Surface Constrained Stochastic Life-of-Mine Production Scheduling

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DEDICATION

This document is fully dedicated, in the very first place, to my beloved wife, Sheila, who supported me unconditionally, understanding my absence and weaknesses during this phase of our life. Secondly, it is dedicated to my mother, Kátia, who brought me to life and kept alive my personal and professional lives in Brazil.

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CONTRIBUTIONS OF AUTHORS

This section states the contribution of the co-author of the papers that comprises the present work. The author of this thesis has done the work herein published, with the normal supervision, advice and orientation of his advisor Prof. Roussos Dimitrakopoulos, who is co-author of the following papers to be published:

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Chapter 4: Marinho, A. and Dimitrakopoulos, R. (2013) Two-stage stochastic surface constrained mine production scheduling with pit discretization.

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ABSTRACT

The maximization of mining project discounted cash flows by defining the best sequence of extraction of underground materials requires understanding the availability of uncertain metal quantities throughout the deposit. This thesis proposes two versions of a stochastic integer programming formulation based on surfaces to address the optimization of life-of-mine production scheduling, whereby the supply of metal is uncertain and described by a set of equally probable simulated orebody models. The first version of the proposed formulation maximizes discounted cash flows, controls risk of deviating from production targets and is implemented sequentially, facilitating production scheduling for relatively large mineral deposits. Applications show practical intricacies and computational efficiency. The second variant extends the first to a two-stage stochastic integer programming formulation that manages the risk of deviating from production targets. The sequential implementation is considered first for pit space discretization and it is followed by the life-of-mine production scheduling at a relatively large gold deposit. The case studies show the computational efficiency and suitability of the method for realistic size mineral deposits, with production targets controlled, risk

postponed to later stages of production and improvements in expected NPV, when compared to deterministic industry practices.

ABRÉGÉ

La maximisation du flux de trésorerie actualisé des projets miniers fait en définissant la meilleure séquence d'extraction de matériaux souterrains exige une bonne compréhension de l'incertitude sur la disponibilité de la quantité de métal provenant du gisement souterrain. Ce mémoire propose deux formulations basées sur des surfaces afin d'optimiser la séquence d'extraction tout au long du projet où la quantité de métal est incertaine et décrite par un ensemble de simulations équiprobables. La première simulation maximise le flux de trésorerie actualisé, contrôle le risque d'écart par rapport aux objectifs de production et est implémentée de façon séquentielle, ce qui facilite la planification pour des gisements relativement grands. L'application de cette formulation sur des problèmes montre une complexité pratique et une efficacité computationnelle. La seconde formulation étend la première en une formulation stochastique en nombres entiers à deux étapes qui permet de gérer le risque d'écart par rapport aux objectifs de production. L'implémentation séquentielle considère d'abord une discrétisation du gisement puis génère une séquence d'extraction annuelle et est appliquée sur un dépôt d'or de grande taille. Les études de cas montrent l'efficacité computationnelle et une adaptation adéquate pour des problèmes de taille réelle avec des

objectifs de production contrôlés, un risque reporté à des étapes ultérieures du développement et une amélioration dans la valeur nette actualisée comparée aux meilleures pratiques déterministes de l'industrie.

Chapter 1 - INTRODUCTION

1.1 Uncertainty in Life-of-Mine Production Scheduling

Mining is the activity of extracting underground materials in a given sequence such as to maximize the discounted cash flow of a mining project (Hustrulid and Kuchta, 2006). Planning this sequence of extraction requires understanding the availability of metal quantities throughout the deposit, which is dominated by uncertainty due to the sparse sampling grids. Estimation methods define single grades at each location (mining block), reproducing overall average grades but do not describe the inherent geological uncertainty over the deposit. Defining a life-of-mine production schedule requires analysing joint distributions of the uncertainty in metal content over mining blocks grouped at different locations over time; therefore, the assumption of perfect knowledge given by a single estimated orebody model is a limited practice, incapable of assessing the joint local geological uncertainty that relates the metal content and economic value of blocks in space. Conventional optimization methods ignore that production scheduling is a “non-linear transfer function”, where averaged grades used as input do not provide schedules with production controlled in average, thus with consequences to all project indicators, including mine production and cash flow forecasts.

Quantifying and using geological uncertainty involved through the use of stochastic simulation methods (Goovaerts, 1997) is a step forward in assessing the risks associated to mine planning and production scheduling. Stochastic mine production scheduling allows for adequate control over mining and processing targets, returning risk profiles to decision makers, instead of single answers given by conventional methods. Stochastic integer programming (Wolsey, 1998; Birge and Louveaux, 1997), or SIP, is a mathematical optimization framework capable of modeling and solving mine production scheduling problems. A limit of SIP approaches in the past (Ramazan and Dimitrakopoulos, 2012; Boland et al., 2008) is the size of the problem, which is addressed herein with the use of surfaces (Goodwin et al., 2005) in a more efficient implementation. A mine schedule can be seen as a set of surfaces in space that divides the orebody into parts to be mined during different periods.

1.2 Goal and Objectives

The goal of this thesis is to address life-of-mine production scheduling under metal content uncertainty through the presentation and development of a new SIP model based on surfaces. The objectives involved in reaching this goal are:

1. Review the literature over conventional and stochastic mine planning optimization.
2. Formulate a new stochastic integer programming model based on surfaces, its implementation and testing for life-of-mine production scheduling under geological uncertainty; then, apply the formulation to a copper deposit.
3. Extend the formulation above to accommodate a two-stage SIP model for mine production scheduling where the recourse actions manage the risk of deviating from production targets; then, apply the formulation to a gold deposit.
4. Provide conclusions and suggest future work on the topic.

1.3 Thesis Outline

The thesis is organized according to the following chapters:

- CHAPTER 1: The subject of this thesis is introduced, including goals, objectives and a thesis outline.
- CHAPTER 2: Literature review on life-of-mine production schedule optimization under uncertainty, including related topics and limitations of conventional methods.

- CHAPTER 3: A new mathematical SIP formulation based on surfaces and a sequential implementation are presented, followed by testing the implementation and application at a copper deposit.
- CHAPTER 4: An extension of the formulation above to a two-stage SIP formulation based on surfaces is presented and tested at a gold deposit.
- CHAPTER 5: Conclusions with future work on this topic are discussed.
- APPENDIX A: Implementation details.
- APPENDIX B: CD with data and programs used.

Chapter 2 - LITERATURE REVIEW

2.1 Conventional Mine Production Scheduling

Mine production scheduling deals with deciding which mining blocks to extract and when, so as to maximize the project NPV and respect physical and production constraints. Mine scheduling is a part of strategic mine planning, which relates to a broader view of a mining complex (King, 2009; Whittle, 2007). Industry best practices optimize separate parts of the mining chain and assume simplifications, for example ignoring geological uncertainty through the use of estimation techniques, which serves to make algorithms incapable of returning truly optimal solutions.

Conventional mine planning starts by finding ultimate pit limits, which is the combination of blocks that maximize the total discounted cash flow of the project and respects slope constraints. This can be modelled using graph theory by assigning one node to each block and associating precedence constraints within directed arcs. The LG algorithm (Lerchs and Grossman, 1965) and its nested implementation (Whittle, 1999) is the most popular method for solving this problem. It starts by creating a dummy root node and connecting every node (block) to it; arcs are initially directed from the root to the nodes. All arcs are labeled as *strong*, if connected to positive nodes, or *weak* if connected to negative. The algorithm iterates connecting strong nodes to their predecessors, in order to respect slope constraints. At each step, arcs are removed in order to maintain the tree structure. The remaining arcs are relabelled (*away* or *towards* the root; *strong* or *weak*). The process stops when all strong nodes connected to the root have their predecessor relations checked; that is, all profitable parts of the mine are selected and slope angles respected. Zhao and Kim (1991) developed a more efficient algorithm that does not require a dummy node and iterates over many directed trees instead of updating a single one. The paper also documents some strategies to improve computational performance, defining templates for slopes management, a maximum bottom pit and a specific search procedure for connecting ore and waste blocks.

Whittle Software (Whittle, 1988, 1999) is known to have the most efficient implementation of the LG algorithm with additional capabilities that allow for discounting, which has led the software to become the reference of “best” practices in the industry. In this implementation, positive blocks are initially flagged to be mined. Their predecessors that are flagged not to be mined must be linked by arcs until there are no more predecessors for the entire branch, or the branch assumes an overall negative value. The algorithm terminates when all branches have been checked. Within ultimate pit limits, Whittle Software economically discretizes the space defining a series of nested pits that generates increasing pit-by-pit discounted cash flow profiles for deposits with millions of blocks (Whittle, 1988). For doing so, it runs the LG algorithm for different "metal costs of mining", which is the quantity of metal that needs to be sold to pay for the cost of mining one tonne of waste. The higher the value of this parameter, the smaller will be the pit generated. These values can be manipulated by defining a series of revenue factors, where factor 1.0 means the actual metal price. A modified version of the 3D LG algorithm based on the algorithm from Vallet (1976) and presented by Seymour (1995), allows for generating a series of nested pits at once, instead of running pit optimization successive times for different parameters. The graph tree algorithm keeps track of the mass, value and strength (value/mass) of each branch; connecting arcs from weaker to stronger branches and pruning others through successive passes. The final solution is a sequence of branches in order of decreasing strength, where each nested pit is found by adding the blocks of each branch.

The main drawback of these discretization strategies is the "gap effect", which is due to the inability of the methods to control capacity constraints, allowing for cases where, for example, the amount of material contained in between two consecutive nested pits is bigger than mining and/or processing capacities. Therefore, nested pits are usually grouped to form mining phases (pushbacks), which are later used for guiding the mine scheduling optimization process. At present, there is no algorithm able to guarantee optimal pushback designs that consider operational geometric constraints.

The question of finding ultimate pit limits is also equivalent to the known maximum closure problem, where each closure represents a set of blocks which necessarily includes all their predecessors. Assigning net values to each node (block) of the graph, the maximum closure gives the optimum pit limits. This problem can be modelled adding source and sink nodes connected to positive and negative valued nodes, respectively (Picard, 1976). The maximum closure is given by the minimal cut of this network, which can be efficiently found by maximum flow algorithms, considering their primal-dual relation. Hochbaum and Chen (2000) comment on the theoretically lower complexity of max-flow algorithms if compared to the LG algorithm; however, Whittle Software (2012) implementation is more efficient. Hochbaum (2001, 2008) proposes the lowest label pseudoflow and the highest label pseudoflow algorithms which adapt the LG algorithm to a network flow model, allowing for the use of capacity constraints for millions of nodes but not guaranteeing flow conservation.

The objective of the open pit mine production scheduling optimization is to assign mining periods to blocks, deciding if each block is going to be mined and when, maximizing the expected net present value and complying with established constraints. For decades, this problem has been studied and is recognizably complex due to the number of binary variables associated to the decision of if/when each block should be mined; therefore, most of the exact current methods fail to return solutions for large deposits. The only reason for finding ultimate pit limits prior to mine production scheduling is computational, issue that has been overcome with recent improved methods and computational capabilities. Approaches have been proposed to reduce the size of the problem without using heuristics, such as Ramazan et al. (2005), but aggregation based simplifications imply loss of resolution, reduces the space of possible solutions and smooth results neglecting selective mining units. The Milawa algorithm (2012) is a heuristic approach from Whittle Software that defines schedules by combining benches inside predefined pushbacks. The method is efficient and practical but has the drawbacks of working within aggregates.

2.2 Mixed-Integer Programming – A Different Class of Methods

A different class of methods, called mixed-integer programming (MIP), allows for the definition of optimal mine production schedules within their ultimate pit limits simultaneously. The actually most efficient software for conventional mine planning is the Blasor owned and developed by BHP Billiton. The algorithm is proprietary with details not published, but the software is capable of dealing with deposits containing billions of blocks for multiple pits, multiple destinations, blending operations, with no prior definition of cutoffs or ultimate pit limits. For doing so, blocks are aggregated by a proprietary fuzzy clustering algorithm according to their location in space and respecting slope constraints. Each aggregation has bins that are allowed to be mined independently. The schedule optimization is performed over aggregations by a MIP formulation (Stone et al., 2007). Other extensions of Blasor include in-pit dumping optimization (Zuckrberg et al., 2007) and the Blondor optimizer for a bauxite mine (Zuckrberg et al., 2010). Other conventional approaches are Johnson (1968), Dagdelen and Johnson (1986), Akaike and Dagdelen (1999), Dagdelen (2007), Bley et al. (2009) and Bley et al. (2012).

