Uncertainty-based production scheduling in open pit mining

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Abstract

Optimization of long-term production scheduling is important for managing the substantial cash flows inherent in open pit mining ventures. In terms of ore grade, tons and quality, discrepancies between actual production and planning expectations arise through uncertainty about the orebody. Traditional methods fail to consider the risk of not meeting production targets caused by the uncertainty in estimated grades. These aspects of uncertainty are integrated in a new optimization formulation for multielement production scheduling, which also takes into account risk quantification, equipment access and mobility and other operational requirements such as blending, mill capacity and mine production capacity. Furthermore, the approach introduces the concept of orebody risk discounting. In a case study of an Australasian nickel-cobalt laterite orebody, this new risk-based approach produced better results in meeting planned production targets than traditional approaches.

Introduction

Production scheduling is a critical mechanism in the planning of surface mining ventures. It deals with the effective management of a mine's production and cash flows in the order of millions of dollars. Long-term production scheduling is used to maximize the net present value (NPV) of the project and focuses on the sequencing of materials to be mined in space over time, under technical, financial and environmental constraints. The importance of incorporating uncertainty and risk from the technical, geological and mining sources in mine production schedules, particularly the possible in situ variability of pertinent orebody grade and ore quality characteristics, is well appreciated. Discrepancies between planning expectations and actual production may occur at any stage of mining. For example, Vallee (2000) reported that 60% of the mines surveyed had an average rate of production that was less than 70% of the designed capacity in the early years. Others (e.g., Rossi and Parker, 1994) reported shortfalls against predictions of mine production in later stages of production. These shortfalls were mostly attributed to orebody uncertainty. Because traditional production scheduling methods do not consider the risk of not meeting production targets caused by grade variability, they cannot produce optimal results.

The detrimental effects of grade uncertainty and in situ variability in optimizing open pit mine design are shown in recent studies. Dimitrakopoulos et al. (2002) showed the substantial conceptual and economic differences of risk-based frameworks compare to the methods ignoring geological risk. Dowd (1997) proposed a framework for risk integration in surface mining projects. Godoy and Dimitrakopoulos (2004) presented a new approach for risk-inclusive cutback designs, which yield substantial NPV increases. Ravenscroft (1992) discussed risk analysis in mine production scheduling, where the use of stochastically simulated orebodies showed the impact of grade uncertainty on production scheduling. Ravenscroft concluded that conventional mathematical programming models cannot accommodate quantified risk, thus there is a need for a new generation of scheduling formulations to overcome infeasible or unrealistic scheduling and account for production risk. Smith and Dimitrakopoulos (1999) showed additional examples using mixed-integer programming to verify the above conclusion in the context of short-term planning. Kumral and Dowd (2001) used stochastic simulations and optimization in short-term planning.

Past efforts to deal with uncertainty attempt to sequentially link stochastic orebody models with conventional optimization formulations, with the exception of Godoy and Dimitrakopoulos (2004). This sequential process is inefficient and, although it assesses risk in a schedule, it does not produce optimal scheduling solutions in the presence of uncertainty. In addition, these efforts do not consider multi-element deposits with complex ore quality constraints, such as nickel laterites, iron ore or magnesium deposits. Furthermore, dealing with orebody uncertainty and in situ variability accentuates the need to consider issues of equipment access and mobility in the related "stochastic" optimization formulations.

In the above context, this paper presents a new, risk-based production-scheduling formulation for complex, multielement deposits. The formulation is based on expected block grades and probabilities of grades being above required cutoffs, both sets of values being derived from jointly simulated deposit models (Dimitrakopoulos, 2002). Expected block grades and

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probabilities are integrated with equipment constraints and the practical feasibility of mining sequencing in a linear programming model. This model typically considers homogenization and blending, mill and mining capacities and performs multi-period optimization. A key effect of such a probabilistic approach is that the more certain areas of the deposit are mined in earlier production periods, leaving uncertain areas for later periods, when additional information usually becomes available. The probabilistic approach followed in this paper introduces a technical link to the new concept of "risk discounting" that explicitly integrates orebody uncertainty in to production scheduling and, inevitably, project valuation.

