Harnessing Uncertainty for Orebody Modelling and Strategic Mine Planning

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Overview

• The economic side of uncertainty

• Models of geological uncertainly

• Limits of traditional mine design optimization

• Shifting the paradigm: Stochastic mine planning

• Using uncertainty to improve project performance

• Conclusions - Uncertainty is great!
Uncertainty Matters:
The Economic Side of Uncertainty

Changing the way we do things
Uncertainty Matters:
Return on Investment is Uncertain, therefore Risky

- Possibility of not making a return on capital (NPV<0)

- Alternative development plans may have different risk profiles and expected values. Example:

  Design - can’t capture high reserves

  Design … can capture…
Risk in Mining: A World Bank Survey

- 60% of mines had an average rate of production LESS THAN 70% of planned rate

- In the first year after start up, 70% of mills or concentrators had an average rate of production LESS THAN 70% of design capacity

- Key contributor to mining risk felt in all downstream phases: Geology and reserves
Uncertainty is not a “Bad Thing”

Many managers believe that uncertainty is a problem and should be avoided.....

... you can take advantage of uncertainty. Your strategic investments will be sheltered from its adverse effects while remaining exposed to its upside potential. Uncertainty will create opportunities and value.

Once your way of thinking explicitly includes uncertainty, the whole decision-making framework changes.

Martha Amram and Nalin Kulatilaka in “Real Options”
Real Options vs DCF View of Value

Real Options View: Current Value of Option to Produce

Traditional DCF View (now or never)

Contingent Decision Payoff Function (future price known)
Accurate Uncertainty Assessment Needed

“The goal of technical evaluation should be to strive for an accurate assessment of uncertainty, not a single precise answer”
Mining Project Valuation

Traditional view

Single estimated model

Orebody Model

Multiple probable models

Risk oriented view

Mining Process or Transfer Function

Mine Design
Production Scheduling

Financial and Production Forecasts

Accurate uncertainty estimation

Unknown, true answer
Single, often precise, wrong answer
Quantitative Models of Geological Uncertainty:

Stochastic or geostatistical conditional simulations
Describing the Uncertainty about a Mineral Deposit

Actual but unknown mineral deposit

Information about the deposit

Probable models of the deposit
Describing the Uncertainty about a Gold Deposit

Model characteristics:

- Large number of blocks
- Multiple domains
- Resource classes with specific sample selection criteria
Lode 1502
Simulation #2
Moving Forward in Optimization:
Limits of Traditional Mine Design

Using Models of Uncertainty
Risk Analysis in a Mine Design

Objective
Quantify the impact of grade uncertainty to tonnage, grades, metal and net present value - net present value vs risk exposure

Methodology
Open Pit Mine Design and Production Scheduling

Intermediate pushbacks

Pit Limit
Limits of Traditional Modelling

The expected project NPV has only 2 – 4% probability to be realised
First 2 years of production likely to be negative cash flow.
Probabilities on Pit Limits

Pit limit determined conventionally

100% probability of falling within the pit for a given metal price
This is Not ...

EARLY EXPERIMENTS IN TRANSPORTATION
Moving Forward ..... Step 1

Exploring existing technologies
Past Work – Open Pit Mine Design

Upside Potential / Downside Risk

\[
\text{Upside or Downside} = \sum (\text{Value} - \text{MAR} \times \text{probability})
\]
### Past Work – Open Pit Mine Design

**Upside Potential / Downside Risk**

<table>
<thead>
<tr>
<th>Pit Design</th>
<th>Upside Potential (m$)</th>
<th>Downside Potential (m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CB-1</td>
<td>CB-2</td>
</tr>
<tr>
<td>2</td>
<td>2.3</td>
<td>2.41</td>
</tr>
<tr>
<td>4</td>
<td>1.3</td>
<td>2.1</td>
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<tr>
<td>6</td>
<td>2.4</td>
<td>2.43</td>
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<tr>
<td>12</td>
<td>2.9</td>
<td>2.40</td>
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</tbody>
</table>
Moving Forward ..... Step 2

Re-writing optimizers
Integer Programming

An objective function

Maximise \( (c_1x_1^1+c_2x_2^1+\ldots ) \) ...

Subject to

\[
c_1x_1^1+c_2x_2^1+\ldots \geq b_1
\]

\[
\vdots
\]

\[
c_1x_1^p+c_2x_2^p+\ldots \geq b_p
\]

\( c = \text{constant} \)

\( X_1^1 = \text{binary variable} \)

Period 1

Period p
Stochastic Integer Programming

The objective function now ....

Maximise \((s_{11}x_1^1 + s_{21}x_2^1 + \ldots + s_{12}x_1^1 + s_{22}x_2^1 + \ldots)\) ....