Other approaches propose strategies to reduce the number of binary variables in MIP formulations, thus improving processing times. Ramazan and Dimitrakopoulos (2004[1]) propose defining waste blocks as continuous variables, considering that negative valued blocks will be chosen to be mined by the optimization process only when overlying profitable material; therefore, slope constraints and the maximizing profit nature of the objective function guarantee that waste blocks will be completely mined in those cases, assuming integer value 1 even though variables are defined as continuous. Caccetta and Hill (2003) presented an MIP formulation for mine schedule optimization and studied the intricacies of its linear relaxation in order to propose a specific branch and cut strategy (Wolsey, 1998). Bley et al. (2010) strengthened this formulation by considering that deeper blocks cannot be included in earlier stages of development, according to upper limits in production constraints.

2.3 Heuristic Approaches

Among heuristic methods available, Tolwinski and Underwood (1996) proposed an approach combining dynamic programming (guided search) and specific heuristics (lookahead values and templates). The scheduling is modelled as a series of mine states, where, from one state to the next, one block is mined until reaching the final pit. The proposed method limits the size of the problem by restricting the state space, allowing for scheduling of average size mines. Although the presented results are better than other given solutions, it was not assessed how far they are to the unknown optimal one. Other heuristic approach based on a genetic algorithm can be found in Denby and Schofield (1995).

2.4 Drawbacks of Deterministic Methods

The conventional mine production scheduling methods presented are based on the strong assumption that all parameters are known with 100% certainty; however, assuming perfect knowledge of the required inputs misleads optimization processes, as the life-of-mine production schedule definition requires analysing joint distributions of metal content over blocks grouped at different locations over time. Conventional methods ignore that production scheduling is a “non-linear transfer function”, where averaged grades used as input do not provide schedules with production controlled in average, thus with consequences to all project indicators, including NPV expectations. Geological uncertainty must be addressed in order to come up with truly optimal mining decisions. Ravenscroft (1992) discusses the shortfalls of using confidence intervals estimation to access geological uncertainty and concluded that conditional simulations are preferred for such a task, since spatial correlations are respected. Dowd (1994) discusses in depth the risk concept in mineral projects, reinforcing the importance of stochastic simulations as input for proper risk analysis. Dimitrakopoulos et al. (2002) discusses these aspects and presents consequences of considering an estimated orebody model as input for a "non-linear transfer function" (mine scheduling) in an open pit case study. The study assessed the risks involved in mining decisions taken with the use of conventional methods, replacing the estimated model used as input by conditionally simulated models. Project indicators, such as the DCF, are then recalculated for each scenario, as presented in Figure 1.

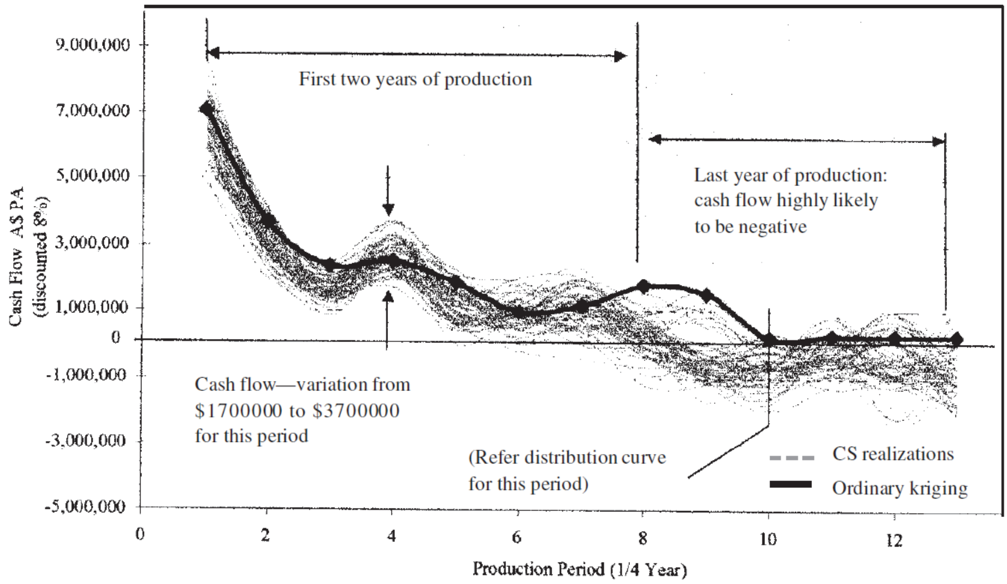


Figure 1: Case study DCF risk profile, Dimitrakopoulos et al. (2002).

Figure 1 suggests that there is considerable risk in not meeting expectations in terms of DCF in the third year, which could not be forecasted without a proper risk analysis. Dimitrakopoulos et al. (2007) proposes a maximum upside potential/minimum downside risk approach, which takes the outputs from each schedule generated, applied to each simulation, and compares them with a point of reference such as the minimum acceptable return. As a result, one of the schedules is selected from those tested, but not necessarily the optimal one. Focus is next given to stochastic techniques for life-of-mine production scheduling.

2.5 Stochastic Mine Planning Optimization

Godoy (2003) developed a long term scheduling approach based on simulated annealing (Kirkpatrick et al., 1983; Geman and Geman, 1984), where multiple schedules were also used to define the probability of each block being assigned to a specific period (transition probability). The objective function

$$O = \sum_{n=1}^{N} \left(\frac{1}{S} \sum_{s=1}^{S} \left| \theta^*(s) - \theta(s) \right| + \frac{1}{S} \sum_{s=1}^{S} \left| \omega^*(s) - \omega(s) \right| \right)$$

minimizes the sum over N scheduling periods of the average deviations in ore and waste production targets over S scenarios, where $\theta^*(s)$ and $\omega^*(s)$ are the actual ore and waste productions and $\theta(s)$ and $\omega(s)$ are the respective target productions. The method starts by freezing mining blocks into their respective scheduled periods when the same decision was taken over all schedules generated. The remaining blocks are considered in the annealing process, where blocks are randomly chosen and swapped to a candidate period (perturbation), respecting slope constraints and evaluating the transition probabilities. Perturbations that improve the objective function are always accepted. The remaining perturbations are accepted according to an exponential probability distribution (Boltzmann distribution), which varies in time with the annealing temperature T and depends on the level of deterioration of the actual solution:

$$\Pr ob(accept) = \begin{cases} 1 & if \rightarrow O_{new} \leq O_{old} \\ \exp\left(\dfrac{O_{old} - O_{new}}{T} \right) & otherwise \end{cases}$$

Higher temperatures and smaller differences between actual and perturbed solutions increase the probability of accepting an unfavorable perturbation. The process should start with higher temperatures to allow the method to explore different parts of the solution space (different feasible mine schedules). Temperatures should decrease within time, allowing less deterioration, that is, with more focus on improvements. Simulated annealing for mine scheduling is further explored in Leite and Dimitrakopoulos (2007) with sensitivity analysis over the set of 25 scenarios considered at a copper deposit and in Albor and Dimitrakopoulos (2009) finding deeper pit limits with sensitivity analysis over the number of scenarios considered for the same copper deposit. Both case studies have shown improve in NPV expectations higher than 26% if compared to results provided by deterministic methods, which shows that the stochastic solutions have higher value, providing schedules with lower risk and higher reward simultaneously. The approach is flexible to be extended to other contexts and simulated models are considered jointly during the optimization process. Its main drawbacks are the inability to control the distribution of risk over time, the demanding work to define schedules for each simulated model and the setup of abstract parameters.

Dimitrakopoulos and Ramazan (2004) introduce the concept of orebody risk discounting, which postpones riskier blocks to later periods of development. A major drawback of this formulation, and other probabilistic based approaches (Ramazan and Dimitrakopoulos, 2004[2]; Dimitrakopoulos and Grieco, 2009), is the prior assignment of risk probabilities for each block, discarding grades over simulations and neglecting that uncertainty has to be evaluated jointly, as groups of blocks in mining periods, and not locally, block-by-block.

Stochastic mathematical programming (Dantzig, 1955; Beale, 1955; Vajda, 1972; Kolbin, 1977; Birge, 1982; Birge, 1997; Birge and Louveaux, 1997; Sen and Higle, 1999) is a modelling framework with some unknown parameters that allows for optimization considering possible scenarios simultaneously. Dantzig (1955) introduced the concept of activities being divided into multiple stages (multistage stochastic programming). In the context of mine production scheduling, Ramazan and Dimitrakopoulos (2012) implements a two-stage stochastic integer programming model, which simultaneously maximizes the yearly discounted profit of the operation and minimizes the risks of not achieving production targets; the formulation is detailed in the next section. Leite and Dimitrakopoulos (2010) presents a case study based on the same formulation, but considering no control over grade and metal productions and no stockpiles. Results showed an expected NPV 29% higher, if compared with a conventional approach fixing the same ultimate pit limits for both. Benndorf and Dimitrakopoulos (2010) shows an application at an iron ore deposit controlling deviations in quality targets over multiple elements and considering penalties for mining configurations that are not smoothed. Menabde et al. (2007) implements a stochastic version of Blasor including a variable cut-off grade approach and considering aggregation of blocks in panels. Jewbali (2006; 2010) integrates long and short-term mine planning, making use of simulated grade control data. Albor and Dimitrakopoulos (2009) combines SIP with a pushback design approach. Using an estimated model, a set of nested pits is generated and intermediate pits are grouped using maximum NPV criteria, depending on the number of pushbacks desired. Finally, SIP is applied following different pushback designs, comparing results and defining the best pushback strategy. Although the approach allows going beyond traditional final pit limits, with 17% of extra ore production, it requires repetitive executions of the SIP procedure, with unreasonable processing time for some cases. Lamghari and Dimitrakopoulos (2012) presented an approach based on Tabu Search that maximizes NPV and minimizes deviations in ore and metal production targets, considering diversification strategies based on the search history and a Variable Neighbourhood Search method. Results are close to the optimal SIP solution but executing dozens of times faster. Boland et al. (2008) proposed a multistage stochastic programming approach considering scenario dependency for processing and mining decisions, where different schedules can be defined according to grouping criteria that depends on the “similarity” among simulated models. Its main drawback is the non-realistic assumption that one of the provided realizations represents the truth, allowing for different mining decisions depending on the model.

For completeness regarding stochastic pit space discretization, something which can facilitate the scheduling of large deposits, further work using the maximum flow framework has been developed considering its ability to cope with real sized instances of mineral deposits. Asad and Dimitrakopoulos (2012) proposed a parametric maximum flow algorithm based on previous work (Meagher et al., 2010) that considers geological uncertainty through the use of stochastic simulations, and market uncertainties through a real options approach, using orebody model realizations with different discounting scenarios (Dimitrakopoulos and Abdel Sabour, 2007).

Recent attempts have been made to associate more parts of the mining chain, incorporating the uncertainty in metal content. Montiel and Dimitrakopoulos (2012) proposes a heuristic approach for mine production scheduling considering multiple material types, dynamic decision over multiple destinations, stockpiles and achieving requirements in terms of processing additives, and Goodfellow and Dimitrakopoulos (2012) does mine supply chain optimization controlling production deviations in mining complexes with multiple mines and processing destinations. In the deterministic context, Whittle Software released its Simultaneous Optimization module (2012) that considers the optimization of the mine sequence, cut-off grades, blending and stockpiling simultaneously, showing a case study with 25% of improvement in NPV expectations, if compared to the same project running optimization procedures sequentially.