In the following sections, the new production-scheduling formulation under conditions of orebody uncertainty is presented and combined with equipment constraints to generate practical scheduling patterns. Subsequently, the formulation is applied to a nickel-cobalt laterite deposit, elucidating the practical aspects of the formulation. Next, the practical differences between this approach and the traditional scheduling approach are discussed. Finally, the conclusions of this study are presented.

Production scheduling under grade uncertainty

The mathematical programming model developed in this section is based on linear programming (LP) and takes into account the geological uncertainty and equipment mobility and access required for scheduling and excavating mining blocks. In this scheduling approach, a probability is assigned to each block to represent the "desirability" of that block being mined in a given period. This probability represents the chances that a block will contain the desired grade and ore quality and quantity, including ore grades above given cutoffs, deleterious elements within required ranges, recovery characteristics and processibility indexes. The probability is calculated from simulated orebody models representing the mineral deposit (e.g., Dimitrakopoulos, 2002). This model can be easily extended to a mixed integer programming (MIP) model (Ramazan, 2001) simply by defining the variables as binary instead of linear, as needed. The model contains an objective function and a set of constraints as follows:

Objective function. The objective function formulation is

$$\text{Minimize } Z = \sum_{t=1}^{p\max}\left[C_1^{\,t} * Y1^t + \left(\sum_{i=1}^{nblock} C_2 * \left(Y2_i^t\right) + C_3 * \left(Y3_i^t\right)\right)\right] \quad (1)$$

where

$p$max is the total time period for scheduling,

nblock is total number of blocks in the model;

$Y1^t$ is the percent deviation from having 100% probability that the material mined in period t would have the desired properties; and

$C_1^{\,t}$ is the cost coefficient for the probability deviation in period t, such that $C_1^{\,1} > C_1^{\,2} > C_1^{\,3} > ... > C_1^{\,p\max}$.

Discussion of C_2, C_3 and $Y2$, $Y3$ is deferred for a subsequent paragraph. In the presence of orebody uncertainty, a number of blocks will have a probability of less than 100%, and thus the schedule will have a deviation from this target probability. This deviation can be seen as the risk of not meeting production targets for the related parameters and has a cost for the objective function. Costs for the objective function are set so that a unit of deviation is more costly in the first period than in the second period, which is higher than the third period, and so on. Thus, the objective function will find the blocks with the highest probabilities for the first period, lower probability blocks for the second period, and so on.

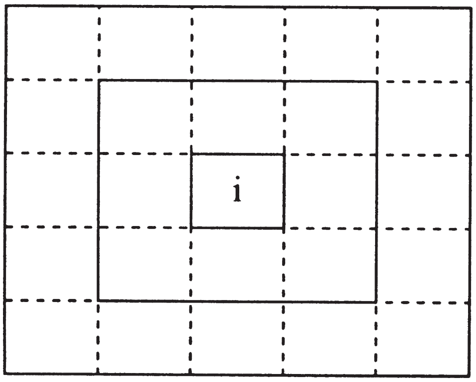

Figure 1 — Inner and outer windows around Block i are set up to establish a smooth schedule for equipment access and a required mining width. (Solid lines show windows; dotted lines show blocks).

Coefficient variables C_2, and C_3 are cost coefficient variables for $Y2$ and $Y3$ percent deviations from mining targets, relating to the smoothness of the mining operation. More specifically, Fig. 1 shows mining blocks and two concentric windows that move as the central Block i moves. The optimization model is set to mine Block i together with the blocks within the inner (smaller) window. If all the blocks within the inner window cannot be mined out, the tonnage of the blocks that cannot be mined is a "deviation" referred to as $Y2$ in percentage, and each percentage costs C_2 for the objective function. The mining blocks within the outer (large) window will be mined, if possible, and $Y3$ and C_3 correspond to the related deviation and cost. The smoothing formulation can ensure minimum mining width for the available equipment access and mobility. If C_2 is set equal to C_3, the objective function will be penalized twice for the deviation of the inner window compared with once for the outer window. This setup means that when Block i is mined it is more desirable to mine it together with the neighboring blocks (in the inner window) than the blocks farther from it (outer window). But it is even better for smoothness of mining, to mine the farther blocks too with Block i, if feasible.

