Managing grade risk in stope design optimisation: probabilistic mathematical programming model and application in sublevel stoping

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Optimising stope design is an intricate element of underground mine planning where optimal designs are expected to integrate multiple technical aspects. Orebody uncertainty is a critical aspect affecting the forecasted performance of designs and is linked to the failing of meeting production targets and project financial expectations. Following recent conceptual developments in open pit mining, the present paper develops and explores a new probabilistic mixed integer programming model developed to optimise stope designs, including size, location and number of stopes under consideration of grade uncertainty and predefined levels of acceptable risk. An application of the method at Kidd Creek Mine, Ont., Canada, demonstrates its practical aspects including risk quantification for contained ore tonnes, grade and economic potential.

Keywords: Optimisation, Stope design, Mathematical programming, Stochastic simulation

Introduction

Optimising stope design plays a critical role in the process of underground mine planning, aiming to effectively integrate multiple technical aspects. These aspects include accessibility and equipment constraints, geomechanical considerations and meeting production requirements when developing a layout that maximises the economic outcomes from the orebody being mined. Geological uncertainty is a key element affecting the performance of mine designs and a topic of investigation as well as new optimisation developments. This is particularly so in open pit mining, where the adverse effects of geological uncertainty on mine designs, production performance and financial forecasts are documented. 1-3 To effectively accommodate mining considerations including geological uncertainty in generating optimal solutions to stope design, mathematical programming methods can be used.

Applications of mathematical programming in underground mining have been employed across various aspects of the underground planning process. More notably, Ovanic⁴ uses mixed integer programming (MIP) of type two special ordered sets to identify a layout of optimal stopes. Trout⁵ uses MIP to schedule the optimal extraction sequence for underground sublevel stoping, and Chanda⁶ employs MIP in conjunction

with simulation to generate a schedule for producing finger raises in an underground block caving situation. More recently, Mayer *et al.*⁷ show a case study on the commercial use of stope optimisation. Optimisation of infrastructure of underground mines has also been considered, particularly through Steiner minimal spanning trees.^{8,9}

A key characteristic of the conventional optimisation approaches, as above, is the use of complex, non-linear optimisation algorithms that assume certainty in their inputs, combined with a single, typically smooth, estimated representation of the orebody considered. This is shown to be a key limitation in the optimisation of open pit mine design and production scheduling, given the presence of uncertainty and in situ variability of orebodies that are also present in underground mining. For example, Dimitrakopoulos et al.1 use stochastic simulation of orebodies, ^{10,11} a geostatistical approach that generates realistic equally probable scenarios of the deposit under study, to quantify these limits. Their study shows predicted project Net Present Value (NPV) from a conventionally optimised mine design to have <5% probability to deliver its predicted NPV, substantial negative differences in expected quarterly discounted cash flows and a shorter life of mine. Recent work in assessing limits of conventional stope optimisation^{12,13} shows comparable results. More specifically, quantification of grade risk through stochastic simulations shows that even with drilling densities about three times the common practice, conventional approaches lead to stope designs with risk profiles that may vary as low as 75% below forecasts and 33% above. In general, the conventional approach arbitrarily under- or overvalues

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the performance indicators of stope designs and neither captures the actual economic potential of the orebody nor allows the user to generate stope designs with a user defined risk tolerance.

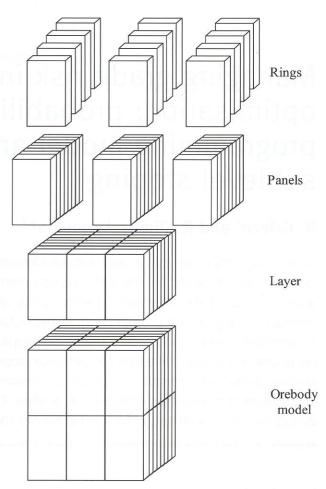
The ability to manage risk in mine designs through new optimisation formulations has been shown to generate substantially better performing mine designs and production schedules in open pit mines. Godoy and Dimitrakopoulos¹⁴ present a multistage optimisation approach that in a case study leads to an improved NPV by 28% compared to conventional approaches, while reducing the risk in meeting production targets. Dimitrakopoulos and Ramazan¹⁵ generate a probabilistic mathematical formulation for production scheduling that controls grade uncertainty through use of probabilistic optimisation and the introduction of the concept of 'risk discounting' that manages the distribution of risk over production time without financial penalties. These are conceptual developments worth further exploration in the context of underground mine design.