2.6 Modelling with Stochastic Integer Programming

The two-stage stochastic integer programming model from Ramazan and Dimitrakopoulos (2012) provides a general formulation for mine production scheduling optimization, controlling ore tonnage, grade and metal production targets with the cost of risk included in the objective function (1). Such a risk is quantified by the excess d_{su}^{t-} and deficient d_{sl}^{t-} amounts in ore production, grade and metal production, over each scenario s, penalizing the objective function ($Part4$) according to pre-defined costs (C_u^{t-}, C_l^{t-}), which include orebody risk discounting rate varying according to period t. The $Part1$ of the objective function accounts for the expected NPV obtained by mining the fractions b_i^t of blocks i over period t and processing the blocks during the same period. $Part2$ compensates for the expected NPV loss from fractions w_j^t of blocks j mined over the same period but sent to the stockpile, with respective cost MC_j^t. $Part3$ stands for the expected NPV obtained from the amounts of ore k_s^t processed from the stockpile during period t, SV^t being the profit per tonne generated. P stands for the number of periods; N the total number of blocks; U the number of blocks considered for stockpiling; and M the number of simulated models.

$$Max\sum_{t=1}^{P}\left[Part1-Part2+Part3-Part4\right] \tag{1}$$

$$Part1=\sum_{i=1}^{N}E\left\{\left(NPV\right)_i^t\right\}b_i^t$$

$$Part2=\sum_{j=1}^{U}E\left\{\left(NPV\right)_j^t+MC_j^t\right\}w_j^t$$

$$Part3=\sum_{s=1}^{M}\left(SV^t/M\right)k_s^t$$

$$Part4=\sum_{s=1}^{M}\left(C_u^{to}d_{su}^{to}+C_l^{to}d_{sl}^{to}+C_u^{tg}d_{su}^{tg}+C_l^{tg}d_{sl}^{tg}+C_u^{tq}d_{su}^{tq}+C_l^{tq}d_{sl}^{tq}\right)$$

Deviations in the objective function are calculated by processing (2), grade blending (3) and metal production (4) constraints. O_{si}, G_{si} and Q_{si} stand for ore tonnage, grade and metal tonnage, respectively, in block i and model s; O_{tar}, G_{tar} and Q_{tar} stand for the ore production, grade and metal production targets, respectively; and $(QST)^t$ is the percentage of metal content at average grade GST in the stockpile.

$$\sum_{i=1}^{N}O_{si}b_i^t-\sum_{j=1}^{U}O_{sj}w_j^t+k_s^t+d_{sl}^{to}-d_{su}^{to}=O_{tar}$$

$$s=1,2,...,M;t=1,2,...,P \tag{2}$$

$$\sum_{i=1}^{N}\left(G_{si}-G_{tar}\right)O_{si}b_i^t-\sum_{j=1}^{U}\left(G_{si}-G_{tar}\right)O_{sj}w_j^t+\left(GST-G_{tar}\right)k_s^t+d_{sl}^{tg}-d_{su}^{tg}=0$$

$$s=1,2,...,M;t=1,2,...,P \tag{3}$$

$$\sum_{i=1}^{N} Q_{si} b_i^t - \sum_{j=1}^{U} Q_{sj} w_j^t + (QST)^t k_s^t + d_{sl}^{tq} - d_{su}^{tq} = Q_{tar}$$

$$s = 1,2,\ldots,M; t = 1,2,\ldots,P \tag{4}$$

An additional set of constraints define the quantity of ore at the stockpile at the end of each period, not exceeding capacity and guaranteeing that blocks are mined before they are stockpiled. Slope constraints guarantee that overlying blocks (predecessors) are mined before underlying blocks (successors), according to a given slope angle; reserve constraints guarantee that blocks are fully mined and not more than once; and mining capacity constraints forces the total production (ore + waste) to be in between a given range. In order to improve performance, some strategies are considered: binary variables associated to waste blocks are treated as continuous; second stage variables are first converted into hard constraints and results are fed back as initial solution; scheduling is divided into two phases for the case study. Results for the gold deposit studied showed a 10% higher expected NPV given by the stochastic solution.

2.7 Surface Based Mine Production Scheduling

An alternative idea to the previous SIP formulations is based on the concept of mining surfaces. A mine schedule can be seen as a set of surfaces in space that divides the orebody into parts to be mined during different periods. Approaches based on surfaces facilitate slope management, improving efficiency in solving SIP formulations.

Goodwin et al. (2005) introduced a new mine state concept in the production scheduling context by incorporating practical mine planning features in order to improve computational efficiency. Mine state was defined as a set of elevations, one for each (x, y) coordinate of a three-dimensional orebody model, representing the pit depth (that is, the distance from a fixed higher elevation) at a given mining period. The evolution from one period to the next is given by downward vertical increments, updating the mine state $z_{ij}(k)$ according to decisions $u_{ij}(k)$ to mine or not mine the area ij (i, j: block indices) at time k. The state model is defined by:

$$z_{ij}(k+1) = z_{ij}(k) + cu_{ij}(k), \quad k \geq 1$$

$$z_{ij}(1) = 0, i = 1,\dots,N_x, j = 1,\dots,N_y, \tag{5}$$

where N_x, N_y are the number of blocks in x and y dimensions, respectively, and c is the constant depth associated to each mining action. Note that having only positive downward increments guarantees the traditional reserve constraints. At each time k, the number of mining states allowed to change is limited by the mining capacity. Slope constraints (6) are controlled over each mine state, comparing only adjacent elevations. This is different than previous approaches that require one constraint for each combination (two-by-two) of blocks and their predecessors, which is an approximation that changes complexity (number of predecessors) depending on slope angles.

$$\left|z_{\ln}(k) - z_{ij}(k)\right| \le h\,, \quad k \ge 1$$

$$\text{for } |l - i| = 1 \text{ and } |n - j| = 1\,, \tag{6}$$

where h is a fixed difference in elevation calculated from the given slope angle. Note that this height should be different at least for states adjacent diagonally, but constraints (6) are reproduced here as in the original manuscript. The authors do not present the remaining constraints involved in the formulation. The objective function is modelled as:

$$Max \sum_{k=1}^{T} \sum_{i=1}^{N_x} \sum_{j=1}^{N_j} d_k V_{ij}\left(z_{ij}(k)\right) u_{ij}(k)\,, \tag{7}$$

where $V_{ij}\left(z_{ij}(k)\right)$ and d_k stand for the economic value given the mining state and its time discounting, respectively, and T is the planning horizon. The authors state the non-convexity of the value function and consider the fictitious linear aspect of the third dimension due to the discrete nature of the input model. No further details are reported and understanding how block values are associated with mining states in the formulation in unclear. Uncertainty is not taken into account. Considering the complexity of the formulation, mine scheduling is performed using the receding horizon control framework, which may allow for a long-term view of the solution space by grouping mining periods.

Chapter 3 - SURFACE CONSTRAINED MINE PRODUCTION SCHEDULING WITH UNCERTAIN METAL AVAILABLE

3.1 Introduction

Mineral projects aim to produce metal available in the Earth's subsurface to meet the needs of society and its development through the sustainable utilization of mineral resources while generating revenue. The longstanding conventional approach of using exploration drilling data to estimate the attributes of interest (grade, material types, density, etc.) in a mineral deposit does not capture the intrinsic geological variability and uncertainty (Dimitrakopoulos et al, 2002; Dowd, 1994). Ravenscroft (1992) concludes that conditional stochastic simulations are preferred for such a task, since spatial correlations are respected. Conventional optimization formulations used to optimize mine production schedules and assess discounted cash flows over production years have been shown to be misleading (Godoy, 2003), causing possibly incorrect assesses of production forecasts, which is reviewed in Dimitrakopoulos (2011).

Two stochastic optimization approaches for long-term mine production scheduling have been developed. The simulated annealing framework was introduced by Godoy and Dimitrakopoulos (2004) and is further explored in Leite and Dimitrakopoulos (2007) and Albor and Dimitrakopoulos (2009). It has been expanded to control production deviations in mining complexes with multiple mines and processing destinations (Goodfellow and Dimitrakopoulos, 2012). Stochastic integer programming with recourse is introduced in Ramazan and Dimitrakopoulos (2005, 2012) to maximize total discounted cash flows, while minimizing deviations from production targets (ore tonnage, grade and metal), as well as deferring risk to latter production periods. The framework considers stockpiles and allows controlling grades, ore and metal productions. Leite and Dimitrakopoulos (2010) presents a case study based on the same formulation, but considering no control over grade and metal productions and no stockpiles. Notable variations of the SIP framework includes long- and short-term mine production scheduling based on simulated future grade control data (Jewbali, 2006; 2010). Benndorf and Dimitrakopoulos (2010) shows an application controlling deviations in quality targets over multiple elements and considering penalties for mining configurations that are not smoothed. Albor and Dimitrakopoulos (2010) use the SIP formulation for pushback design, demonstrating that stochastically generated pit limits are larger than the corresponding conventional ones. Menabde et al. (2007) proposes an alternate formulation that uses a variable cutoff grade and relies on aggregations of blocks to ensure the problem is computationally tractable. Boland et al. (2008) propose a multi-stage stochastic programming approach that considers both processing and mining decisions. To address the computational and size limits of SIP mine scheduling formulations, Lamghari and Dimitrakopoulos (2012) introduce tabu search and variable neighborhood search to replace the need to solve SIP formulations with conventional mathematical programming solvers, assisting computationally efficient solutions. Regarding stochastic pit space discretization, something which can facilitate the scheduling of large deposits, Asad and Dimitrakopoulos (2012) proposed a parametric maximum flow algorithm based on previous work (Meagher et al., 2010) that considers geological uncertainty through the use of stochastic simulations, and market uncertainties through a real options approach.

The well-known drawback of any mixed-integer programming based approach, in the mine scheduling context, is computational (Hustrulid and Kuchta, 2006). As the number of binary variables increases with the number of mining blocks being scheduled, the amount of time required to generate an optimal solution often becomes impractical. Ramazan and Dimitrakopoulos (2004[1]) proposes defining waste blocks as continuous variables, considering that negative valued blocks will be chosen to be mined by the optimization process only when overlying profitable material.

An alternative approach to past SIP developments is based on surfaces that facilitate a divide and conquer approach for scheduling. Related to this is the work of Goodwin et al. (2005), who proposes a new “mine state” concept for open pit production scheduling. A mine state is defined as a set of elevations, one for each coordinate of a three-dimensional orebody model, which are used to represent the pit depth (the distance from a fixed higher elevation) at a given mining period. The evolution from one period to the next is given by downward vertical increments, updating the mine state each time. Slope constraints are controlled over each mine state, comparing only adjacent elevations. Each state is always below previous ones, as only positive increments are allowed. Their formulation has a non-convex objective function, but no further details are reported. In addition, uncertainty in input parameters is not accounted for. The mine state concept is the same as the surface concept used herein and is presented in the subsequent sections, with the difference that surfaces carry exact elevations in space, instead of measures of depth from a fixed elevation.

The present paper builds upon the previous work by proposing a SIP formulation to address the optimization of mine production scheduling, defining constraints over surfaces so as to reduce the number of constraints in the mathematical formulation, while uncertainty and risk are controlled through additional hard constraints. It is important to note that, in the proposed formulation, the management of slope constraints is simpler and more general. In the following sections, the concepts and stochastic scheduling formulation based on surfaces are detailed. Following this, a sequential implementation of the proposed method is presented and tested so as to elucidate performance and related intricacies. An application at a copper deposit demonstrates the method's capability to deal with relatively large deposits. Conclusions follow.

3.2 A Stochastic Programming Formulation Based on Surfaces

3.2.1 Surfaces and Related Concepts

Surface relations used herein are based on the fact that mining blocks describing a deposit are not independently distributed in space and can be grouped into vertical columns. More specifically, surfaces are defined as sets of elevations in which mining periods in the production schedule are divided, similarly to Goodwin et al. (2005). Each column of blocks (fixed x and y coordinates) can be partitioned by T surfaces into $T+1$ groups of blocks, which then becomes the T mining periods in addition to the blocks that are not extracted. For each surface (or period) T, cell c is defined by a fixed pair of coordinates (x, y), and each cell c has an elevation $e_{c,t}$ associated with period t. Variables $e_{c,t}$ are continuous and assume values from the origin up to the highest elevation allowed in the orebody model. Using the concept of surfaces, slope angle constraints can be controlled over surfaces, rather than blocks. This is a trade-off, where new variables $e_{c,t}$ are included but fewer constraints are required, as shown in subsequent sections.

Typical approaches for production scheduling in terms of mathematical programming (Hustrulid and Kuchta, 2006) are based on binary variables $x_{i,t}$, where $x_{i,t} = 1$, if block i is mined in period t, and 0 otherwise. Here, the notation of $x_{i,t}$ is modified to $x^{z}{}_{c,t}$, where each i corresponds to a pair (c, z), for correlating $e_{c,t}$ (elevation; continuous) with $x^{z}{}_{c,t}$ (block; binary). The $x^{z}{}_{c,t}$ variable assumes the value 1 only if block (c, z) is the deepest block being mined in period t and in column c; otherwise, the $x^{z}{}_{c,t}$ variable is zero.