The model in Eq. (1) requires suitable cost coefficients ($C_1^{\,t}$) for deviations from 100% probability and a smooth schedule, and these are derived through a trial-and-error approach as follows. At the start, a low cost penalizes the deviation from the smoothness in Eq. (1) to result in a widely spread mining pattern for various periods, while the probability will be expected to be relatively high in the first period. Incremental cost increases over time will lead to a required mining width and suitable equipment access in the schedule. This schedule is considered as optimum for maximizing probabilities, given the degree of the smoothness obtained.

The objective function in Eq. (1) does not directly maximize net present value (NPV). Rather, it opts to provide a feasible scheduling pattern and ensure a desired grade and quality of the ore produced. The reason is that feasible scheduling patterns and the amount of ore having the desired quality to be sent to the mill need to be priorities, indirectly leading to a practically maximum NPV that is realistic. Otherwise, the generated NPV would only be optimal in the mathematical sense and not in mining practice. Furthermore, the risk of producing adequate ore having the desired properties is integrated in the process, to maximize the chances of delivering to the mill the amount and quality of ore required during mining operation. Risk minimization and feasible patterns result in practically maximum NPV.

Model constraints. The proposed scheduling optimization model in Eq. (1) contains a series of constraints. These include probability targets and equipment accessibility and mobility, as well as the more traditional constraints of grade blending mill requirements, mill capacity, upper and lower bounds for ore quality parameters, mining capacity and others that depend on the conditions of the given mine, such as stripping ratio and wall slope. The constraints considered here and used in the subsequent case study are as follows.

Probability constraints.

$$\sum_{i=1}^{nblock} \left(P_i - 100.0\right) * OT_i^t + Y1^t * TO = 0 \qquad (2)$$

where

P_i is the probability of Block n having a grade within a desired interval, $P_i \leq 100$;

the constant 100.0 is the target probability for the schedule;

OT_i^t is the ore tonnage scheduled from Block i to be mined in Period t;

TO is a constant number representing total ore tonnage to be scheduled in Period t; and

$Y1^t$ is the percent deviation from the probability target at Period t.

Note that $Y1^t$ is penalized at a rate of $C_1^{\,t}$ in the objective function. In the first period, each unit of $Y1^1$ will cost $C_1^{\,1}$, a unit $Y1^2$ will cost $C_1^{\,2}$ and a unit $Y1^3$ will cost $C_1^{\,3}$ for the objective function. Because the objective function is a minimization, the blocks with the highest probabilities will be scheduled in the first period to have the minimum possible $C_1^{\,1} * Y1^1$ due to the fact that $C_1^{\,1}$ is larger than $C_1^{\,2}$ and $C_1^{\,3}$.

Constraints for equipment access and mobility. The two windows discussed in the previous section (Fig. 1) are used to set up the objective function and constraint formulations. Constraints for the inner window and for mining Block i at Period t are

$$-\sum_{j=1}^{nb1} K1_j * OT_j^t + K2_i * OT_i^t - Y2_i^{\,t} \leq 0 \qquad (3)$$

where

$K1_j = 1/TO_j$ and $K2_i = nb1/TO_i$ are the coefficients to convert ore tons to percentage;

TO_j is the total ore tonnage available in mining Block j;

$nb1$ is the total number of blocks within the inner window excluding the central block, which is eight in the Fig. 1; and

Y parameters are deviations from smoothness for this inner window.

The constraint formulation for the outer window is similar to the inner window

$$-\sum_{j=1}^{nb2} K1_j * OT_j^t + K2_i * OT_i^t - Y3_i^{\,t} \leq 0 \qquad (4)$$

where

$Y3$ parameters are the equivalent to $Y2$ parameters the outer window and

$nb2$ is the total number of blocks within the outer window.

In this case, $K2_i = nb2/TO_i$.

Grade blending constraints. Upper bound constraints require the average grade of the material sent to the mill to be less than or equal to a certain value, Gr_{max}

$$\left(\sum_{i=1}^{nblock} \left(Gr_i - Gr_{\max}\right) * OT_i^t\right) \leq 0 \qquad (5)$$

where

Gr_i is the average grade of Block i.

Lower bound constraints require the average grade of the material sent to the mill to be greater than or equal to a certain value, Gr_{min}

$$\left(\sum_{i=1}^{nblock} \left(Gr_i - Gr_{\min}\right) * OT_i^t\right) \geq 0 \qquad (6)$$

where

$t = 1, 2, \ldots, p\max$.

