Pushback Design of Open Pit Mines Under Geological and Market Uncertainties

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ABSTRACT

Strategic open pit mine planning is carried out under multiple sources of technical and financial risks. Given the uncertainty about ore grade, metal prices and foreign exchange rates, assigning dollar values to mining blocks at the planning time in order to optimise block destinations (mill, stockpile or waste) is a challenge. In such an uncertain environment, there is a probability that the actual grade, metal prices and exchange rate at the production time will be different from those based on which an optimum block destinations have been chosen at the planning time. In such cases, the mine management has the flexibility to revise the original decision regarding the block destination that has been made at the planning time and send the block to the optimum destination based on the new information. Another issue that affects the block value and consequently the optimisation process is the fact that the value of a block extracted sooner is greater than the value of the same block extracted later. This time value of money is usually ignored in block valuation. However, it could have a significant impact on the optimisation process.

This paper aims to quantify the value of management flexibility to revise or alter original decisions and integrate it into the dollar value assigned to each mining block at the planning time, while addressing the time value of money concept. In this respect, this work outlines real options valuation (ROV) approach for jointly handling stochastically described geological and market uncertainties and integrating the block destination flexibility in the process of assigning a value to mining blocks at the planning time. The output of this ROV model is a time-dependent discounted expected value for each block that captures the value of management flexibility rather than a single static value. A parametric minimum cut algorithm is then applied to produce single pushback designs. The algorithm used intrinsically leverages the uncertainty in a set of simulations of the orebody to produce low risk open pit pushback designs. The method used for optimising over multiple realisations of an orebody, as opposed to single geostatistically estimated block values and optimising over a single orebody model, leads to an increase in the probability that the pushbacks produced for the earlier stages of the mine meet their expected value.

INTRODUCTION

The aim of open pit mine planning is to define optimum pit limits and an optimum life-of-mine (LOM) production schedule that maximise the pit value under some technical and operational constraints. The basic input to this process is a set of block values representing the net economic worth of each block. Based on the estimated block values, the optimiser selects the optimum destination of each block so as to maximise the overall pit value under some given technical constraints. A dollar value is usually assigned to each block by estimating the revenue of recoverable metal at a given fixed metal price and subtracting applicable mining, processing and other costs. Therefore, the value of each mining block depends, among other factors, on its metal content, metal prices and foreign exchange rates.

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In conventional open pit planning, block values are estimated using a single point estimate of metal content (ore grade), a fixed flat metal price and a fixed exchange rate (for example, Caccetta and Giannini, 1988; Ramazan, 2007). An example of commonly used formulas for estimating block value can be expressed as (Whittle, 1988, 1999; Hochbaum and Chen, 2000):

$$V = T_o GRP - T_o C_p - TC_m \qquad (1)$$

where:

V = block value, $

T_o = tonnes of ore in the block

G = grade, unit/tonne

R = recovery

P = unit price, $/unit

C_p = processing cost, $/tonne

T = total amount of rock (ore and waste) in the block

C_m = mining cost, $/tonne

Block value estimations using current, common conventional procedures are based on three main implied assumptions:

1. the ore grade or metal content of each block is known with certainty,
2. market variables such as metal prices and exchange rates are known with certainty, and
3. at the project evaluation stage it is assumed that there are no possible future revisions of decisions related to optimum block destinations.

This means that the decision whether to send a block to the mill or to the dump has to be taken at the time of planning for all blocks and no future revisions to these decisions are allowed. In other words, at the project evaluation stage, it is assumed that all decisions regarding block destinations have to be made upfront. These observations have also been reported by Henry, Marcotte and Samis (2004).

The first assumption indicates that there is perfect knowledge about metal content of mining blocks. This clearly contradicts the fact that some, but not complete, information about metal content is provided by borehole data and other exploration techniques. These data are then interpolated and/or extrapolated to develop orebody models using geostatistical modelling techniques. This usually results in a single, average value estimate for the metal content of each mining block. Given the limited information and the embedded assumption about the mineralised material between boreholes, the probability that the estimated metal contents will be realised is small. Accordingly, there is a high probability that the actual metal content will be different from that used to calculate block values. Figure 1 shows the estimated average copper grade of a mining block at a copper deposit, as well as 20 equally possible realisations. It is obvious that the actual block grade could be significantly higher or lower than the estimated average. Given the highly variant possible outcomes in Figure 1, expressing block grade in a single number oversimplifies reality and could lead to erroneous block valuations.