Block attributes such as total/ore/metal tonnages are accumulated starting from the topography down to the last block over each column, with cumulative values being stored at each level; attributes for single blocks are discarded for the optimization process, which allows quick operations between surfaces by taking differences. A key aspect of this approach is the need to associate blocks with surface cells and is performed by comparing their elevations in space (Figure 2): a block (c, z) will be considered the last block mined over c in a given period t if its centroid elevation $E^{z}{}_{c}$ (in grey) lies between $e_{c,t}$ and $e_{c,t} + \Delta z$, where Δz is the block dimension in z. Figure 3 shows, in a sectional view, cells representing the end of first and second periods, with blocks valued accordingly. Note that for each column (given x and y) and each period, $\sum_{z} x^{z}{}_{c,t} = 1$, which is a strong constraint given by the approach proposed herein that reduces processing time.

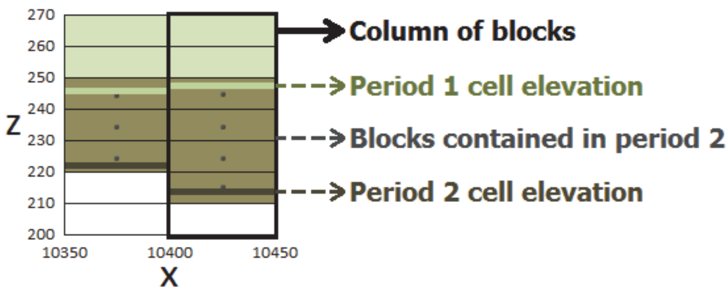


Figure 2: Example of a cross-section of blocks being represented by cells.

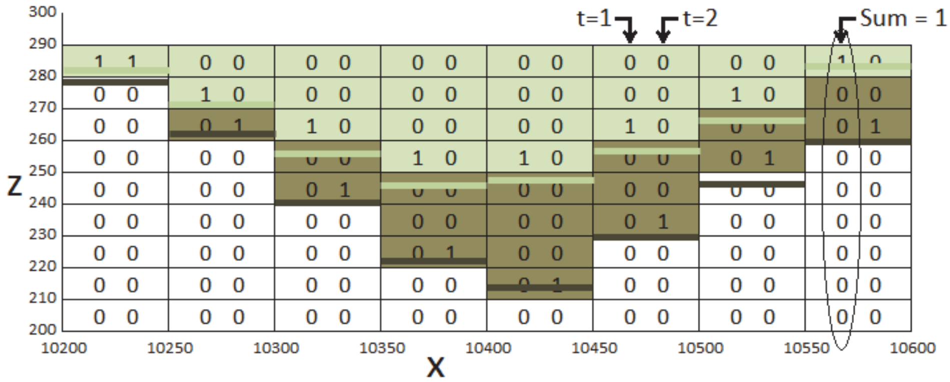


Figure 3: Assigning a period to a block according to surface elevations.

3.2.2 Notation

The following notation is used herein.

3.2.2.1 Indices

1. M: number of cells in each surface; where $M = x \times y$ represents the number of mining blocks in x and y dimensions.
2. c: cell index corresponding to each (x, y) block/cell location, $c = 1, \ldots, M$.
3. Z: number of levels in the orebody model.
4. z: level index, $z = 1, \ldots, Z$.
5. T: number of periods over which the orebody is being scheduled and also defines the number of surfaces considered.

6. t: period index, $t = 1,\ldots,T$.

7. S: number of simulated orebody models considered.

8. s: simulation index, $s = 1,\ldots,S$.

3.2.2.2 Constants and sets

9. E_c^z: elevation of the centroid for a given block (c,z).

10. H_x: maximum difference in elevation for adjacent cells in contact laterally in the *x* direction, calculated by $H_x = \Delta x \times \tan(\theta)$, where Δx is the block size in x and θ is the maximum slope angle.

11. H_y: maximum difference in elevation for adjacent cells in contact laterally in the y direction, calculated by $H_y = \Delta y \times \tan(\theta)$, where Δy is the block size in y.

12. H_d: maximum difference in elevation for adjacent cells in contact diagonally, calculated by $H_d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \times \tan(\theta)$.

13. X_c, Y_c and D_c: equivalent to H_x, H_y and H_d concept, the sets of adjacent cells, laterally in x, in y and diagonally, for a given cell c, respectively.

14. T_c^z: cumulative tonnage of block (c,z) and all blocks above it (scenario independent).

15. $O_{c,s}^z$: cumulative ore tonnage of block (c,z) and all blocks above it in scenario s.

3.2.2.3 Parameters

16. $\underline{T_t}$ and $\overline{T_t}$: lower and upper limits, respectively, in total tonnage to be extracted during period t.

17. $\underline{O_t^R}$ and $\overline{O_t^R}$: lower and upper limits, respectively, on ore tonnage to be processed over period t, where R is used to denote “risk”.

18. $\underline{O_t^A}$ and $\overline{O_t^A}$: lower and upper limits, respectively, on expected ore tonnage to be processed over period t, where A is used to denote “average”.

19. $V_{c,t,s}^z$: cumulative discounted economic value of block (c,z) and all blocks above it in scenario s and period t.

3.2.2.4 Variables

20. $e_{c,t}$: scenario-independent continuos variables associated with each cell c for each period t, representing cell elevations.

21. $x_{c,t}^z$: binary variable that assumes 1 if block (c,z) is the last block being mined in period t over c, and otherwise. $x_{c,0}^z = 0, \forall (c,z)$.

3.2.3 Mathematical Model

3.2.3.1 Objective function

The proposed objective function in Equation (1) maximizes the expected net present value from mining and processing selected blocks over all considered mine production periods. Recall that all values are cumulative and $x_{c,t}^z - x_{c,t-1}^z$ is associated with the difference between the surfaces, which results in 0, 1 or -1, depending on location (c,z) and period t.

$$\max \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{c=1}^{M}\sum_{z=1}^{Z} V_{c,t,s}^{z}\left(x_{c,t}^{z}-x_{c,t-1}^{z}\right) \tag{1}$$

3.2.3.2 Constraints

The constraints presented in Equations (2) to (8) are scenario-independent, while constraints in Equations (9) and (10) are scenario-dependent stochastic constraints.

Surface constraints: the following set of constraints (Equation (2)) guarantee that each surface t has, at maximum, the same elevation as surface $t-1$, which is used to avoid crossing surfaces and blocks being mined more than once. $e_{c,0}$ are constant elevations defined by the actual topography of the deposit.

$$e_{c,t-1}-e_{c,t}\geq 0 \qquad c=1,\ldots,M; \quad t=2,\ldots,T \tag{2}$$

Slope constraints: the maximum surface slope angle is guaranteed herein by Equations (3) to (5). Each cell elevation is compared to the elevation of the 8 adjacent cells, which therefore represents a set of $8\times nx\times ny\times T$ continuous constraints. Note that adjacent cells are compared twice, guaranteeing upward and downward slopes. The number of slope constraints controlled by surface relations does not depend on slope angles and require fewer constraints than conventional formulations, as follows:

$$e_{c,t}-e_{x,t}\leq H_x \qquad c=1,\ldots,M; \quad t=1,\ldots,T; \quad x\in X_c \tag{3}$$

$$e_{c,t} - e_{y,t} \le H_y \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T; \quad y \in Y_c \tag{4}$$

$$e_{c,t} - e_{d,t} \le H_d \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T; \quad d \in D_c \tag{5}$$

Link constraints: mining blocks and surfaces are linked in the formulation by comparing the elevation of each block centroid with the elevation of each surface. Variables $x^z_{c,t}$ will assume value 1 only for block centroids that are in the same elevation or exactly above the correspondent surface (index t). Constraints (6) guarantee this link, where Δz is the block height, and constraints (7) guarantee that there is only one block defining the end of each period t over each column c of blocks z,

$$0 \le \sum_{z=1}^{Z} \left(E^z_c x^z_{c,t} \right) - e_{c,t} \le \Delta z \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T \tag{6}$$

$$\sum_{z=1}^{Z} x^z_{c,t} = 1 \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T \tag{7}$$

Mining constraints: constraints (8) ensure ore and waste production requirements are respected during each mining period:

$$\underline{T_t} \le \sum_{c=1}^{M} \sum_{z=1}^{Z} T^z_c \left(x^z_{c,t} - x^z_{c,t-1} \right) \le \overline{T_t} \qquad t = 1,\ldots,T \tag{8}$$

Variability constraints: constraints (9) guarantee that processed ore tonnages are not outside lower and upper bounds, $\underline{O^R_t}$ and $\overline{O^R_t}$. These hard constraints guarantee that deviations from production targets are within required specifications.

$$\underline{O^R_t} \le \sum_{c=1}^{M} \sum_{z=1}^{Z} O^z_{c,s} \left(x^z_{c,t} - x^z_{c,t-1} \right) \le \overline{O^R_t} \qquad s = 1,\ldots,S; \quad t = 1,\ldots,T \tag{9}$$

Target constraints: constraints (10) guarantee that expected processed ore tonnages are not outside lower and upper bounds, $\underline{O_t^A}$ and $\overline{O_t^A}$, keeping results close to defined targets. The absence of such constraints implies in results closer to the upper bound defined in (9). Other constraints over other additive variables can be modelled using the same approach.

$$\underline{O_t^A} \leq \frac{1}{S}\sum_{s=1}^{S}\sum_{c=1}^{M}\sum_{z=1}^{Z} O_{c,s}^{z}\left(x_{c,t}^{z} - x_{c,t-1}^{z}\right) \leq \overline{O_t^A} \qquad t = 1,\ldots,T \qquad (10)$$

The variables involved in the formulation are defined as following. Note that elevations are continuous and can be initially constrained by the limits of the orebody models.

$$e_{c,t} \in \Re \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T \qquad (11)$$

$$x_{c,t}^{z} \in \{0,1\} \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T; \quad z = 1,\ldots,Z \qquad (12)$$

3.3 Sequential Implementation

This section proposes a sequential implementation of the formulation, where periods are included in the optimization process in a stepwise fashion. Once the method returns its best solution for a mining schedule of t periods, a new period $t+1$ is added to the optimization process. The proposed formulation is executed successive times and results for one optimization process are used as limiting assumptions for next processes. The main steps of the sequential implementation are as follows:

1. Take the actual topography of the deposit as a top limiting surface; no other surface can extend above this limit.
2. Similarly, define a bottom limit by eliminating external waste volumes, vertically and horizontally, according to slope angles, but guaranteeing that every block with some probability of being ore is inside this limit. No surface in any period can go outside such limits and blocks located outside are not considered in optimization processes.
3. Find an initial solution for period 1 with successive runs of the mathematical formulation proposed in Section 3.2, given pertinent considerations (please see Section 3.3.2).
4. Iteratively improve the initial solution, using the same formulation with other pertinent considerations (Section 3.3.3).
5. Find an initial solution for periods 1 and 2, with similar considerations of step 3 (Section 3.3.4).
6. Improve the solution considering all included periods jointly, extending considerations of step 4 (Section 3.3.4).
7. Loop over steps 5 and 6, including new periods, until there is no more profitable material available (Section 3.3.4).

8. Freeze all periods except by the last and run the last optimization process, based on the same formulation, looking for extra deeper blocks to be included (Section 3.3.4).

3.3.1 Efficiency Aspects

The formulation proposed in Section 3.2 has a specific characteristic that can be exploited for reasons of computational efficiency. More specifically, integer programming methods require solving the linear relaxation of the formulation, when unavoidable situations like the ones presented in Figure 4 occur. The sum over all columns, for each period, is always equal to 1, as guaranteed by constraints (7).

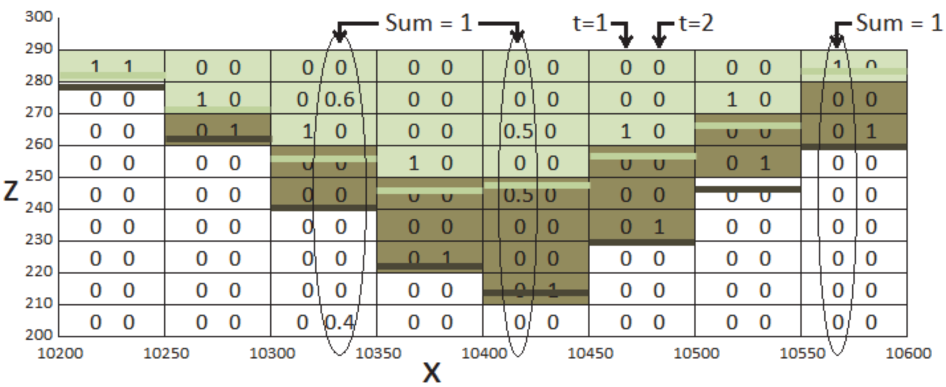


Figure 4: Linear relaxation of binary variables.