Reserve constraints. The set of reserve constraints is used to require all the available ore tons in a block to be mined. The following formulations are written for each block.

$$\sum_{t=1}^{p\max} OT_i^t = \left(Tot.Ore\right)_i \qquad (7)$$

where

$i = 1, 2, \ldots, nblock$.

Processing capacity constraints. Processing is constrained by the maximum production capacity of the plant ($PCap_{\max}$) and the minimum production requirement ($PCap_{\min}$). These upper and lower bounds are necessary to ensure a smooth feed of ore to mill.

- *Upper bound constraints for each period:*

$$\sum_{i=1}^{nblock} OT_i^t \leq PCap_{\max} \qquad (8)$$

- *Lower bound constraints for each period:*

$$\sum_{i=1}^{nblock} OT_i^t \geq PCap_{\min} \qquad (9)$$

Mining capacity constraints. Mining capacity constraints represent the actual available equipment capacity ($MCap_{\max}$) during each production period and are

$$\sum_{i=1}^{nblock}\left(OT_i^t + WT_i^t\right) \le MCap_{\max} \qquad (10)$$

where

WT_i^t is the waste tonnage scheduled from Block i to be mined in Period t.

Table 1 — Average values in the simulation-based model (SM) and traditional model (TM).

Model	Ore,[1] 10^6 t	Tonnage,[2] 10^6 t	UEV,[3] $\$10^6$	Ni, %	Co, %	Mg, %	Al, %
SM	28.91	47.45	1,640.56	1.29	0.090	4.50	0.58
TM	28.83	48.32	1,655.20	1.30	0.088	4.70	0.67

[1]Ore represents ore tonnage
[2]Tonnage is total tonnage
[3]Total undiscounted economic value

Production scheduling under uncertainty in a Ni-Co laterite deposit

The case study considers a part of a typical laterite nickel deposit in Australasia. The deposit is expected to produce around 30,000 t (33,000 st) of nickel metal and 3,000 t (3,300 st) of cobalt metal per year, with a mine life estimated to exceed 20 years. The operation is expected to recover high-purity nickel and cobalt by electrowinning. Important metallurgical issues are the response of the ore to pressure acid leaching, given the magnesium and aluminum content in the ore being processed and the forecasting of acid consumption in the mill due to this content. Orebody variability and uncertainty are considered critical in achieving "multivariable" mine optimization and production scheduling.

Deposit models and constraints. The geology of the orebody shows a layer of waste material on top of limonite and saprolite layers, here combined to a zone (LS), with rocky saprolite (RS) below. Both LS and RS may contain high-grade nickel. For classification of ore and waste, the cutoff grade is set at 0.5% Ni. The deposit is characterized by seven attributes: Ni, Co, Mg and Al grades, volume of percent rock (Vol%R), thickness of LS and thickness of RS. The main mineral considered for profit in this project is nickel. Cobalt is a byproduct with limited contribution to overall mine cash flows. Magnesium and aluminum are relevant to the acid consumption at the processing plant and have a major influence on processing costs.

For the purpose of scheduling, the deposit is represented by 2,030 blocks, each 40 x 40 m (130 x 130 ft). An orebody model is generated using the technique of joint conditional simulation, as detailed below. The model comprises total tonnage, economic value, % tons of the LS layer, % tons of the RS layer at -2 mm, Ni, Co, Mg, Al, volume % rock and % total ore tons (% tons of LS + % tons of RS at –2mm) within each block.

The seven deposit attributes mentioned above are simulated, jointly and conditionally, using a 5 x 5-m (16 x 16-ft) grid for 35 realizations. The 5 x 5-m grids are then reblocked to the 40 x 40-m block size. Each set of joint simulation models generated is equally likely to be the real deposit, given the available information. The joint conditional simulation of these attributes is based on the so-called simulation with minimum/maximum autocorrelation factors. This is an approach that spatially decorrelates the variables involved to noncorrelated factors. The independent factors are individually simulated and back-transformed to the conditional simulations of the correlated deposit attributes that reproduce the cross-correlations and individual correlations of the original variables (Desbarats and Dimitrakopoulos, 2000). Simulated representations of the orebody are used to generate average block grades and probabilities of values of different attributes to be within given ranges of interest and as needed for the scheduling optimization formulation in the previous section.