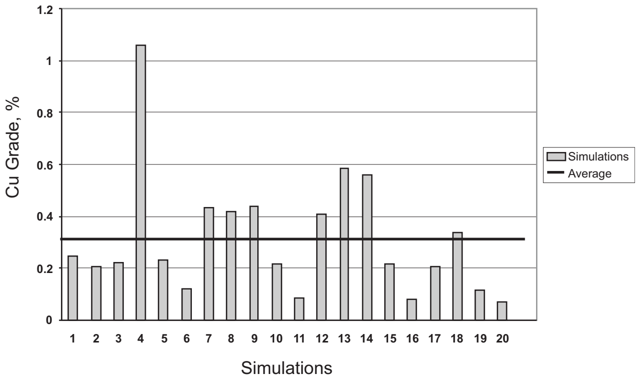

FIG 1 - Possible Cu grade of a mining block.

Ignoring grade uncertainty could lead to suboptimal mine plans with significant deviations from production targets. The ultimate outcome has an adverse effect on the project bottom line since project value was optimised on the basis of inaccurate inputs. The significance of modelling and integrating geological uncertainty into open pit mine planning has been emphasised in Dimitrakopoulos Farrelly and Godoy (2002) and Dimitrakopoulos, Martinez and Ramazan (2007). Uncertainty about ore grade was identified as a critical source of risk affecting mining project viability. Godoy and Dimitrakopoulos (2004) and Leite and Dimitrakopoulos (2007) show through case studies that integrating geological uncertainty in open pit mine planning significantly reduces the risk of deviation from production targets and could result in 26 - 28 per cent increase in project value.

The second implied assumption in conventional open pit mine planning is that market variables such as metal prices and exchange rates are fixed, ie do not change throughout life-of-mine (LOM) and are known with certainty. Obviously, this assumption is far from realistic – looking at the past history of metal and currency markets, it is not difficult to conclude that the probability of metal prices and exchange rates remaining unchanged is null in both the short and the long-term. Therefore, assuming that these variables will be constant throughout a moderate LOM of, for instance, five to ten years will most likely result in either over- or under-valuation of mining blocks. In both cases, deviations from scheduled capacity and suboptimised project value are likely outcomes. It is worth noting here that when evaluating projects in countries other than the USA, it is important to model the uncertainty of both metal prices and the exchange rate. Modelling uncertainty of metal prices expressed in the local currency and assuming that this also covers uncertainty about exchange rate might be inappropriate. This is because metal prices are governed by the global supply and demand in the world markets, while the exchange rate is governed by economic variables specific to the two countries.

The third assumption indicates that mining blocks throughout LOM will be sent to the destinations optimised at the planning time with no possible future revisions. This is obviously a logical result of the first and second assumptions about certain ore grade and market variables. In other words, if ore grade and market variables throughout LOM were known with certainty at the planning time, it would be possible to optimise block destinations and consider this a final decision. In such a case, it would be unnecessary to review the preoptimised block destinations in the future since no changes were made to the input data based on which these destinations were optimised. Therefore, it could be concluded that the third assumption would be realistic if the first and the second assumptions are realistic. Since, as described above, block grades and market variables are highly uncertain and consequently block values are uncertain, the assumption that there is no revision to previously optimised block destinations could be erroneous. To clarify this point, assume for example that both the copper price at the optimisation time and its expected long-term level equal US$2.00/lb. Figure 2 shows ten paths over a period of five years. Each path represents a possible scenario for the future copper prices. Based on conventional mine planning procedures, a fixed copper price of US$2.00 will be used to calculate block values based on which optimum decisions will be made at Year 0, for each block, whether to send it to the mill or to the dump. Consider blocks scheduled to be extracted in Year 5 and classified as waste based on a constant copper price of US$2.00/lb – will they be sent to the dump in Year 5 even when the copper price follows the path represented by Realisation-4 in Figure 2? On the other hand, for blocks that are scheduled for the mill in Year 5 at the price of US$2.00/lb, will the operator proceed with this schedule without re-checking even if the actual copper price follows the path in Realisation-6? Obviously, what happens in reality is that mine operators usually revise previously taken decisions with time based on the new information. This is usually carried out by re-running the optimiser at a regular time intervals or when major changes to the key variables take place. It is important here to note that, standing at the early project evaluation stage, the conventional optimiser cannot bring such a flexibility to modify block destinations in the future.

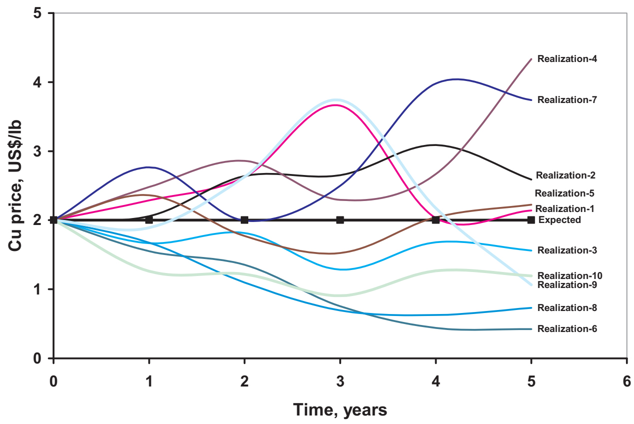

FIG 2 - A sample of possible copper price realisations.