The present paper introduces a new probabilistic mathematical programming formulation integrating and managing risk when optimising underground open stoping situations. Acknowledging the significance of other sources of risk, including the prediction of underbreaking and overbreaking, the focus of risk in the present work is associated with grade uncertainty. The formulation aims to enable the management of grade risk so that designs respond to a predefined acceptable level of risk, while capturing the potential upside of the orebody. Geotechnical constraints relating to stope size limitations are also considered. The method proposed requires that the orebody under study is stochastically simulated¹¹ to facilitate the quantification of the related uncertainty and its subsequent integration to optimisation.

In the next sections, a new probabilistic mathematical programming formulation for stope design optimisation is first detailed. Then, an application of the formulation and its related intricacies are presented using data from Kidd Creek Mine, Ont., Canada, an underground open stoping operation. Comments and conclusions follow.

Stope design considerations

In establishing an optimal design layout of open stopes, the problem definition must aim to identify the optimal location, size and number of stopes within a given orebody model. In doing so a number of optimisation objectives can be defined including goals to achieve maximum dollar value, contained metal, contained tonnes or contained grade. To achieve any of these goals, an understanding of the stope layout and mining constraints are essential. Determining a minimum and maximum allowable stope size is largely based on the surrounding geotechnical issues as well as the production requirements such as material and equipment capability and access constraints. These sizes must allow for the safe extraction of ore, produce the required production with minimum dilution and reflect equipment capacities. Secondary extraction in the form of pillar recovery is another geotechnical issue that should be considered in a design, if relevant, and could be an influencing factor in determining stope size.



1 Components of required orebody model: modified after Ovanic, 1998

With the orebody model regularised to mineable rings, all major geotechnical and production concerns are been assumed incorporated. Such concerns include maximum stope span and required tonnes from a blasted ring. This allows geometrical constraints to be in terms of the minimum and maximum number of rings allowable for defining a given stope and pillar. In understanding the mathematical model in the next section, it is important to note the configuration of the orebody model as illustrated in Fig. 1, where each layer is composed of a number of rows or panels with each panel consisting of a series of rings. In addition, it should be noted that when considering a number of multiple simulated scenarios of the orebody, as in the present study, each ring can be represented by the probability of being above a specified cut-off and by the average grade of all conditional realisations of that ring above the cut-off; these are calculated from the simulated orebody models and are used to incorporate grade uncertainty in the approach developed herein. The approach allows for the generation of risk based stope and pillar designs based on varying levels of acceptable risk.

Mathematical model for generating risk based designs

The mathematical formulation presented herein encompasses the above operational constraints to generate a mathematical programming model that identifies an

optimal stoping layout in the presence of grade uncertainty. Identifying the optimal stoping layout involves selecting rings that together satisfy the constraints while maximising the objective. As a ring is either identified as part of the stope or not, each ring within the orebody model is itself a decision variable that can be represented in binary form. By varying the level of acceptable risk for which a stoping panel must satisfy, multiple risk based designs can be generated and analysed. Risk profiles of each design for a variety of project parameters can be compared by running them through all simulated orebody models, as in Dimitrakopoulos *et al*, and discussed in terms of upside potential and downside risk, as applied to any pertinent project indicator(s).

The objective function and constraints of the mathematical model developed herein are presented next.