Recall that the binary variables in the formulation are related to surface elevations by constraints (6) and (7). These constraints guarantee that the weighted average of the block elevations fall just above the respective cell elevations. For example, in the third column of mining blocks in Figure 4, considering the second mining period, there are two blocks with fractional values assigned. One block is assigned with value 0.6 and located at elevation 275m. The second block has value 0.4 and elevation 205m. The weighted average is calculated by: $0.6 \times 275 + 0.4 \times 205 = 247$m. The surface for the second period in the same column is located at elevation 240m. Hence, constraint (6) is respected in the linear relaxation, as it must be, considering that the weighted average is in between 240m and 250m. Figure 4 presents just an illustrative example to facilitate understanding, but any combination of fractional values for variables $x_{c,t}^{z}$ that respect (6) and (7) will be part of a valid solution. However, fractional values must exist both above and below the cell elevation, otherwise the weighted average of block elevations will not comply with constraints (6). If other surfaces are used to split the deposit into parts and blocks are fixed to zero in one side of the surface, there will be a considerable reduction in the number of possible combinations of fractional solutions, which consequently reduces processing time. These limiting surfaces are the basis for the sequential approach proposed herein and will be called as top or bottom limits.

3.3.2 Engineering Aspects

The sequential implementation of the proposed method considers some engineering aspects of a mine's production schedule to improve efficiency of the optimization processes involved. The first objective of the sequential implementation is to find the most profitable available limits when mining a single period. The original topography serves as the initial upper limit for this single period, while the bottom limits may be defined, as pre-processing step, such as to reduce the number of blocks involved in each optimization process; more specifically, the implementation considers the following.

3.3.2.1 Bench limits

Mining operations have practical limits on minimum/maximum benches (levels of the orebody models) to be mined over a production year, productivity and operational constraints related to long-term plans and mining capacities. For this reason, bench limits are established for each period to define the maximum meaningful depth for a surface. The example depicted in Figure 5 to 9 is a simplified illustration used to help explaining the sequential steps. Assuming not more than 3 benches can be mined per period, the mining surfaces of periods 1 and 2 are not permitted to mine below the sixth and third benches, respectively, while the surface of the third period is left unconstrained (*free*). Note that, in Figure 5, the schedule deals only with period 1, therefore bench limits for periods 2 and 3 are not considered at this step.

3.3.2.2 Maximum mining depth

While the bench limit does not allow mining below a fixed elevation, the maximum mining depth parameter defines the maximum distance that each cell of the surface can move downwards, creating a bottom limit parallel to the top (please see arrows in Figure 5). This parameter is related to the practical assumption that it is not possible to mine more than a given depth in a fixed amount of time, depending on fleet, productivities and other mining considerations.

3.3.2.3 Fractional periods

For further efficiency improvements, the pit space is discretized dynamically. Instead of running one optimization process for the entire first period at once, including all accessible blocks in the optimization process, the approach runs the formulation for fractions of the period (fractional periods) sequentially, rescaling full period targets and limits, until achieving targets for the entire period. For example, consider all mining targets and limits for quarters of production; the algorithm finds four consecutive smaller schedules, always using the previous result as an upper limit for the next; the bottom of these four quarters is considered a surface for the entire mining period. Figure 5 illustrates the schedule of two fractional periods of production. The final objective of these fractional steps is to find one feasible solution that will be later improved (Section 3.3.3).

The combination of these strategies for defining bottom limits to each optimization process (bench limits, maximum depth and fractional periods) improves efficiency, as decisions are taken locally and sequentially. Figure 5 shows an example of two fractional periods considering maximum depth of 20 meters for each. The maximum depth is represented by arrows for each column until reaching the limit 20 meters below the actual surface. The bottom of the last fractional period is considered the surface representing the first period of the schedule. Note that, for clarification purposes, only mining blocks and limits are presented in the figure, however all operations are related to cells elevations, which are continuous variables and are not required to be at the bottom elevation of the blocks.

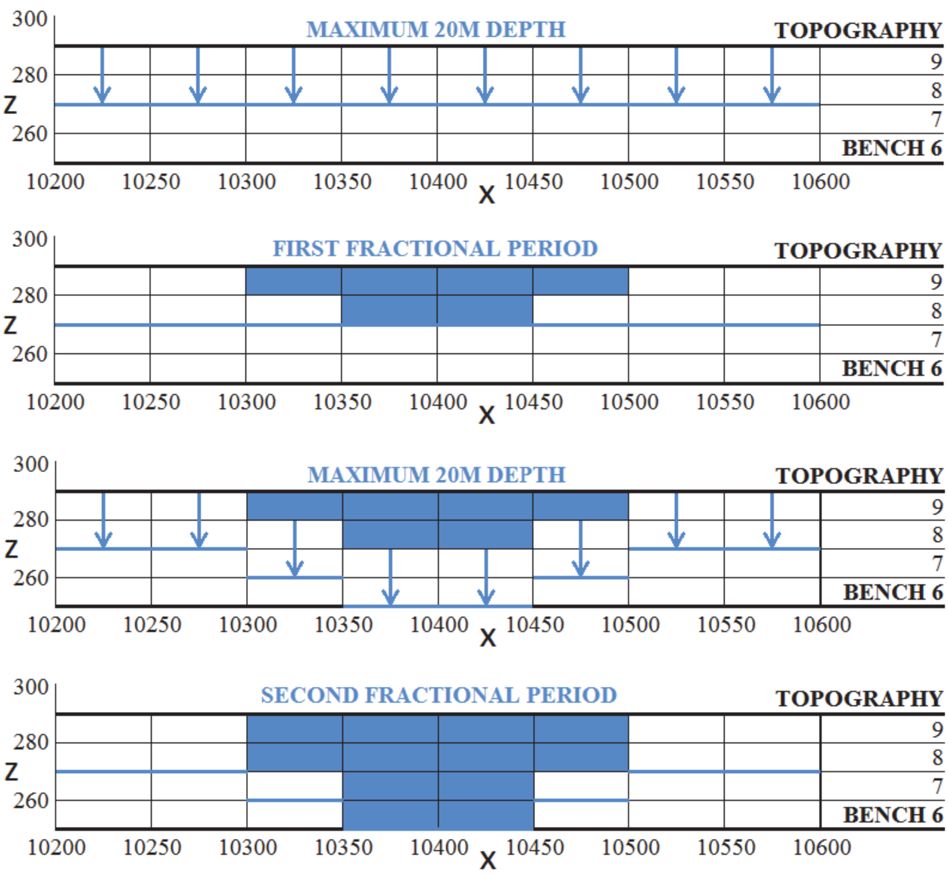


Figure 5: Two sequential fractional period optimizations within limits.

3.3.3 Schedule Improvement Steps

In order to assess potential further computational improvements, a local search strategy is performed (Aarts and Lenstra, 2003). A distance range is considered above and below the first period surface found, defining new top and bottom limits, and the optimization is executed again. The algorithm loops over this process until no other better solution (Equation (1)) is found. In this step, the surface is allowed to move inside given ranges, and blocks are combined accordingly. Bench limits are still respected, but maximum depth limits are not defined for entire mining periods, being used only as a tool for improving efficiency in finding an initial solution. Figure 6 illustrates one loop over this step with a range of 10 meters (dashed lines) over the surface of period 1 (continuous line), where elevations are changed, including two blocks and excluding the other two. Recall that blocks are assigned to periods as a consequence of surface elevations.

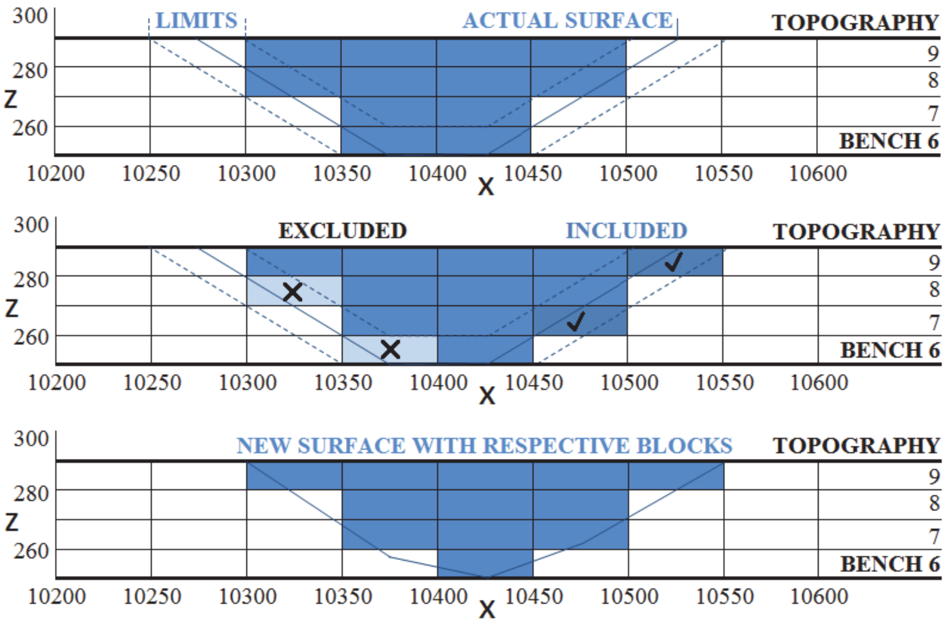


Figure 6: Tolerance range (one loop) applied to results for period 1.

3.3.4 Multiple-period Optimization

With the best solution (Equation (1)) for period 1, after the last step of Section 3.3.3, period 2 is then considered. Using the surface for period 1 as top limit, the same steps presented in Figure 5 are repeated for period 2. Figure 7 shows solutions being defined sequentially, first for 1.5 periods and then for 2 periods. The result will be a feasible initial solution for the surface of period 2.

The schedule found separately for periods 1 and 2 in previous steps are then combined into a joint optimization for two periods. The distance range defined in the previous section is now considered simultaneously to both periods, allowing combined adjustments to increase the expected net present value (NPV) of the project, as presented in Figure 8, where two blocks swap periods, allowing the inclusion of a new block in the schedule. The steps presented are repeated until the end of the schedule.

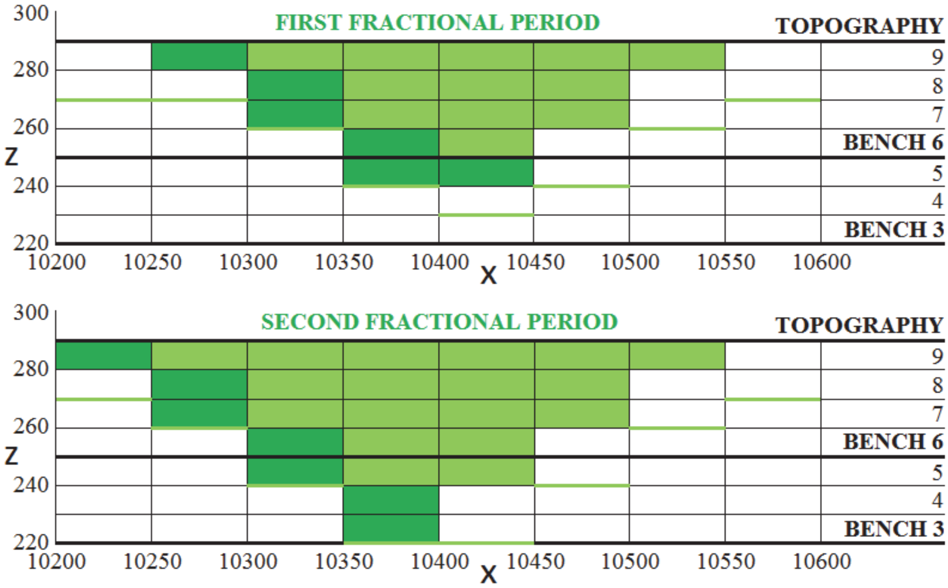


Figure 7: Optimization process achieving targets for the two first periods.