Previous work of integrating uncertainty in open pit mine planning has been carried out by Dowd (1997), Dimitrakopoulos, Farrelly and Godoy (2002), Godoy and Dimitrakopoulos (2004), Leite and Dimitrakopoulos (2007) and Dimitrakopoulos, Martinez and Ramazan (2007). Their focus was on integrating geological uncertainty into long-term mine planning. Handling both metal price and exchange rate uncertainty has been dealt with in literature focusing on real options applications to mine project valuations, which includes, among others, Brennan and Schwartz (1985), Trigeorgis (1993), Moyen, Slade and Uppal (1996), Kelly (1998), Moel and Tufano (2002), Monkhouse and Yeates (2007), Abdel Sabour and Poulin (2006) and Samis *et al* (2006). The focus of this real options work was on market uncertainty – geological uncertainty was not considered. Work that focuses on both geological and market uncertainty includes Henry, Marcotte and Samis (2004); Dimitrakopoulos and Abdel Sabour (2007).

This paper aims to integrate geological and market uncertainty as well as operating flexibility to revise block destination in open pit mine planning. This will contribute to enhancing the planning process in a number of ways. First, stochastic models for market variables that capture both variability and volatility will be adopted in mining block valuation rather than using a fixed price and exchange rate as in conventional analysis. Second, the value of management flexibility to change decisions regarding block destinations in the future based on prevailing market conditions is integrated and included in block value estimates. Third, pit planning will be carried out using discounted block values that account for the time value of money. The next section outlines the proposed methodology, followed by a case study which provides a practical application, and finally, conclusions and recommended future extensions are presented.

UNCERTAINTY-BASED MINE PLANNING APPROACH

The proposed approach consists of two optimisation stages:

1. a dollar value is assigned to each mining block, considering the different aspects of uncertainty and management flexibility to revise block destination at the production time according to the realised market variables; and
2. the second stage includes a new method for designing a lower risk long-term mine plan based on a generalisation of the minimum cut algorithm for ultimate pit design.

The method uses the simulation and block valuation data from the first stage to decide when best to extract a given block to reduce risk and increase profit.

Block valuation

As described above, ore grade and market variables are highly uncertain and cannot be defined with a point estimate. However, modelling uncertainty of these variables does not provide much useful insight nor does it make significant differences in open pit mine optimisation unless accompanied by modelling management actions and responses to uncertainty resolutions. To illustrate the effect of integrating uncertainty and flexibility into mining block valuation, consider the simple case provided in Table 1 for a mining block of 10 000 tonnes scheduled for production after five years. For simplicity, assume that copper grade, copper price and CAN$/US$ exchange rate can be one of the two possibilities outlined in Table 1. Figure 3 shows valuation results for this block in four cases. In Case 1, no uncertainty is considered and the block has been valued using conventional methods where ore grade and market variables are assumed to be known with certainty. Block value corresponding to this case is represented by the straight line called 'expected, static' in Figure 3. In this case, there is only a single value estimate for the block since all variables are assumed to be known with certainty. In Case 2, only ore grade uncertainty is considered while both copper price and exchange rate are assumed to be known with certainty. Since uncertainty is considered, flexibility of management to take action is integrated as well. As shown in Figure 3, the expected block value in this case is 62 per cent higher than that in Case 1. This result agrees conceptually with the findings of Godoy and Dimitrakopoulos (2004) and Leite and Dimitrakopoulos (2007). Also, in this case, possible block value is spread over a range rather than a single

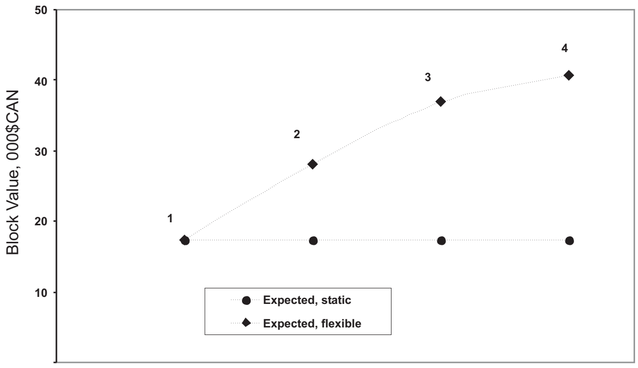

FIG 3 - Comparison between the conventional and the uncertainty-based block valuations.

value, as depicted in Figure 4. In Case 3, both ore grade and copper price uncertainties are considered while exchange rate is assumed to be certain. In this case, the expected block value is 112 per cent higher than that in Case 1, as indicated in Figure 3 and the range of block value is wider than that in Case 2, as shown in Figure 4. Moving to Case 4, where uncertainty related to all variables is considered, the expected block value is 134 per cent higher than that in the deterministic case. It is worth noting here that since management has the flexibility to revise block destination according to real time information, minimum block value is limited to the mining cost, incurred to give access to valuable block underneath, while maximum value is unlimited and is directly related to the three uncertain variables.