Objective function

The objective function formulation is

Maximise
$$\sum_{j=1}^{m} \sum_{i=1}^{n} g_{ij} p_{ij} B_{ij}$$
 (1)

where m is the number of panels within the orebody model, n is the number of rings within a panel, g_{ij} is the average grade of ring i in panel j above a specified cutoff, p_{ij} is the probability of ring i in panel j being above the specified cut-off and B_{ij} is the decision variable representing ring i in panel j.

Assuming the density of the material is constant throughout the orebody model, this objective function essentially maximises metal content.

Model constraints

The objective function represented by equation (1) is subject to a series of system constraints. These include stope and pillar size constraints and an acceptable level of risk restriction.

The minimum stope size constraints ensure that a given stope consists of at least the minimum number of rings

$$\sum_{i=h}^{MN-2+h} B_{ij} - B_{(h+MN-1)j} - B_{(h-1)j} \leq (MN-2)$$
 (2)

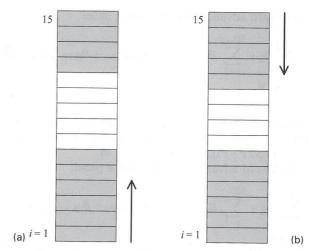
where h=1,...,(n-1); j=1,...,m; and MN is the minimum number of rings required in a stope. Any decision variable representing a ring outside the defined orebody model is considered a value of zero (e.g. B_{oj}).

The maximum stope size constraints restrict the number of rings within a given stope, so it does not exceed the maximum allowable

$$\sum_{i=h}^{MX+h} B_{ij} \leqslant MX \tag{3}$$

where h=1,...,(n-MX); j=1,...,m; and MX is the maximum number of allowable rings within the stope.

The acceptable level of risk constraint states that the probability of selected rings within a panel must be



a forward movement through given stoping panel; b backward movement through panel

2 Development of pillar constraints

greater than or equal to a specific probability representing the minimum acceptable level of risk

$$\sum_{i=1}^{n} (p_{ij} - PL) B_{ij} \geqslant 0 \tag{4}$$

where n is the number of rings within a panel j, p_{ij} is the probability of ring i in panel j above the specified cut-off and PL is the minimum acceptable level of risk for selecting stopes within a panel guaranteeing all rings within the stopes that have a combined probability greater than this value. This constraint allows multiple designs for a single cut-off to be created by varying the value of PL.

The size of the pillars (in terms of number of rings) to be left unmined between two primary stopes is dependent on the size of the adjacent stopes. The following pillar constraints have been formulated in such a way that for each of the potential stopes' sizes, between the minimum and maximum, an assigned number of subsequent rings must be left unmined. The number of rings is determined by comparing each allowable stope size to the average acceptable stope size. If a stope size is less than the average, the size of the pillar is equal to that of the stope, else it is the average between the size of the stope and average stope size rounded up.

As optimising the stope layout for a given panel could vary depending on which direction the set of constraints begins, equations written in reference to progressing through a given panel from both directions, forward and backward are required to ensure the larger pillar of two consecutive stopes is selected. This concept is illustrated through an example shown in Fig. 2 where a stope consisting of six rings requires a pillar of five rings and a stope composed of five rings demands a pillar of four rings. Under these conditions the stoping panel layout based on the forward advancement is selected as a stope size of six rings calls for a larger pillar than that of the five ring stope.

The formulations for each direction are written in terms of the number of rings within a given stope. When the number of contained rings are even (STP=2, 4, 6,..., MX), the formulations are written as follows.

Forward advancement through a panel

$$\sum_{i=h}^{h+1} B_{ij} + \sum_{i=1}^{\frac{STP}{2}-1} B_{(h+1+2i)j} - B_{(STP+h)j} + B_{(STP+PIL+h-1)j}$$

$$\leq \frac{STP}{2} + 1$$
(5)

where STP is the number of rings within a stope, PIL is the number of rings within a pillar, h=1,...,(n-STP-PIL+1) and j=1,...,m.

Backward advancement through a panel:

$$\sum_{i=h-1}^{h} B_{ij} + \sum_{i=1}^{\frac{STP}{2}-1} B_{(h-1-2i)j} - B_{(h-STP)j} + B_{[(h+1)-(STP+PIL)]j}$$

$$\leq \frac{STP}{2} + 1$$
(6)

where h=n, (n-1), ..., (STP+PIL) and j=1,..., m.