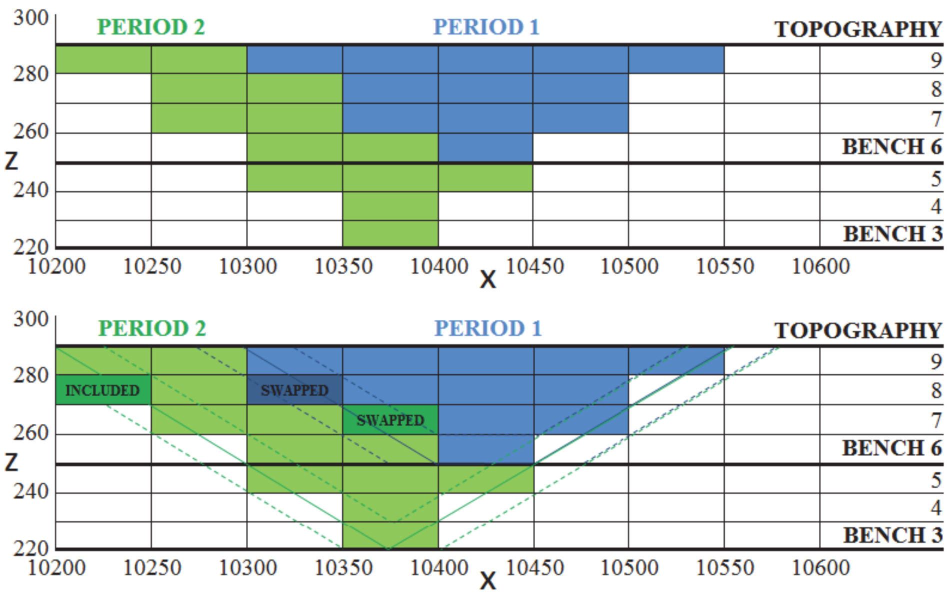


Figure 8: Two-period joint optimization, applying tolerance ranges to both.

3.3.5 Finding Deeper Pit Limits

In the last step, a new search is performed beyond the limits of the last period to explore if additional mining blocks can be included in the final schedule. This step is performed by fixing all previous periods and setting the surface of the last mining period as the top limit for running again the last optimization process, that is, new blocks can be included but not excluded. Note that the proposed approach does not require any given final pit surface, being all remaining blocks of the model available to be included in the last period of the production schedule. At this point of the scheduling process, fewer blocks are available in between the surface of the last period and the bottom limit defined in Step 2, allowing for an optimization without the use of bench and maximum depth limits. The top of Figure 9 shows the resulting 3-period schedule, after following the same steps presented in Figures 7 and 8; the bottom shows the inclusion of one extra block, resultant from searching the simulated orebody models deeper. After this step, the optimization process terminates.

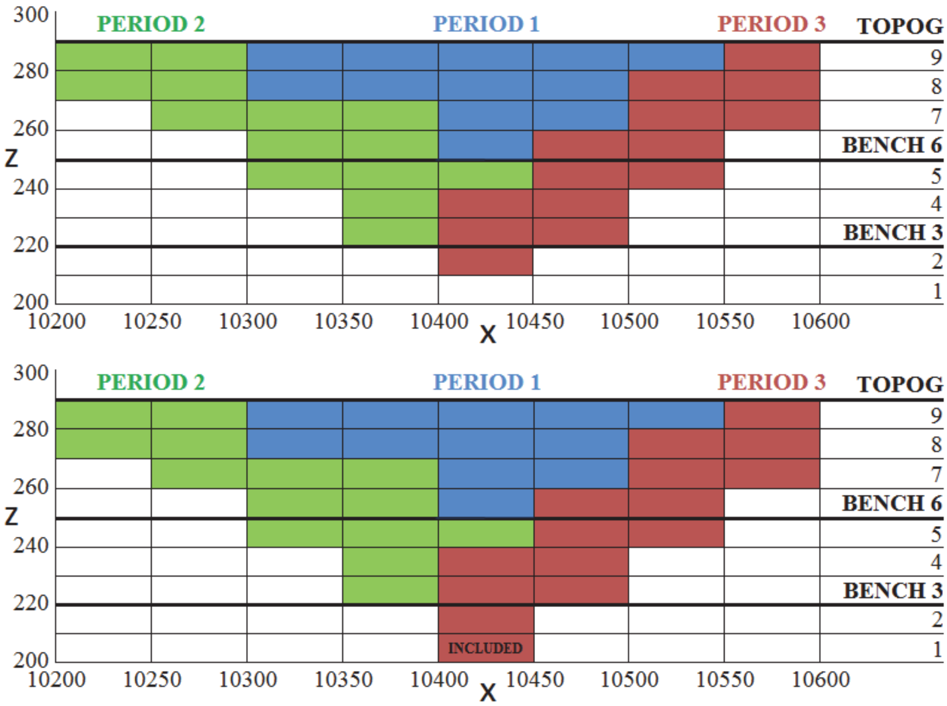


Figure 9: Final schedule including the third period and looking deeper.

The sequential implementation of the SIP formulation in Section 3.2, as discussed above, raises the questions of optimality of the related production sequence. This topic is addressed and explored next through an application.

3.4 Testing the Formulation and the Sequential Implementation

This section presents the application of the mathematical formulation introduced before. Subsequently, Case 1 introduces the use of fractional periods and maximum depth limits; Case 2 establishes bench limits; Case 3 tests the sensitivity of the maximum depth parameter; lastly, Case 4 explores variability constraints.

The small copper deposit simulated in Leite (2008) is considered here for testing purposes, allowing comparisons in terms of performance and sensitivity regarding input parameters. The orebody model consists of 9953 blocks of $20\times20\times10$ m³ size and simulated using the direct block simulation method (Godoy, 2003; Boucher and Dimitrakopoulos, 2009). The parameters considered for scheduling are given in Table 1. The annual ore production variability is controlled with a set of hard constraints defined experimentally and assuming a life of mine of eight years, as known from previous studies (Albor and Dimitrakopoulos, 2009; 2010), from where parameters were taken.

Copper price, $/lb	1.9
Selling cost, $/lb	0.4
Mining cost, $/t	1.0
Processing cost, $/t	9.0
Recovery, %	90
Discount rate, %	10
Mining capacity, Mt/year	28
Expected ore production, Mt/year	7.5
Slope angle, degrees	45
Cut-off grade, %	0.3

Table 1: Economic and technical parameters for testing

Graphs in the next sub-sections report the average forecasts of ore, waste, metal and cumulative cash flows along with the corresponding deciles $P10$, $P50$ and

$P90$. Results are based on 15 simulated realizations of the deposit, while the production forecasts reported and their risk assessment are based on 20 simulations.

3.4.1 Scheduling with the "Full SIP"

The life-of-mine production schedule results for the "Full SIP" are summarized in Figures 10 and 11. On average, ore production is close to the specified ore production target of 7.5 Mt per year. The waste risk profile presents low variability through all scheduled periods. Metal production starts at higher levels at the beginning of the life of mine, remains stable from period 2 to period 6, and decreases in the end, as expected. Figure 11 presents the cumulative NPV for this project showing a 90% probability of achieving at least 254 million dollars with expected NPV of $\$277$ million. Figure 12 shows, in a vertical section, the physical schedule obtained, presenting a shape relatively easy to be operationally designed.

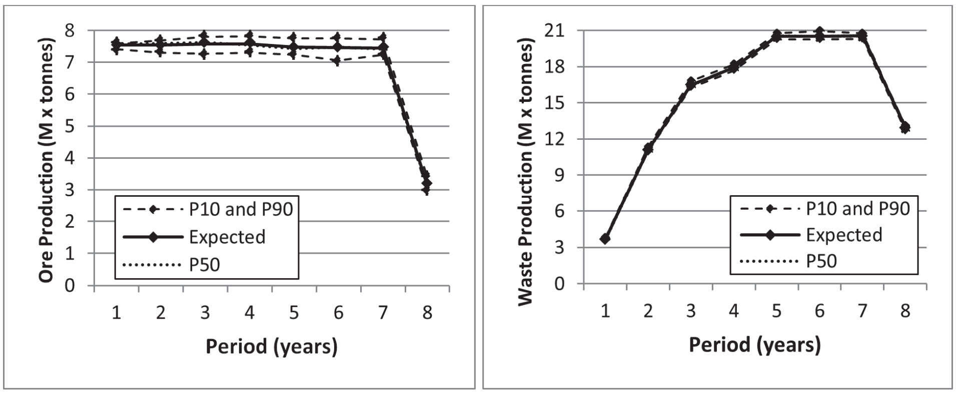


Figure 10: Risk profile for ore and waste tonnages for "Full SIP" solution.

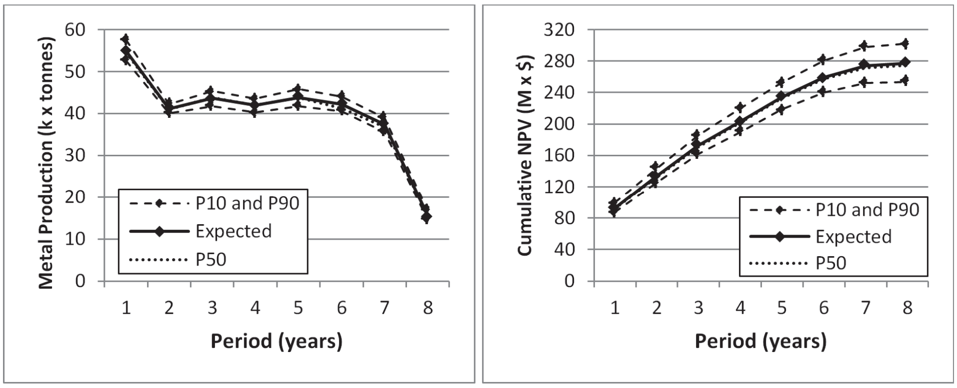


Figure 11: Risk profile for metal tonnages and NPV for “Full SIP” solution.

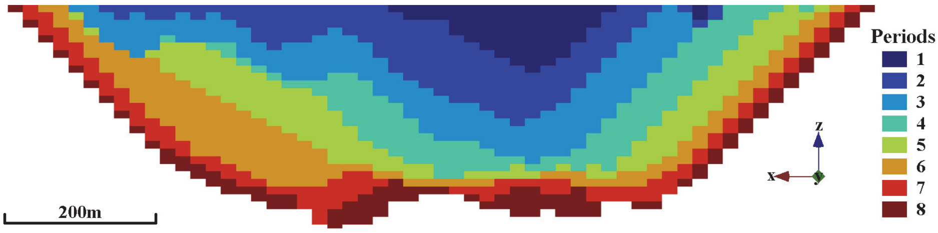


Figure 12: East-West deepest vertical section (N10290) for “Full SIP”.

3.4.2 Sequential Implementation

3.4.2.1 Case 1

In Case 1 no bench limits are imposed and the maximum mining depth is defined as 20m for each fractional period. The resulting schedule is identical to the schedule generated for the “Full SIP” case, as it can be observed in Figures 13 to 15.

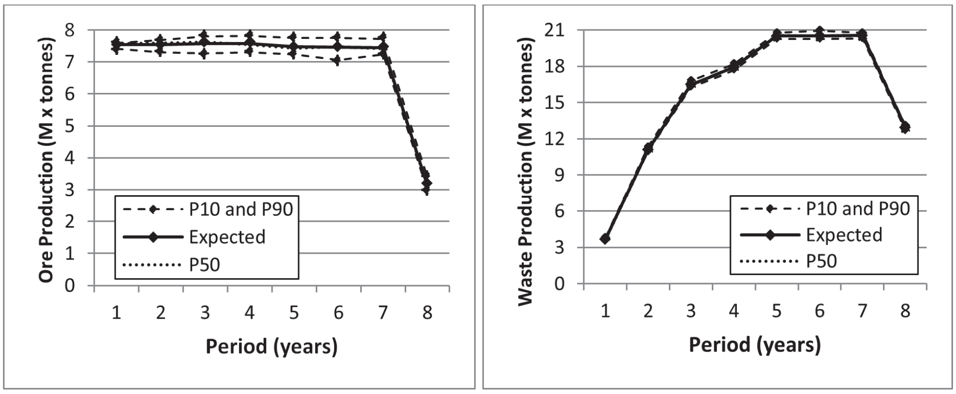


Figure 13: Risk profile for ore and waste tonnages for Case 1 solution.