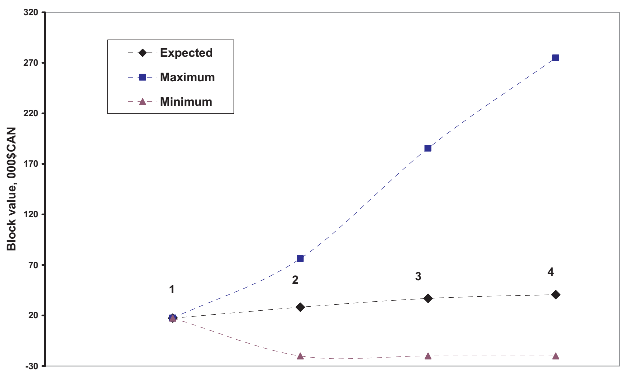

FIG 4 - Valuation of a mining block in a hypothetical copper deposit.

TABLE 1

An illustrative example for a hypothetical block in a copper deposit.

Mining cost, CAN$/t	2
Processing cost, CAN$/t ore	10
Process recovery	0.9
Treatment, CAN$/t concentrate	80
Refining, CAN$/lb copper	0.1
Concentrate grade, %	30
Ore grade, % Cu	0.2 or 0.5 (Expected = 0.35)
Cu price, US$/lb	1.00 or 3.00 (Expected = 2.00)
Exchange rate	0.7 or 1.25 (Expected = 0.9)

The first step in block valuation is to model market uncertainty. In this respect, a variety of stochastic models are used to describe the evolution of metal prices and exchange rates with time. Among them, the mean-reversion model proposed by Schwartz (1997) is widely used since it reflects the cyclic nature of metal and currency markets and estimating its parameters statistically using historical data is straightforward. Schwartz's model can be expressed as:

$$dS\ /\ S = \eta(\mu - \ln S)dt + \sigma dz \qquad (2)$$

where:

S is the spot value of market variable

μ is the logarithm of the long-term equilibrium level

η is the reversion speed

σ is the standard deviation

dz is an increment in a standard Wiener process

More details about stochastic models can be found in Schwartz (1997) and Dixit and Pindyck (2004), among others. Based on the stochastic model described above, a large number of correlated realisations of market variables are generated at each period.

The second step is to evaluate each mining block, given the generated realisations of market variables, simulated orebody models, mining cost, processing cost and all other refining and smelting terms. At this time, a first level optimisation process is carried out to define the optimum destination of each block that maximises block value. In this respect, ROV can be applied to integrate the value of management flexibility to choose block destinations at the actual extraction time so as to maximise block value. A comparison is carried out between the value of sending each block to the dump or to the processing plant. The value of a block if it is sent to the dump is simply the mining cost:

$$V_D = C_M T \tag{3}$$

where:

V_D is the block value if it is sent to the dump

C_M is the unit mining cost

T is the block tonnage

The block value if it is sent to the processing plant is a function of metal(s) price(s), exchange rate, and metal content, in addition to mining, processing, smelting and refining costs such as:

$$V_P = F(S, M_C, R, FOREX, T, C_M, C_P, C_S, C_R) \tag{4}$$

where:

V_P is the block value if sent to the processing plant

S is the metal price

M_C is the metal content

R is the recovery

FOREX is the exchange rate

C_M is the unit mining cost

C_P is the unit processing cost

C_S is the smelting cost

C_R is the refining cost

The decision regarding optimum block destination at the extraction time is taken so as to maximise block value:

$$V_B = \max\left(V_D, V_P \middle| S, FOREX, M_C\right) \tag{5}$$

It is worth noting here that the block value V_B is conditional on the simulated realisation of metal price and exchange rate at time t and the simulated orebody model. This process generates distribution for block value at all possible extraction times. At this stage, no operational or technical constraints such as mill capacity and angle of slope are considered. All of these constraints will be taken into account in the second stage of optimisation, explained below.

Parametric minimum cut for multiple realisations and block valuations

Traditional methods of long-term mine planning are geared towards a single orebody model. From the single orebody model, ultimate pit limits are calculated using the slope constraints and a fixed economic value per block by the Lerchs-Grossman algorithm. The ultimate pit is then broken up into continuous smaller sections known as pushbacks, phases or cut-backs these sections have the property that if removed in the appropriate order they obey the engineering slope requirements for the pit. These pushbacks are often produced using a Lerchs-Grossman ultimate pit type algorithm with the orebody model scaled by some parameter. Through scaling, the ultimate pit algorithm can produce a series of nested pits which can be used as possible choices for pushbacks.

