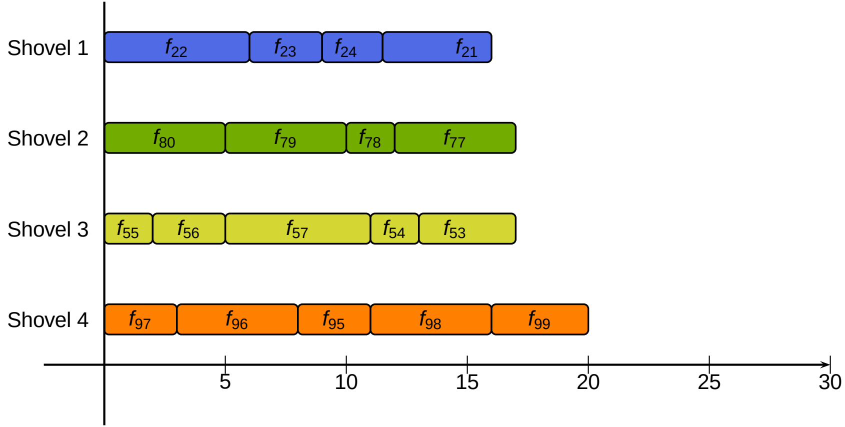
The heuristic schedule



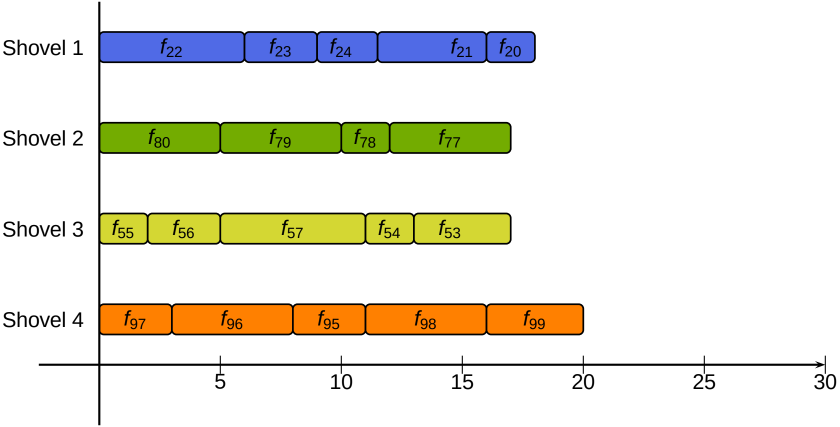
The heuristic schedule



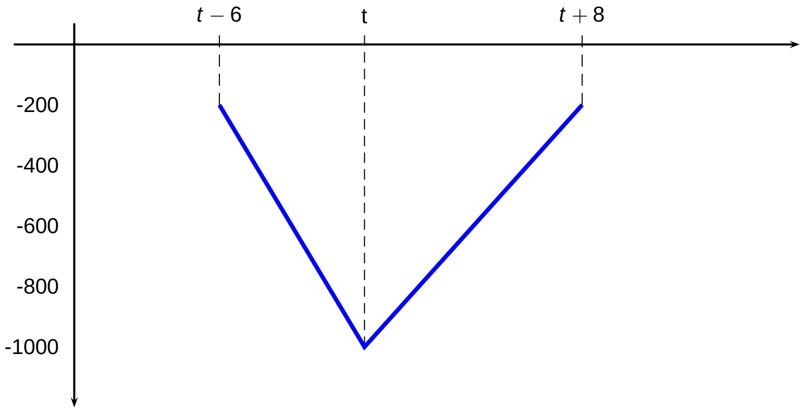
The heuristic schedule



The heuristic schedule



Reduction



Tests

- **Instance T1:**

	Number Constraints	Number Variables	Number Nodes	Times Z_{IP}^{opt} (sec)
M_4	1718	830	22145	31
Heuristic	1765	1290	494	9

- **Instance T2:**
  - 30 periods;
  - 90 faces;
  - 310000 variables;
  - 84000 constraints.

Solved in 29 seconds (4 seconds if some variables are set to zero)