This study considers that ore material sent to the processing plant during each production period should have an average Ni grade in the range of 1.3 ± 0.1%. The probability of each block having the Ni grade in the desired feed range for the mill is used to minimize the geological risk in mine optimization, as discussed earlier. The total ore tonnage, total tonnage, total undiscounted economic value, average Ni, Co, Mg and Al grades in the simulation based model (SM) and the probabilities are shown in Table 1, together with the corresponding values for a traditional model (TM) discussed in a subsequent section.

The lower and upper bound constraints on Ni grade for each period are 1.2% and 1.4%, respectively. Ore production is limited to between 9.5 and 10 Mt (10.5 and 11 million st) per period, because the scheduling model is designed over a period of three years and average periodical ore tons is around 9.64 Mt (10.6 million st). Overall, average Mg and Al are around 4.5% and 0.6%, respectively. Minimum and maximum periodical ranges are selected as 4.0% to 5.0% for Mg and 0.6% to 0.7% for Al. Following established practices, the economic value of each block is calculated to include clearance, mining, processing and administration costs, recovery and price for Ni and Co, overburden and suitable densities.

The steps followed in this project can be summarized as:

- *Step 1:* Provide jointly simulated models of the deposit attributes of interest: Ni, Co, Mg, Al, Vol%R, thickness of LS, and thickness of RS.
- *Step 2:* Assign probabilities to each block for a Ni grade between the desired bounds (1.2% and 1.4%) from the jointly simulated models of the deposit in *Step 1*.
- *Step 3:* Generate the orebody SM by averaging the joint simulations in each scheduling block.
- *Step 4:* Schedule the orebody SM using the formulations in Eqs. (1) through (10).
- *Step 5:* Quantify the risk in the optimal production schedule using the jointly simulated, equally probable deposit models of pertinent attributes in *Step 1*.

Application. The production scheduling results obtained by applying the optimization formulation in Eq. (1) and constraints in Eqs. (2) through (9) to the orebody SM of the Ni laterite deposit (Table 1) are shown in Fig. 2 and summarized in Table 2. The table includes ore and total tonnages mined, undiscounted economic value (UEV) and NPV and average grades per scheduling period. In the schedule, UEV is higher for the first period than the second period ($543 million compared with $536.5 million) and is highest in the last period ($561 million). The total UEV is estimated to be around $1,640 million. Although the economic value is high in the last year, the probability of meeting the required average grade is low, reflecting high risk in achieving the planned metal production in the last period. At about $503 million, NPV is the highest in the first period, and decreases to about

Periods, years	Ore, 10^6 t	Tonnage, 10^6 t	UEV,[1] $\$10^6$	NPV, $\$10^6$	Ni, %	Co, %	Mg, %	Al, %	Prob.,[2] %
1	9.53	14.98	543.03	502.81	1.28	0.091	4.23	0.62	88.29
2	9.58	16.25	536.53	459.99	1.30	0.090	4.65	0.53	84.28
3	9.81	16.22	561.00	445.34	1.31	0.090	4.60	0.59	78.83
Tot/avg[3]	28.91	47.45	1,640.56	1,408.14	1.30	0.090	4.50	0.58	83.75

Table 2 — Summary results of production scheduling using the simulation-based model (SM).

[1]Total undiscounted economic value.
[2]Probability.
[3]Total of columns for tonnage and economic values and averages for the remaining columns.