With recent advances in simulation techniques, new methods for producing ultimate pit limits and pushbacks are needed for multiple realisations of the same deposit. Averaging the multiple realisations into a single model and using the traditional techniques does not leverage some of the upside available from the simulations. The method described is an extension of the network flow/minimum cut approach to ultimate pit design (for example, Hochbaum and Chen, 2000).

Given an orebody model and economic values associated with each block and a designation as either waste or ore, the minimum cut algorithm for producing an ultimate pit begins by constructing a directed graph G (for a background in graph theory and a definition of term, see Diestel, 2005). The graph G consists of a node for each block in the orebody model and two extra nodes, a source node s and a sink node t. If a block bi is designated as ore an arc from the source node s to the node representing bi is present in G, the capacity of the arc (s, bi) is equal to the economic value of block bi. If a block bi is designated as waste an arc (bi, t) from the node representing block bi to the sink t is in G, the capacity of this arc is the absolute value of the economic value of mining block bi. Slope constraint are represented as arcs from a node vi to a node vj if block bj must be removed prior to block bi. Slope constraint arc have an infinite capacity.

Figure 5 shows a small 2D example of a potential block model with economic values associated with the blocks. Figure 6 shows the graph that would be constructed from that example, not the slope constraint arcs have infinite capacity (and no labels).

1	-2	-2	-2	-2
	5	6	-3	
		4		

FIG 5 - A 2D vertical cross-section of a block model.

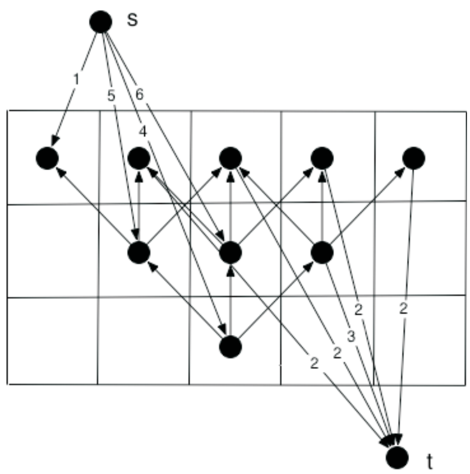

FIG 6 - The graph G constructed from the example in Figure 5.

A cut of a directed graph is a set of edges such that after the removal of these edges no directed path exists between the source node s and the sink node t. A minimum cut is the set of edges where the sum of capacities is as small as possible over all cuts in the graph. Many efficient algorithms are known for computing the minimum cut of a graph in polynomial time and they perform well in practice.

A minimum cut, F, in the graph G corresponds to a valid pit, no slope constraint can be contained in the set F or else the sum of capacities would be infinite but choosing the set of edges leaving the source s is a smaller cut, a contradiction to the minimality of F. This implies that the set of blocks corresponding to nodes not reachable from s in G - F forms a valid pit in terms of slope constraints. By the way the arc capacities were specific, the minimum cut corresponds to the pit where the sum of the capacities of arcs from the source to ore nodes left outside the pit (s, bi) plus the sum of capacities of arcs from the waste nodes inside the pit to the sink (bj, t) is minimised. This minimises the sum of ore left outside the pit plus waste inside. Since the total sum of ore inside the whole orebody model is a constant, this is equivalent to maximising the ore inside the pit minus the waste inside the pit. Figure 7 shows the minimum cut in our example, the minimum cut depicted has value 4 + (2 + 2 + 2) = 10 and minimises the value ore left outside the pit plus the cost of the waste in the pit.

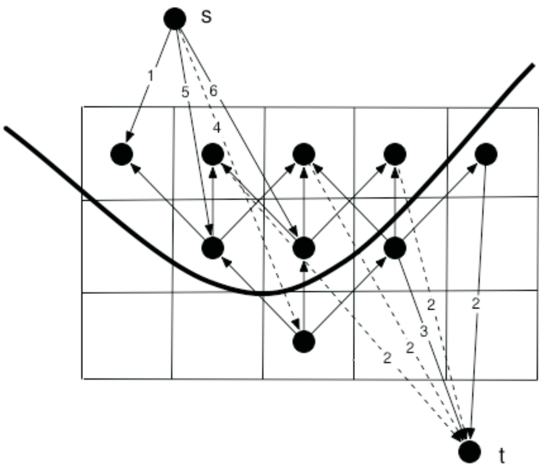

FIG 7 - The minimum cut in the graph G.