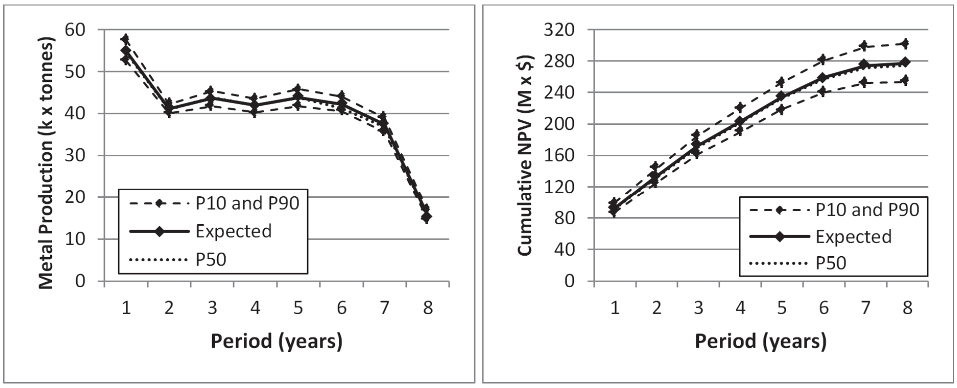


Figure 14: Risk profile for metal tonnages and NPV for Case 1 solution.

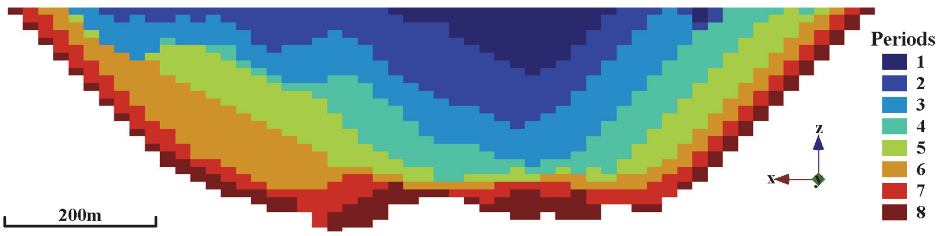


Figure 15: East-West deepest vertical section (N10290) for Case1.

3.4.2.2 Case 2

In Case 2 bench limits are setup. The number of benches allowed to be mined from periods 1 to 8 has the following configuration: $9,6,5,3,2,1,1, free$. Results are presented in Figures 16 to 18. The solution indicates that an additional average of 0.4 Mt of ore and 3 Mt of waste being mined in this case with the same average NPV of 277 million dollars, with 90% chance of being higher than 252 million dollars. The differences in waste quantities are mostly related to periods 4 and 8. Comparing these numbers, vertical sections and risk profiles, the schedules in Cases 1 and 2 are identical for all practical purposes. The negligible physical differences are mostly located in periods 4, 5 and 6, when comparing Figures 13 and 15.

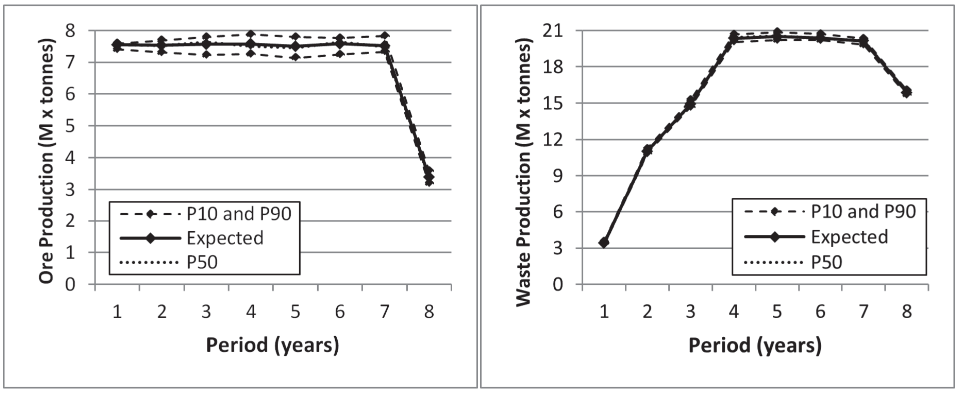


Figure 16: Risk profile for ore and waste tonnages for Case 2.

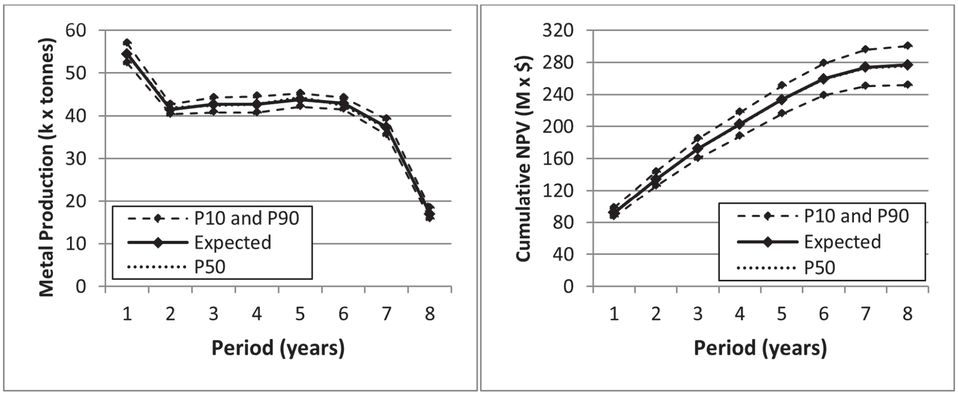


Figure 17: Risk profile for metal tonnages and NPV for Case 2.

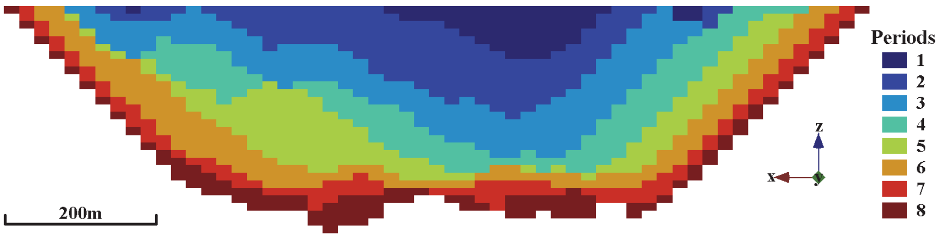


Figure 18: East-West vertical section (N10290) for Case 2 schedule.

3.4.2.3 Case 3

Case 1 shows the same results of the “Full SIP” for no bench limits setup, while Case 2 includes bench limits with the same maximum depth setup. Case 3 relaxes the maximum depth from 20m to 30m and uses the same parameters as Case 2, returning slightly less waste over period 3. Figures 19 to 21 indicate that there are negligible differences in the risk profiles. Vertical sections of Cases 2 and 3 are also similar, with differences mostly located over periods 3, 4 and 5.

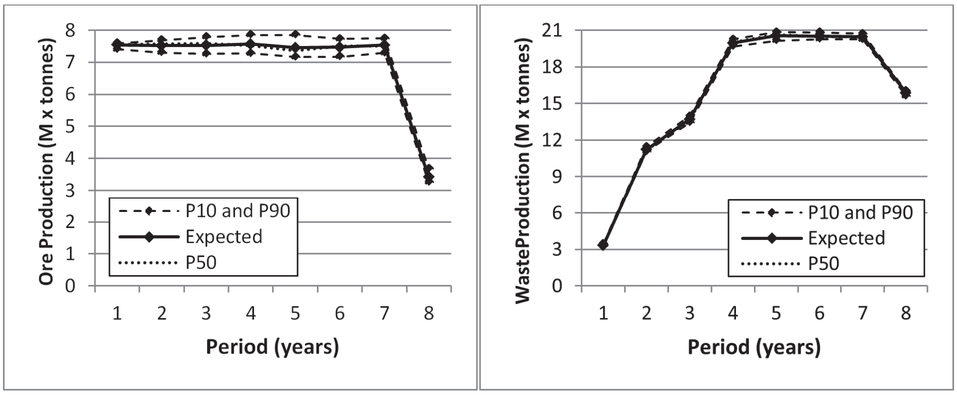


Figure 19: Risk profile for ore and waste tonnages for Case 3.

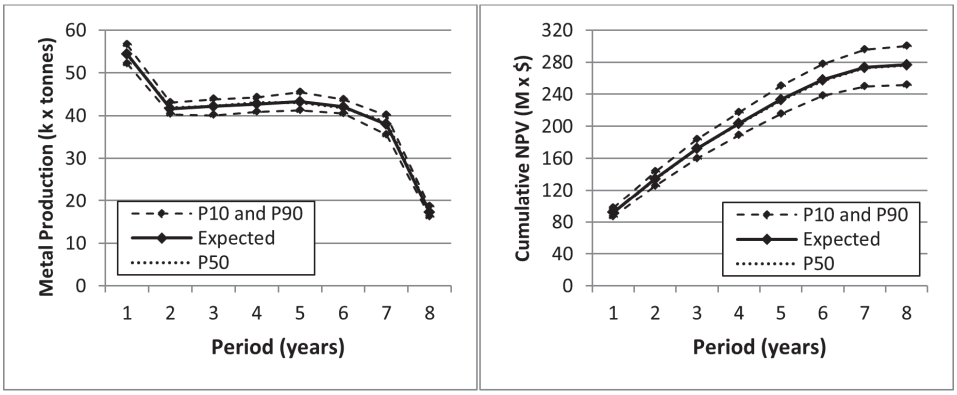


Figure 20: Risk profile for metal tonnages and NPV for Case 3.

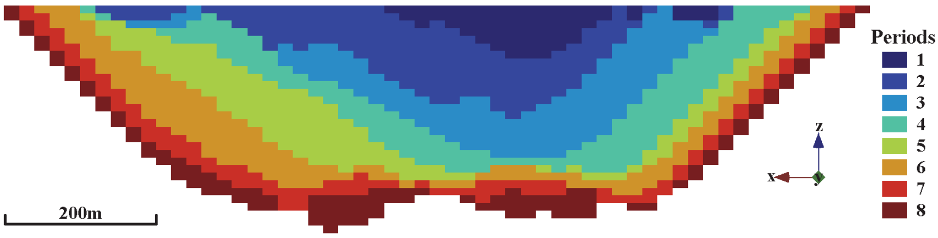


Figure 21: East-West vertical section (N10290) for Case 3 schedule.

3.4.2.3 Case 4

Case 4 defines new limits for $\underline{O_t^R}$ and $\overline{O_t^R}$ (equation 9) with remaining parameters equal to Case 2. Previous risk profiles for ore production have shown stable behaviour after period 2, which indicates the possibility of finding a feasible schedule within "tighter" limits. Defining "tighter" limits (Case 4) is not recommended before evaluating possible outcomes from "freer" setups (Cases 1 to 3), as it can incur in infeasibility or risk averse solutions. Results obtained show similar physical schedule, with slightly differences from period 2 to 7, and risk profiles when compared to previous cases, with exception of the waste profile that reduces from period 3 to period 4. There is improved control in the ore production variability, which increases over time, as presented in Figures 22 to 24. It can be noted that the same level of variability in period 2 for Cases 1 to 3 is being respected up to period 4 for Case 4, guaranteeing lower risk for a longer time horizon.

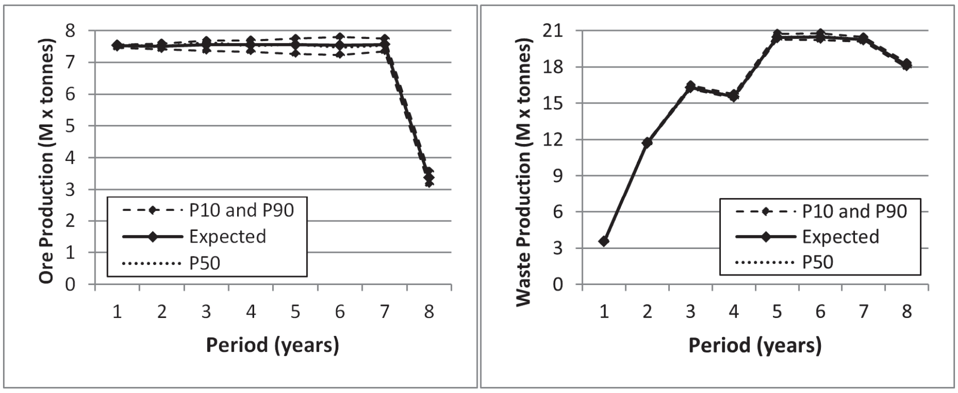


Figure 22: Risk profile for ore and waste tonnages for Case 4.