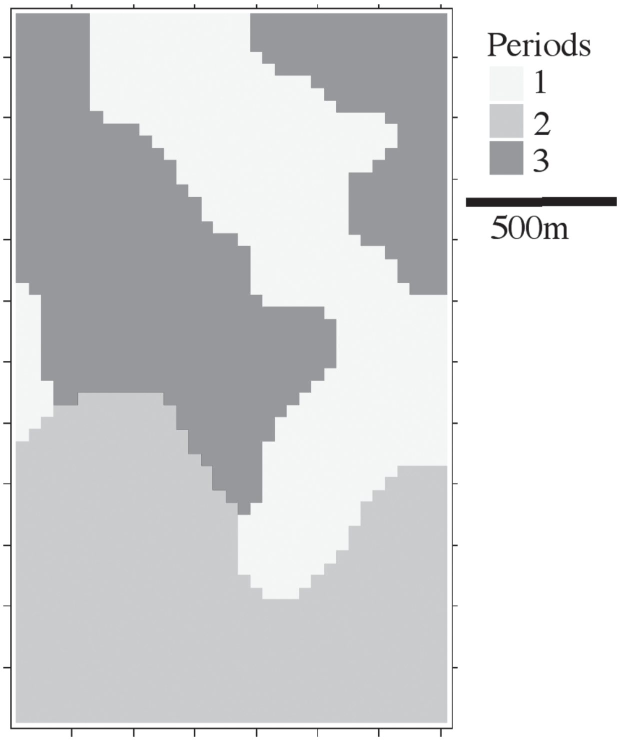

Figure 2 — Production scheduling results incorporating orebody risk and equipment access (from SM).

$445 million in the last period. Total project NPV is about $1,408 million, which is less than 2% different from the optimization using an objective function directly maximizing NPV.

As shown in Table 2, the schedule has the highest probability to achieve the desired properties of the ore produced in the first year (88.3%), a lower probability in the second year (84.3%) and the lowest in the last year (78.83%), exactly as intended in the scheduling optimization model. This shows that the available risk of not achieving the production targets could be distributed over the different time periods by controlling their costs in the objective function. In the present case study, an 8% "risk discount rate" is used to discriminate the costing between time periods. This risk discount rate can be viewed as a parameter controlling the "orebody/grade" risk distribution over time, which is distinctly different from the discount rate conventionally applied to economic values. If a higher rate is used, the differences in the probabilities between different periods are expected to be higher.

Table 2 shows that the LP model is scheduling a little over 9.5 Mt (10.5 million st) of ore in the first period, barely satisfying the processing minimum capacity constraint. This low rate in the ore production leads to lower NPV at the end of the first year. The reason for this is that the objective function is refusing to mine the ore tonnage with lower probabilities to meet the grade requirements; this refusal is because of the relatively high cost assigned for probability deviation factors in the objective function. The reduction of the cost of probability deviations will result in the LP model producing more ore tonnage with higher NPV at the end of first production period. However, the probability of meeting production targets would be lower. Thus, if the decision maker is willing to tolerate additional risk, the cost on the deviations of the probabilities can be reduced. This example suggests a tradeoff between risk and targets as well as between NPV and the utility of risk quantification.

Additional characteristics of the mine production schedule include the effect on the project economics of small variations in the average Ni grade, as well as the average grade of Co, Mg and Al, in the three production periods. The LP model did not produce significant partial block mining, because the costs of deviations are different between periods. Almost 8% of the blocks were partially mined, with most of the 8% scheduled in a single period.

Figure 2 shows the scheduling patterns generated from the proposed LP model. They suggest that the scheduled blocks can be mined at two faces. The production schedule allows equipment access and mining in a continuous manner once mining is started from a certain location. For example, during the first year, some equipment may start mining downwards from the top of the deposit, while other equipment may start mining the bottom part of the first year's scheduling pattern and mining upwards before moving to the small patch on the left side of the deposit. The second and third years' scheduling patterns are also easy to mine continuously.

Comparison of risk-based and traditional optimal scheduling. In this section, the production schedule generated in the previous section is compared with the schedule generated by a traditional approach (TM). The TM uses an estimated model of the deposit (as summarized in Table 1), commonly generated through an approach such as kriging (David, 1988). The scheduling optimization does not include probabilities and the corresponding penalties for related deviations. Figure 3 shows the optimal production schedule for TM, and Table 3 summarizes the results.

In Table 3, the probabilities of meeting the schedule are calculated by comparing the TM schedule with each of the equally possible SM representations of the deposit, in a way similar to *Step 5* in the previous section. Evidently, the effect of not factoring risk in the scheduling optimization formulation generates lower probabilities for meeting production targets. Furthermore, the nonuse of orebody "risk discounting" leads to the ordering of probabilities being the reverse of that of the SM schedule. In the TM schedule, the probability

Table 3 — Summary results of production scheduling using the traditional model (TM).									
Periods, years	Ore, 10^6 t	Tonnage, 10^6 t	UEV,[1] $\$10^6$	NPV, $\$10^6$	Ni, %	Co, %	Mg, %	Al, %	Prob.,[2] %
1	9.78	14.98	660.89	611.94	1.394	0.104	4.327	0.666	82.19
2	9.66	15.77	555.51	476.26	1.292	0.088	4.812	0.741	82.91
3	9.44	17.56	438.85	348.37	1.210	0.072	5.023	0.621	85.59
Tot/avg[2]	28.83	48.32	1,655.2	1,436.57	1.300	0.088	4.718	0.676	83.55