When the number of contained rings is odd, formulations are further subdivided in terms of progression through a panel beginning on even number rings and odd number of rings. The formulations for both the forward and backward advancement of odd size stopes on even numbered rings (STP=3,5,6,...MX and i=2,4,6,...,n) are written as follows

$$\sum_{i=h}^{\frac{1}{2}(STP+1)+(h-1)} B_{2ij} - B_{(STP+2h)j} + B_{(STP+PIL+2h-1)j}$$

$$\leq \inf\left(\frac{PIL+1}{2} + 0.5\right)$$
(7)

where

$$h=1,\ldots,\inf\left(\frac{n-\text{STP-PIL}}{2}+0.5\right)$$

and j = 1,, m.

Backward advancement through a panel

$$\sum_{i=h}^{\frac{1}{2}(STP+1)+(h-1)} B_{(RNG-2i+2)j} - B_{[RNG-STP-(2h-2)]j} +$$

$$B_{[RNG-(STP+PIL-1)-(2h-2)]j} \leq \operatorname{int}\left(\frac{PIL+1}{2} + 0.5\right) \quad (8)$$

where

$$h = 1, 2, \dots, \left(\frac{n - \text{STP-PIL}}{2} + 0.5\right)$$

and j = 1, ..., m.

When progression of odd size stopes is on odd numbered rings (STP=3,5,7,...,MX and i=1,3,5,...,n), the formulations are written as follows.

Forward advancement through a panel

$$\sum_{i=h}^{\frac{1}{2}(STP+1)+(h-1)} B_{(2i-1)j} - B_{(STP+2h-1)j} + B_{(STP+PIL+2h-2)j}$$

$$\leq \inf\left(\frac{PIL+1}{2} + 0.5\right)$$
(9)

where

$$h = 1, ..., int(\frac{n - STP - PIL + 1}{2} + 0.5)$$

and $j = 1,..., m$.

Backward advancement through a panel

$$\sum_{i=h}^{2(STR+I)+(h-I)} B_{(RNG-2i+1)j} - B_{(RNG-STP-2h+1)j} + B_{[RNG-(STP+PIL-1)-(2h-1)]j} \le int \left(\frac{PIL+1}{2} + 0.5\right)$$
(10)

where

$$h = 1, 2, \dots, \left(\frac{n - \text{STP} - \text{PIL} + 1}{2} + 0.5\right)$$

and j=1,..., m.

With the integer programming (IP) formulation just discussed defining an optimal stope design layout while considering grade variability and geotechnical limitations, the potential for secondary extraction of pillars is possible. Assuming the complete production of the primary stopes, the IP formulation can be rerun to determine a feasible pillar recovery layout by assigning all decision variables representing stope rings as zero. This will force the IP to consider only the pillar rings when selecting a layout in a similar manner as the primary stopes.

Application at Kidd Creek Mine, Canada

The practical aspects of the mathematical model presented above are shown in this section in an application using data from Kidd Creek Mine, Ont., Canada. Kidd Creek is one of the largest and richest copper–zinc–silver volcanic massive sulphide deposits in the world¹⁶ producing approximately 6000–7000 tonnes per day.¹⁷ The deposit has been divided into two major orebodies consisting of three main ore types: the stringer ore, massive banded and bedded ores, and breccia ore. Since its discovery in 1963 the mine has produced close to 160 000 000 tonnes of ore.¹⁷