The same algorithm can be used to produce a series of nested pits similar to those produced by Whittle (1999). A parametric minimum cut algorithm replaces the capacities on the arcs leaving the source and arcs entering the sink with functions of a single parameter λ. The capacities on arcs leaving the source s must be non-negative non-decreasing functions of λ and the arcs entering the sink t must be non-negative non-increasing functions of λ. With these properties, a series of nested pits can be produced for $\lambda_1, \lambda_2, \ldots, \lambda_k$ where $\lambda_1<\lambda_2<\ldots<\lambda_k$ efficiently. For a small value of λ a small pit will be produced, as λ is increased larger and larger nested pits will be created until the ultimate pit is reached.

Given multiple orebody realisations, one would ideally like a long-term mine plan that met production targets and achieved a high net present value (NPV) over all realisations. A standard approach to produce a mine plan from multiple simulations is to average the realisations into a single orebody model and produce the long-term mine plan in the traditional way. The problem with this approach is that much of the information on risk due to geologic local variability is lost when the realisations are averaged together. For example, if you had a block worth -$3000, -$3000 and $9000 in three different realisations then the average value would be $1000. Similarly, a different block may be worth $1000 in all three realisations and would be equal to $1000 when averaged together. There is no way to distinguish that there is a lower risk associated with the second block than the first. Consider a small 2D example constructing the graph associated with each simulation and merging the source nodes s and sink nodes t, the resulting graph would look like Figure 8 (the slope constraints have been omitted from the drawing).

Simple parametric functions on the arcs from the source will be considered, mainly the economic value of the associated block multiplied by λ. Since the same decision must be made in the mine plan for a given block across all simulations, the nodes in the different simulations should be on the same side of the minimum cut, this can be modelled by placing bidirectional infinite arcs between corresponding nodes. Since these bidirectional arcs have infinite capacity they will never be in the minimum cut, so the nodes can be merged into a single node. Arcs from s to nodes that were merged can be replaced by a single arc with the sum of the capacities. Arcs to t from the merged nodes can also be replaced by a single arc with the sum of the capacities. The graph on the left of Figure 9 depicts the merged graph from Figure 8. The graph on the right of Figure 9 is the graph if the simulations were averaged into a single orebody model.

The graph constructed in the fashion described contains more information from the simulations than the graph constructed from the averaged simulations. If one had to decide between the block b1 that had values worth $-3000, $-3000 and $9000 in three different realisations and the block b2 worth $1000 in all three realisations, b2 would be chosen since the difference between including b1 and not including it is (9000λ) - (6000), while the difference between including b2 and not including it is 3000λ, for $0<\lambda<1$, $9000\lambda-6000<3000\lambda$. The goal of this

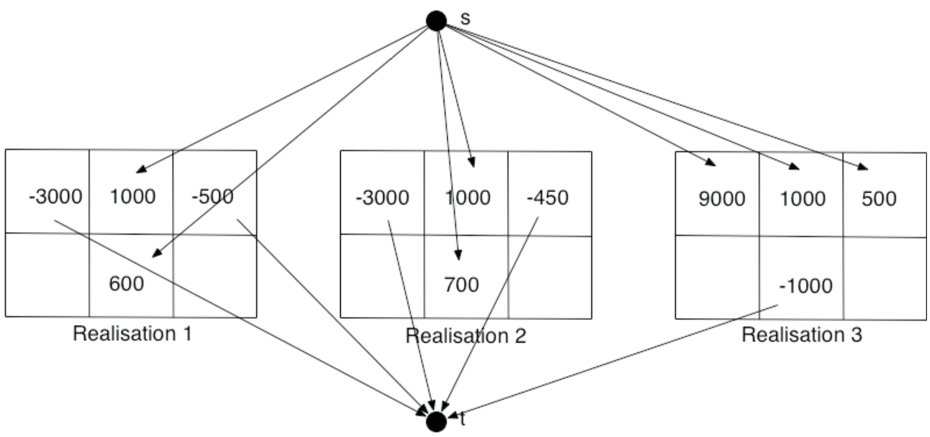

FIG 8 - Minimum cut graphs with source S and sink T merged.

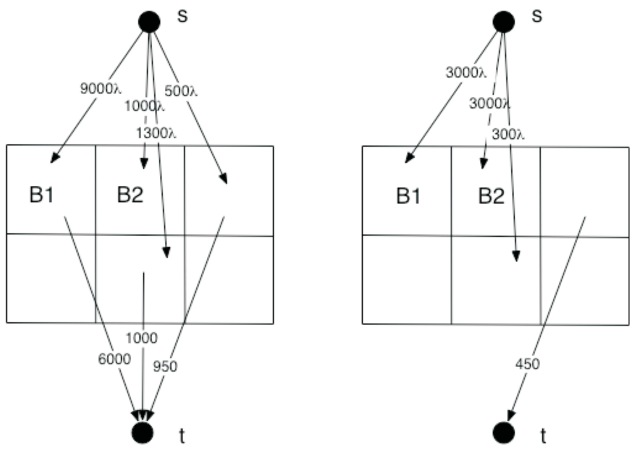

FIG 9 - The graph on the left is constructed using the simulation data, the graph on the right is the graph constructed if the simulations were averaged together.

modelling is to favour blocks that have a higher probability of being ore, this approach may be too conservative in some cases. For instance, a block could have a greater than 50 per cent chance of being more valuable than another but also a greater probability of being waste, this approach will favour the block with the higher probability of being ore. An interesting area of further research would be to try and find a way to control the balance between risk and reward.