[1]Total undiscounted economic value.
[2]Probability.
[3]Total of columns for tonnage and economic values and averages for the remaining columns.

of achieving the desired properties of the ore produced is lowest in the first year (82.2%), higher in the second year (82.9%) and highest in the last year (85.6%). This trend is usually not desirable, as it is expected that more information will be available as a result of experience gained with the deposit as the mining operation proceeds. Uncertainty in the riskier areas would therefore be expected to decrease, enabling a decision maker to improve decisions on short-term production scheduling and blending processes in the future. In addition, the objective is usually to secure production characteristics at early stages of a project, so as to secure cash flows and loan repayment, as well as improve overall financial aspects of a project.

In comparing the scheduling patterns of TM in Fig. 3 and SM in Fig. 2, the SM scheduling pattern appears practical for mining in two phases, whereas the TM pattern is spread over the deposit and does not appear feasible in practice. This is a common concern with traditional MIP/LP scheduling models. The spread of scheduling patterns in Fig. 3 means that mining equipment would need to be moved often in a given period. In addition, mining blocks may not provide access to equipment, as may be the case for the blocks on the top and center part of the deposit scheduled for the second period. Either their excavation will have to be in the third period or other blocks scheduled for later periods will have to be mined first in order to reach them. These issues are not considered in traditional optimization. The changes that are not considered by the MIP optimizer in an operation may cause infeasibilities in the model constraints and in terms of ore tonnage, grade and quality and sub-optimal NPV or in a NPV that will not materialize.

Figure 4 summarizes the comparison of the risk-based (SM) and traditional (TM) formulations, showing the average deviations per mining period from expected "optimal" production targets, and the probability of deviations in ore production per mining period occurring. The values plotted in the figure are generated by calculating the deviations of each schedule with respect to the 35 jointly simulated orebody models. Figure 4 (a) shows the average of these deviations. Figure 4 (b) is obtained by finding the ratio of the number of models in which the ore tonnage constraints are violated to the total number of simulated orebody models. There is little difference in the first year in the probability of deviation occurring and the amount of deviations between two methods. TM produced slightly lower deviations in ore production for the first year. This is because of that the proposed LP model's objective function doesn't only optimize in minimizing the deviations, but also considers the feasibility of mining patterns in optimization. During the second year of production, the risk-based SM schedule has about 28% (100,000 t) less deviation in expected ore production compared with the traditional schedule. Furthermore, the probability of deviation in ore production occurring is around 10% less than in the traditional schedule as shown in Fig. 4. The ore tonnage is directly related to Ni grades, and increasing the probabilities to meet Ni grade constraints increases the chance of producing the required ore tons. There are no significant deviations in grades, which means that grade constraints are not as tight as processing capacity constraints. The proposed risk-based LP schedule performs substantially better than the traditional schedule when comparing the overall deviations in ore production during the first two periods that the LP model considers.

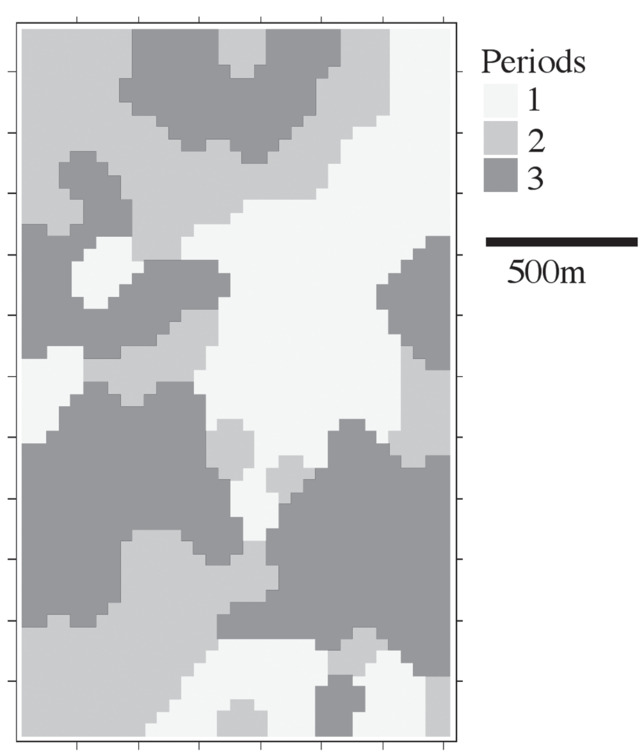

Figure 3 — Production scheduling results from the traditional scheduling approach (TM).