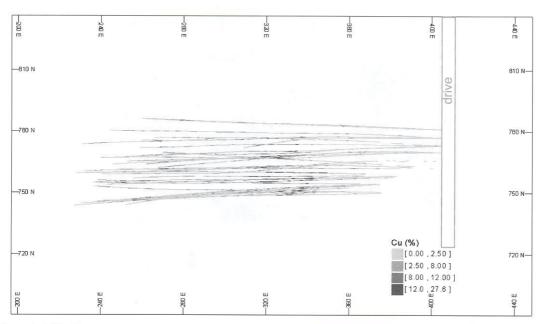
Various underground mining techniques are employed including sublevel caving, open stoping and sublevel stoping. The mine in which the data set was obtained is located 1400 m below the surface in phase 1 of the no. 3 mine. Mining this stage is via open stoping methods with stope sizes typically 15 m wide by 20 m long by 40 m high. Blasting rings above a 2.5% copper cut-off are generally spaced every three metres with a minimum of two rings and a maximum of seven rings defining a single stope. Following the convention used for determining pillar sizes, the subsequent number of rings contained within a pillar PIL for each acceptable stope STP size is indicated in Table 1.

Stochastic simulation of copper grades

The data used for the present study consists of copper assays taken from a comprehensive drilling campaign investigating the copper concentrated stringer ore type where 37 diamond drillholes were composited every 1.5 m. Drillhole fans extend ~100 m in the westerly direction, span 40 m in the northerly direction and reach heights close to 100 m (Fig. 3). These data were used to conditionally simulate 40 realisations utilising the

Table 1 Subsequent number of rings contained within pillar PIL for each acceptable stope STP size

50000000						
STP	2	3	4	5	6	7
PIL	2	3	4	5	5	6



3 Plan view of drillholes

generalised sequential Gaussian simulation method¹¹ on a 1.5 m by 1.5 m by 1.5 m grid. Figure 4 presents plan view images of 3 realisations for comparison. In validating the simulated realisations, Fig. 5 shows the reproduction of the data histogram and Fig. 6 the reproduction of variograms in the east-west, northsouth and vertical directions. Simulated realisations are reblocked to mineable rings producing orebodies with block sizes of 15 m wide by 20 m long by 40 m high. A single orebody where each ring is represented by the average grade of all simulated orebodies for that ring above the 2.5% cut-off and the associated probability of the rings being above the cut-off provides a means of quantifying grade risk for use in the mathematical model. Figure 7 represents the orebody in ring layout for the study area showing the model with its two layers each comprising of eight potential stoping panels which themselves contain 18 rings.

Design analysis

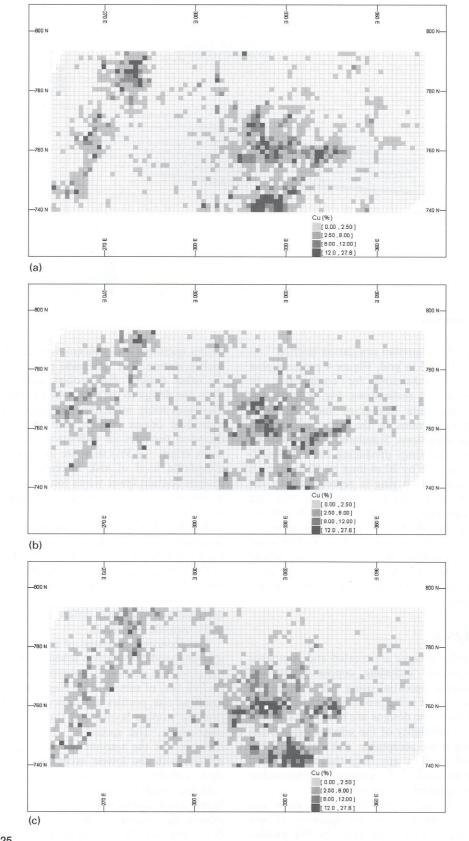
Five stope and pillar designs are created and compared for project parameters including copper grade, ore tonnes and economic potential using risk profiles. To observe the sensitivity of using probabilistic information for grade uncertainty, the designs are based on a 40, 60, 80, 90 and 100% probability of the selected rings being above these minimum acceptable risk levels. When producing the risk profiles, any rings that are less than the cut-off in all the simulated orebodies are removed to provide an analysis based on ore rings only.