By modelling the graph appropriately, parametric minimum cut algorithms can be used to incorporate distinct prices based on the period of extraction. Given a series of decreasing block prices p(1,i)>p(2,i)>...>p(k,i) for an ore block i and periods 1 to k, the graph G will have one node per period for each block i. Let v(j,i) be the node representing block i in period j, if j < k then arc (s, v(j,i)) will have capacity p(j,i) - p(j + 1,i) which is positive since p(j,i)>p(j + 1,i). Let arc (s,(v(k,i)) have capacity p(k,i). If an ore block i is removed in period j the arcs from s to v(l,i) for l<j will cross the cut and the sum of the capacities on these arcs represents the loss of revenue associated with waiting until period j to remove the ore. For a waste block i and decreasing costs associated with removing the block p(1,i)<p(2,i)<…< p(k,i) for periods 1 to k, the arcs from v(j,i) to the sink t will have capacities $|p(j,i) - p(j+1,i)|$. The arc from node v(k,i) to t will have capacity $|p(k,i)|$. If the waste block i is removed in period j, the sum of the arcs that cross the minimum cut associated with block i equals the cost of removing block i in period j. The graph G contains infinite capacity arcs from every node to the corresponding node in the subsequent period, ie (v(j,i), v(j+1,i)) for all i and j<k. This ensures that a block that is removed in a period stays removed in subsequent periods. Infinite slope constraint arcs exist between nodes in the same period.

Consider the following example, where we are given the horizontal cross-section of an orebody block model's values in three different periods. Figure 11 shows the arcs associated with the upper left most block. If we remove it in the first period, no cost is added to the minimum cut. If it is removed in period 2 the arc with value 2000 - 1800 crosses the cut and represents a loss of a potential $200. Similarly, if we remove it in period 3 the arcs with capacity 2000 - 1800 and 1800 - 1600 cross the cut representing a loss of $400. If the block is left in the ground the block all three arcs from s to the block cross the cut representing a loss of $2000.

By converting the network described into a parametric network by multiplying the capacity of arcs from s to a node in period i by λ_i and running a parametric minimum cut algorithm choosing appropriate values of λ_i for each period, one can produce a series of nested pits over the multiple periods and choose the pits that best meet the production schedule requirements as the pushback design. The choice of λ_i's can be accomplished by a Lagrangian relaxation approach. Where the λ_i's for which the pushbacks closely match the desired requirements are chosen. The two generalisations of the minimum cut algorithm, to multiple realisations of the orebody model and to block evaluations based on time of extraction, can combined to produce a set of pushbacks with a lower risk than the traditional single orebody model approach.

CASE STUDY – APPLICATION TO A COPPER DEPOSIT

This section provides an application of the proposed procedures explained above to a copper deposit. The deposit is located in a typical archean greenstone belt. The region consists predominantly of mafic lavas with lesser amounts of intermediate to felsics volcaniclastics. Rocks are moderately deformed with a prominent cleavage subparallel to what is considered to be the original bedding, an E-W trend with average 64° south. The deposit itself is in a sequence of moderately to strongly foliated, sulfidic, mafic to intermediate volcanic rocks, which have been intruded by numerous subvolcanic felsite and feldspar porphyry and/or intermediate volcanic tuff, with size ranging from lapilli to agglomerate, within a strongly chloritic and biotitic matrix. It can be traced over a strike length of 1.5 km with a thickness varying from a few metres to more than 75 m. Mineralisation consists of about ten per cent sulfides, mostly chalcopiryte, pyrite and pyrrhotite, occurring as disseminations, streaks and stringers apparently controlled by the strong rock cleavage. The geological database is compounded by 185 drill holes with ten metre copper composites in a pseudo-regular grid of 50 m × 50 m covering an approximately rectangular area of 1600 × 900 m^2 – the average dip is 60° north. Using the geological information, one mineralisation domain is defined and modelled through a geostatistical study. In this example, there are two sources of market uncertainty, related to copper price and US$/CAN$ rate. This market uncertainty is modelled by generating 20 000