Subject to

\[s_{11}x_1^1 + s_{21}x_2^1 + \ldots \geq b_1\]

\[\vdots\]

\[s_{11}x_1^p + s_{21}x_2^p + \ldots \geq b_1\]

\[s_{12}x_1^p + s_{22}x_2^p + \ldots \geq b_1\]

\[s_{1r}x_1^p + s_{2r}x_2^p + \ldots \geq b_1\]

Period 1
Simulated model 1
Simulated model 2
Simulated model r

Period p
“Uncertainty Will
Create Opportunities and Value”

Higher NPV for less risk
Base Case:
Geological Risk Assessment of Ore Production

Uncertainty in Ore Production - Base Case Schedule

<table>
<thead>
<tr>
<th>Period</th>
<th>Target Ore Production</th>
<th>Maximum Ore</th>
<th>Expected Ore</th>
<th>Minimum Ore</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.9%</td>
<td>18.2%</td>
<td>12.7%</td>
<td>8.7%</td>
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<tr>
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<tr>
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<td>10.8%</td>
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<tr>
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<td>12.5%</td>
<td>11.4%</td>
<td>12.4%</td>
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<tr>
<td>5</td>
<td>10.8%</td>
<td>10.4%</td>
<td>12.3%</td>
<td>10.4%</td>
</tr>
<tr>
<td>6</td>
<td>11.4%</td>
<td>9.0%</td>
<td>11.9%</td>
<td>9.0%</td>
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<tr>
<td>7</td>
<td>12.4%</td>
<td>11.9%</td>
<td>12.3%</td>
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<tr>
<td>8</td>
<td>12.3%</td>
<td>14.5%</td>
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<tr>
<td>9</td>
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<td>12.9%</td>
<td>12.3%</td>
<td>12.9%</td>
<td>12.3%</td>
</tr>
</tbody>
</table>

Average Deviation (%)

- Avrg. Deviation
- Target Ore Production
- Maximum Ore
- Expected Ore
- Minimum Ore
Risk-based: Assessment in Ore Production

Uncertainty in Ore Production - Risk-based Schedule

Average Deviation (%)

Period

Ore Production

Avrg. Deviation
Target Ore Production
Maximum Ore
Expected Ore
Minimum Ore

Mt

0.5% 3.0% 1.2% 0.7% 3.5% 1.9% 0.2% 2.7% 1.6% 0.0% 0.6% 0.4% 0.1% 0.0% 0.7% 0.0%

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Uncertainty is Good: “Base case” vs “Risk-based”

Multistage combinatorial optimization

Risk-Based

Difference 28%

Traditional “Expected”
Uncertainty is Good:
Discounting Geological Risk

The discounting goes along
with production sequencing
Objective function

\[
\text{Max } \sum_{t=1}^{P} \left[ \sum_{i=1}^{N} E\{(NPV)_i^t\} \ b_i^t \right]
\]

\[
- \sum_{i=1}^{U} E\{(NPV)_i^t\} + MC_i^t * S_i^t
\]

\[
+ \sum_{s=1}^{M} (SV)_s^t (P) q_s^t
\]

\[
- \sum_{s=1}^{M} (c_u^{ty} d_{su}^{ty} + c_l^{ty} d_{sl}^{ty})
\]

Part 1: Mill & dump
Part 2: Stockpile input
Part 3: Stockpile output
Part 4: Risk management
Stochastic Integer Programming - SIP

A production schedule

Orebody Model 1
- Deviation 1
  - Ore Grade 1 Metal
  - Ore Grade 2 Metal
  - Ore Grade R Metal
  - TARGET [ ]

Orebody Model 2
- Deviation 2
  - Ore Grade 1 Metal
  - Ore Grade 2 Metal
  - TARGET [ ]

Orebody Model R
- Deviation R
  - Ore Grade R Metal
  - TARGET [ ]
Cross-Sectional Views of the Schedules

SIP

Whittle Four-X

Periods

1
2
3
4
5
6
Managing Risk Between Periods

Deviations from metal production target

Metal quantity (1000 Kg)

C^t = C^{t-1} \times RDF_{t-1}

RDF_t = 1/(1+r)^t

RDF – risk discounting factor

r – orebody risk discount rate
# Case Study on a Large Gold Mine

The SIP specific information

<table>
<thead>
<tr>
<th><strong>Orebody risk discounting rate</strong></th>
<th>20 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of shortage in ore production</td>
<td>10,000 /t</td>
</tr>
<tr>
<td>Cost of excess ore production</td>
<td>1,000 /t</td>
</tr>
<tr>
<td>Cost of shortage in metal production</td>
<td>20 /gr</td>
</tr>
<tr>
<td>Cost of excess metal production</td>
<td>20 /gr</td>
</tr>
<tr>
<td>Number of simulated orebody models</td>
<td>15</td>
</tr>
</tbody>
</table>
Deviations from Production Targets

Metal Production

<table>
<thead>
<tr>
<th>Periods</th>
<th>Metal quantity (1000 Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>-12</td>
</tr>
<tr>
<td>4</td>
<td>-16</td>
</tr>
<tr>
<td>5</td>
<td>-20</td>
</tr>
<tr>
<td>6</td>
<td>-24</td>
</tr>
</tbody>
</table>

- SIP model
- WFX
Stockpile’s Profile

Available ore at the end of each period

- SIP model
- WFX

Ore taken out from the stockpile

Periods

<table>
<thead>
<tr>
<th>Period</th>
<th>Available ore (million)</th>
<th>Ore taken out (million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
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<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Uncertainty is Good: Traditional vs Risk-Based

Stochastic Integer Programming

Cumulative NPV values

- SIP model
- WFX

Average NPV values

- SIP model
- WFX

Geological Risk Discounting = 20%

Difference = 17%

$723\ M\ Risk\ Based

$609\ M\ Traditional

Periods

1 2 3 4 5 6
Some conclusions

- “…. uncertainty is (not) a problem and should be avoided ?”
- “… you can take advantage of uncertainty…. ”
- “….uncertainty will create opportunities and value.”
- “ …once your way of thinking explicitly includes uncertainty, the whole decision-making framework changes.”
- We need:

  Stochastic mine planning and NEW mathematical models
And

• It is all about good people:

   Education and training in a long term sense
Please join us!