The difference in size/volume of the five stope and pillar designs is illustrated by the quantity of ore (tonnes) within each. Figure 8 provides the risk profiles of each stope and pillar design respectively for the present study parameter, with the black diamonds indicating the size of the designs before all waste rings were removed. These graphs show how the spread of the risk profiles decreases with increasing acceptable levels of risk. Although the design with 100% average probability of selected rings being >2.5% is the most confident in the amount of ore it expects to contain, this

design sacrifices a significant volume of ore to achieve such a stature. The downside potential experienced by the other designs' risk profiles show small differences from one design to the next, whereas the upside potential illustrates more influential differences. Comparing the individual design profiles with their associated black diamond provides an indication as to the quantity of waste (rings <2.5%) they are likely to contain. As the acceptable level of risk increases, the corresponding designs contain fewer waste tonnes as the selected rings in the majority of simulated orebodies are above the cut-off.

Another key parameter in terms of meeting mill requirements is the uncertainty in the copper grade. Figure 9 demonstrate the increase in the average grades of the stope and pillar designs respectively with increasing acceptable risk levels. The risk profiles for designs in Fig. 9 individually illustrate a similar spread and demonstrate negligible differences within their upside and downside potentials.

The economic potential of each risk based design is also analysed. Production scheduling is not considered; therefore the economic potential represents the profit (before tax) of mining each ring within the design. The resulting risk profiles are presented in Fig. 10a for the primary stoping layout and the pillar recovery outline respectively. The spread of the designs' risk profiles observed for the amount of contained ore showed the designs with the lowest acceptable level of risk to be more variable and contained the most waste compared with the other designs. Despite the potential to achieve higher ore quantities with these designs, the burden of mining the extensive waste has proven too influential. Not only do these designs have an average value noticeably less than the others, but also there is a potential to lose money if they are mined. The design based on 100% average probability of selected rings being >2.5% has illustrated how a design, though very conservative in terms of ore tonnes, can in fact produce a design that is comparable in worth to the other designs based on a high level of acceptable risk. The designs with 80, 90 and 100% average probability of selected rings

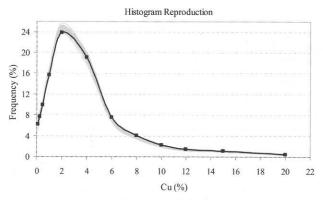


a 3; b 18; c 25
Simulated copper realisations

being >2.5% all have very similar average values and vary slightly in their upside and downside potentials. Although the design with 100% average probability of selected rings being >2.5% exhibits risk profiles with the smallest spread and the most desirable downside

potential, it does not allow for the potential to earn as much as the designs with 80 and 90% average probability of selected rings being >2.5%.

The variability experienced by the parameters of the pillar recovery designs was generally more substantial



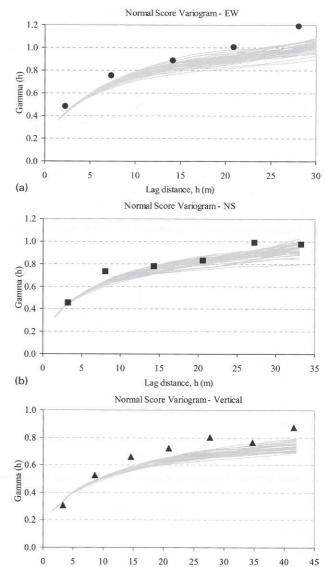
5 Data histogram (black squares) and histograms of 40 simulated copper realisations (grey lines)

with reference to the overall size of the risk profiles, due to the limited selection of remaining rings to choose from. This observation has been accentuated by the risk profiles of their economic potential shown in Fig. 10b. All designs except the design with 100% average probability of selected rings being >2.5% are most likely to produce negative cash flows, which would force the engineer to reconsider their possible extraction.