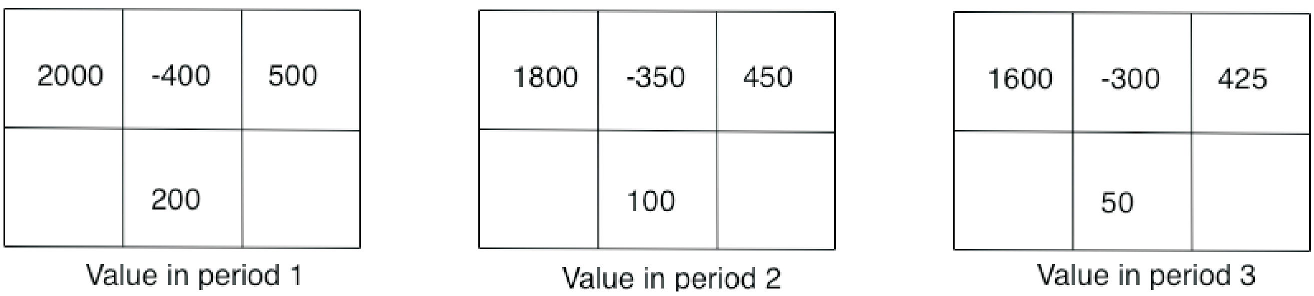

FIG 10 - Values over three different periods.

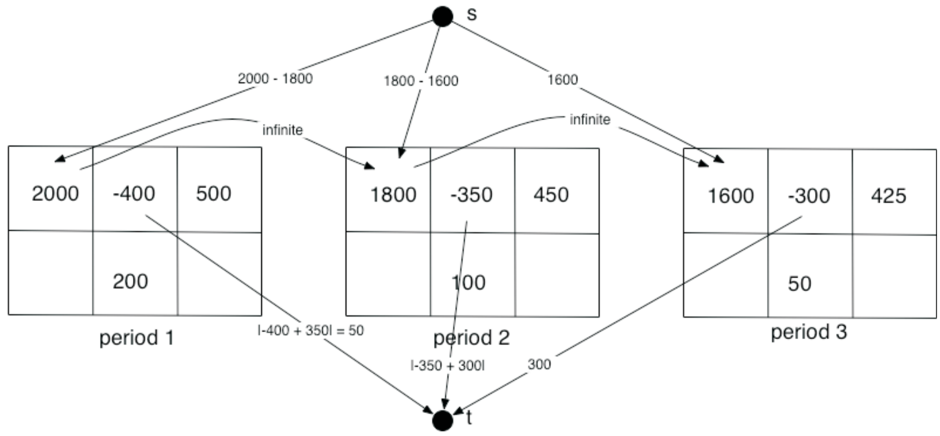

FIG 11 - Arcs associated with the upper left most block.

correlated realisations of copper prices and exchange rates using the mean-reversion model in Equation 2. Uncertainty about orebody is modelled by simulating 20 orebody models using conditional simulations. Block valuations are carried out using the procedures explained above and the data in Table 2. Figure 12 shows discounted values for a sample of mining blocks using both the conventional and the proposed uncertainty-based procedure provided that these blocks will be extracted in year 10. It is obvious, in this case study, that conventional block valuation method tends to undervalue mining blocks since it ignores flexibility to revise block destination. As shown in Figure 12, the difference between the two sets of valuation ranges, approximately, from ten per cent to 50 per cent and it is inversely related to block value. However, it is worth noting here that in some cases, conventional procedure might overvalue mining blocks.

TABLE 2

Data of the illustrative copper mine.

Number of blocks	21 3001
Block tonnage, t	10 800
Mining cost, CAN$/t	2
Processing cost, CAN$/t ore	10
Recovery	0.9
Treatment, CAN$/t concentrate	80
Refining, CAN$/lb copper	0.1
Concentrate grade, %	30
Discount rate, %	8
Long-term copper price, US$/lb	1.60
Long-term exchange rate	0.9

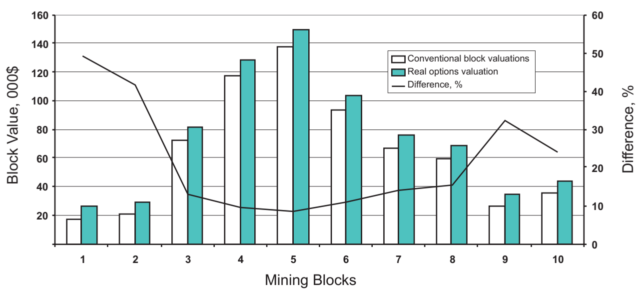

FIG 12 - Valuation results for a sample of ten blocks.

CONCLUSIONS

This paper outlined a simulation-based procedure for integrating both geological and market uncertainties into open pit mine planning. This two-level optimisation procedure combines a number of uncertainty modelling, advanced financial valuation and open pit planning algorithms. As a first level optimisation, management flexibility to change or revise block destination was integrated into block values. The described pushback design algorithm can use the geological and market uncertainty to produce lower risk long-term mine plans. Application to a case study of a Canadian copper deposit showed significant differences in block value estimates between the conventional and the proposed procedures.

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