Conclusions

This paper presents a new, risk-based optimization formulation for long-term production scheduling in open pit mines. It is particularly suitable for complex, multielement orebodies, such as Ni laterites, iron ore and magnesium mines. The mathematical programming formulation integrates orebody uncertainty in respect of grade, ore quality and quantity and risk quantification as well as equipment access and mobility and other typical operational requirements.

A key part of the formulation is that it is based on the probabilities of grades of different elements to be above

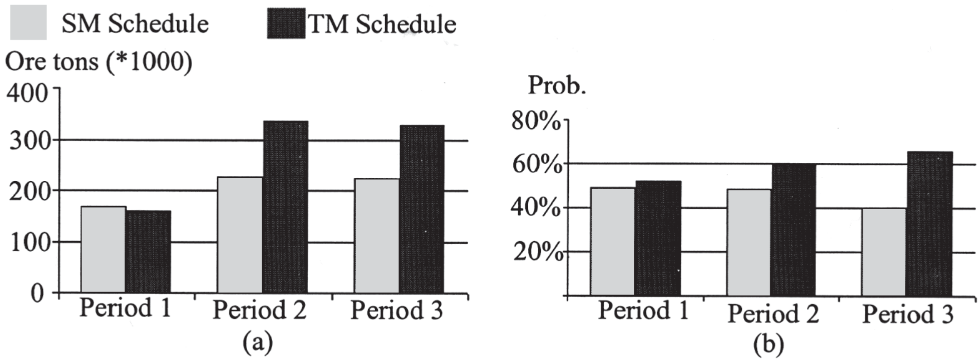

Figure 4 — (a) Average deviations in periodical ore production in tons and (b) average probability of the deviations occurring in the SM and TM schedules.

relevant cutoffs or within a given range. This provides the opportunity to:

- Generate schedules that aim to reduce risk at early production stages when secure cash flows are most critical. And later production periods will benefit from additional information that becomes available as mining operations proceed.
- Explicitly set up the approach to reduce risk in meeting production expectations.
- Introduce the concept of an "orebody risk discount rate" that can account for orebody uncertainty and be used in combination with the common approach of employing discount rates in dealing with mining project uncertainty.
- To generate production schedules with feasible mining patterns, the formulation is coupled with equipment accessibility and mobility constraints, which aim to minimize inefficiencies in the utilization of mining equipment.

The practical aspects of the risk-based approach were shown in an application at a Ni-laterite deposit. Relevant attributes and input to the scheduling formulation (Ni, Co, Mg and Al grades, volume of percent rock, thickness of layer LS and thickness of layers) were jointly simulated, conditional to all available drilling information. Thirty-five equally possible simulated representations of the deposit were used to generate probabilities and averages for the optimization, as well as quantify the risk in meeting production schedules.

The comparisons of the results with traditional long-term production scheduling based on NPV optimization verifies the expectations for the new risk-based formulation that risk in meeting production targets is minimized and risk is lower in the first period, than the second and so forth. In addition, the scheduling patterns generated from the proposed approach are feasible and superior to those from the traditional optimization.

Although the proposed approach is not set up to explicitly maximize NPV, it generates a realistic NPV, which is the best under the scheduling considerations. It is obvious that NPV can be increased by forcing the probabilistic LP model to mine high-grade blocks in the early periods through the use of high and tight grade constraints in those periods. However, increasing NPV will generally increase the risk of not meeting production targets. The traditional model shown in this study produced 2% higher total NPV due to the high cash flows in the first scheduling period of the model. However, the risk of not meeting production targets in the first period was about 6% higher than for the proposed risk-based LP model. There are also the practical mining issues mentioned above to take into consideration.

Future work could consider additional testing as well as a more direct integration of orebody uncertainty in production scheduling formulations. This LP model needs to be extended to other three-dimensional deposits with large vertical extensions.

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