Conclusions

The present paper has presented a new probabilistic optimisation approach to stope design that accounts for orebody uncertainty and risk in mine planning. The development of a mixed IP formulation to optimise stope design and in terms of size, location and number of stopes, provided a platform which allowed the consideration of grade uncertainty and the concept of a predefined level of acceptable risk. Unlike any conventional stope optimisation approach, the stope designs based on the concept of acceptable risk gives the mine planner control over the final stope layout and its potential future performance while considering grade uncertainty. The application of the proposed approach is based on the ability to stochastically simulate equally probable representations of the deposit.

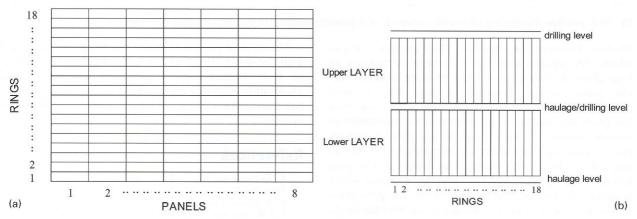
The application of the proposed approach at Kidd Creek Mine, Ont., Canada demonstrated how it might assist the planner in considering various pertinent project parameters and how to assess their effect on several aspects of the mining operation. Production rates



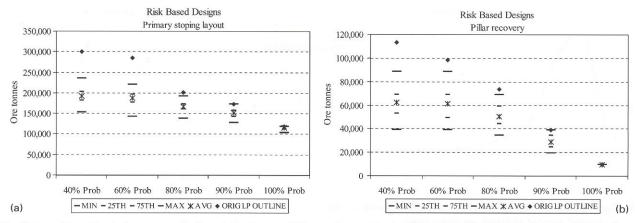
6 Reproduction of variograns by simulated copper grades: conditioning data (black circles or squares) and simulated realisations (grey lines) in a east-west direction, b north-south direction and c vertical direction

Lag distance, h (m)

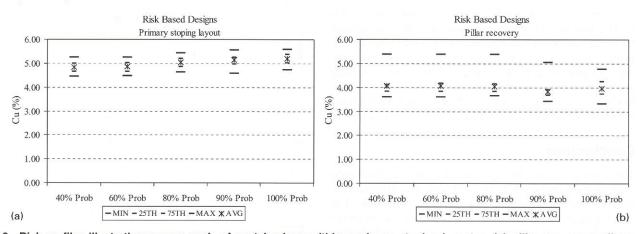
and average grades for processing throughput can play a vital role in the design of a stoping layout as fluctuations outside the desired manageable range can lead to



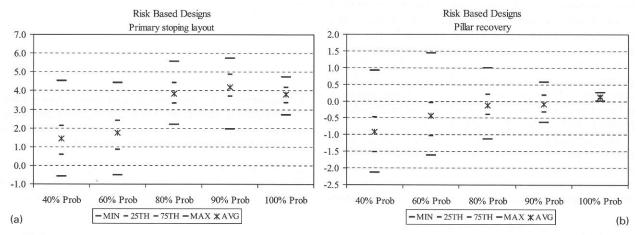
a plan view showing rings within panel for single layer; b section view showing rings within two layers of Kidd Creek model
 Schematic representation of ring configuration



B. Risk profiles illustrating amount of contained ore within a primary stoping layout and b pillar recovery outline



9 Risk profiles illustrating copper grade of contained ore within a primary stoping layout and b pillar recovery outline



0 Risk profiles illustrating economic potential of a primary stoping layout and b pillar recovery outline

financial losses in the long run. High grading, such as mining the design with 100% average probability of rings above a desired cut-off may produce an economically viable design; however, it may fail to meet production demands as a result. Blending high with low grade ore may be an accepted practice and therefore various amounts of waste may be tolerable. A percentage of waste may be required from production stopes to help meet a backfilling quota. The design of a stoping layout is not as simplistic as selecting a design which produces the highest dollar value. Last, the perception of risk will vary from one company to another hence the importance of understanding and interpreting the concept of risk in the design process.

The method presented herein can be further advanced with the direct use of all available simulated orebodies, so as to further capture geological information and uncertainty, consider sequencing and thus accommodate risk management as part of a more complex stope design process.

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