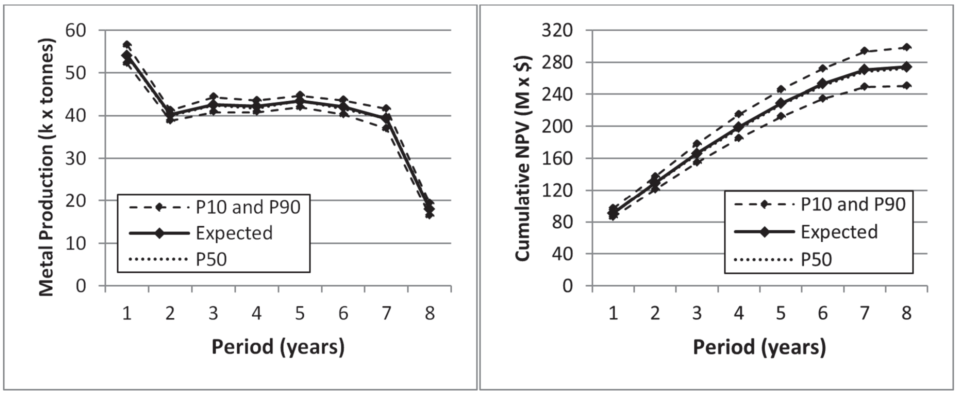


Figure 23: Risk profile for metal tonnages and NPV for Case 4.

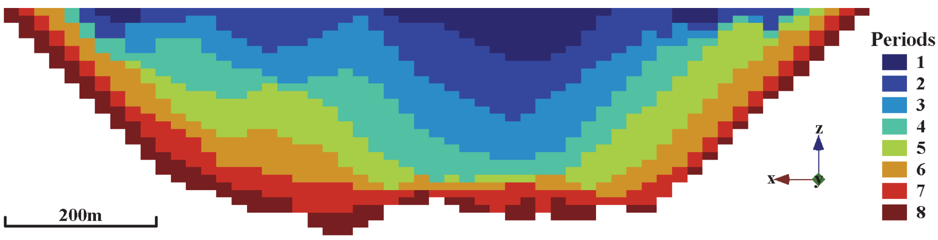


Figure 24: East-West vertical section (N10290) for Case 4 schedule.

3.4.3 Comparison

All cases presented in the previous section show similar risk profiles and physical schedules if compared to the “Full SIP”. The study suggests that the proposed approach converges to a similar solution, regardless of when different parameters are used. There are, however, substantial differences in computing times from the “Full SIP” to the other cases. The following reporting is based on runs over a 64-bit PC with Intel® Core™ i7-2600S CPU @ 2.80GHz, 8 GB RAM, 8 processors and using the IBM ILOG CPLEX 12.4 optimization software callable library. The proposed alternative sequential implementation reduces the processing times from days to minutes. Results for Cases 1 to 4 were obtained in ~0.6% of the processing time necessary for the “Full SIP”.

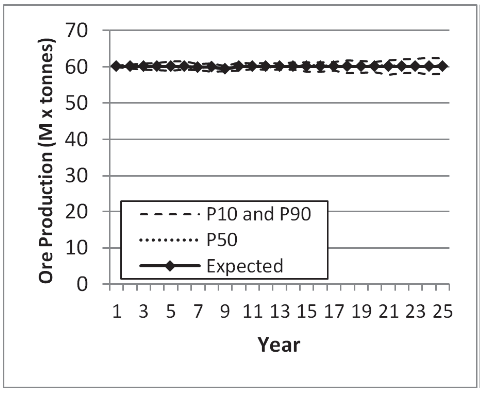
3.5 Application at a Copper Deposit

The proposed method is applied to a copper orebody, represented by 176,138 mining blocks of $25\times25\times15\text{m}^3$ size, to demonstrate the implementation of the proposed formulation at a reasonable sized deposit. Table 2 shows the parameters considered for the yearly-based schedule.

The results in terms of risk profiles and physical design for a 25-year schedule are presented in Figures 25 to 28. The optimization process considers all 176,138 blocks of the deposit, scheduling 90,825 blocks for 25 years of production.

Copper price, $/lb	2.00
Selling cost, $/lb	0.23
Mining cost, $/t	1.67
Processing cost, $/t	6.00
Recovery, %	80
Discount rate, %	8.5
Mining capacity, Mt/year	100.0
Expected ore production, Mt/year	60.0
Slope angle, degrees	41.0
Cutoff grade, %	0.2

Table 2: Economic and technical parameters for the case study.



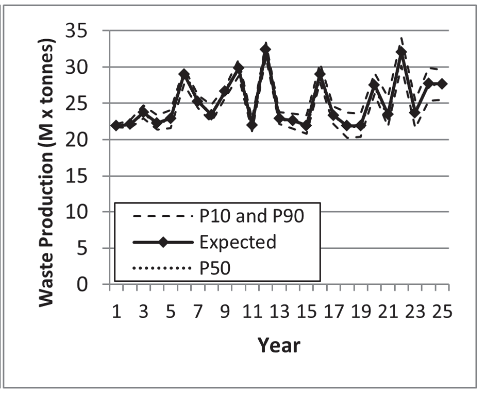


Figure 25: Risk profile for ore and waste tonnages for yearly schedule.

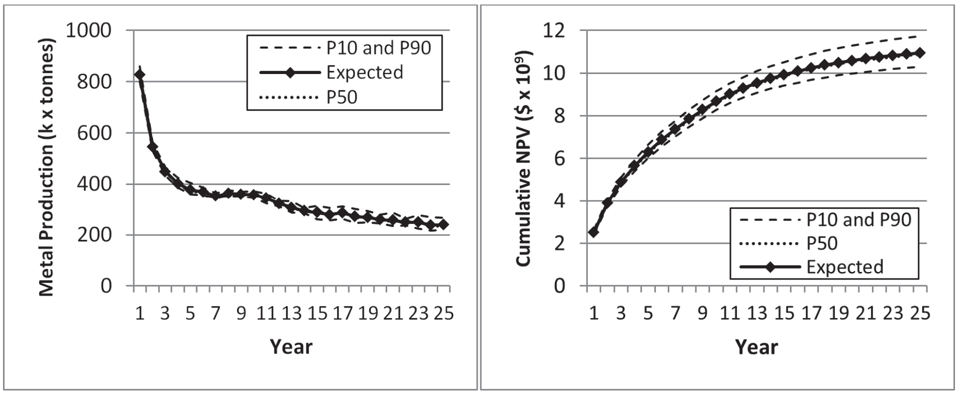


Figure 26: Risk profile for metal tonnages and NPV for yearly schedule.

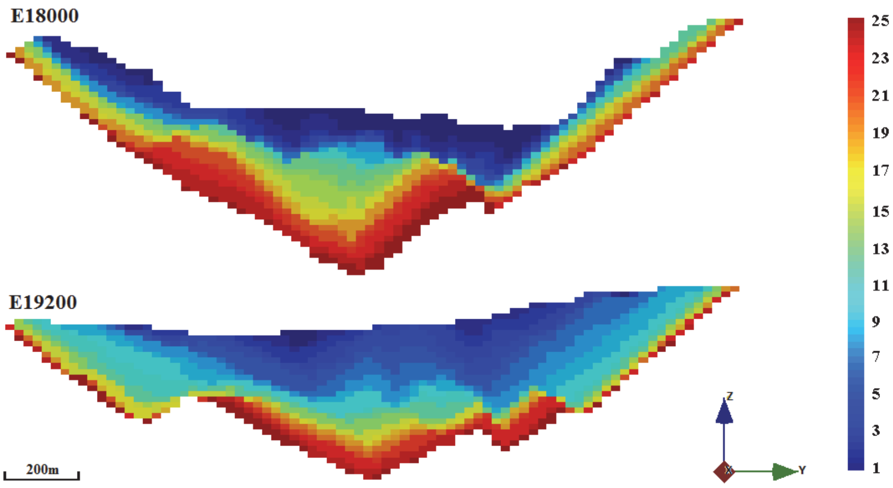


Figure 27: North-South vertical sections for yearly schedule.

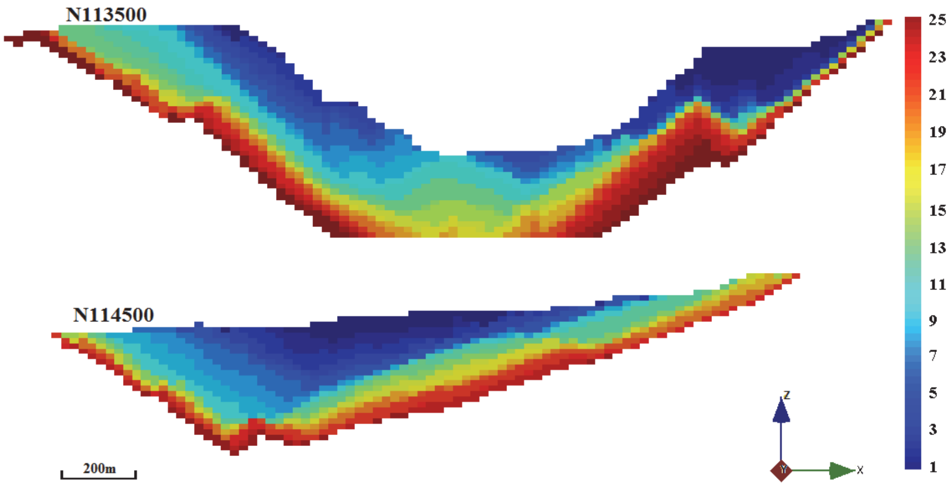


Figure 28: East-West vertical sections for yearly-based schedule.

The ore production risk profile shows stable behaviour around the target with low uncertainty. The waste profile oscillates in approximately ± 5Mt around 27Mt of production also with low risk associated, being above 30Mt for 2 periods. These variations do not allow for a practical fleet management, but production picks and valleys can be smoothed by anticipating waste during the schedule design. The expected metal production started with 828 kt reducing to a level slightly above 350kt in year 5, when it was keep almost constant until year 10. More than 50% of the expected NPV is obtained in the first 4 years of production and 90% is achieved within 15 years. Figures 27 and 28 present vertical sections of the physical schedule obtained with complex shapes that need to be properly designed. Slope constrains are respected, remembering that vertical sections are presenting only blocks, but angles are controlled over surface elevations.

3.6 Conclusions

A stochastic mathematical programming formulation based on surfaces was proposed herein for mine production scheduling optimization. The proposed sequential implementation divides the scheduling problem into sub-problems, using surfaces based on relevant engineering aspects of multiple-period scheduling. Fractional periods are defined to increase efficiency and guide the developments of the mine to regions where short-term periods also tend to respect constraints. Each multiple-period solution is further processed by allowing iterative changes in surface elevations of all periods simultaneously. The sequential implementation has been shown to generate the same practical results of the "Full SIP" case, even for different parameters. A case study over a full field copper deposit addresses the question of size, proving that the method can be efficiently applied to relatively large instances.

3.7 Chapter Appendix

In order to check the robustness related to the number of realizations considered in the optimization process, the method was performed for a 25-years schedule using sets of 15, 20 and 25 simulations. The behaviour of the resulting risk profiles over Figures 29 and 30 is the same, showing that running the optimization process with 15 simulations would be enough to describe the uncertainty over this deposit and would drive the process to almost the same results.

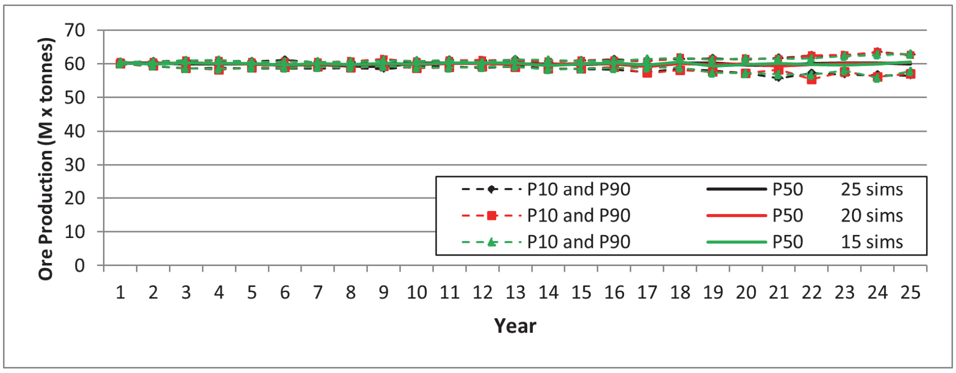


Figure 29: Risk profile for ore, considering 15, 20 and 25 simulations.

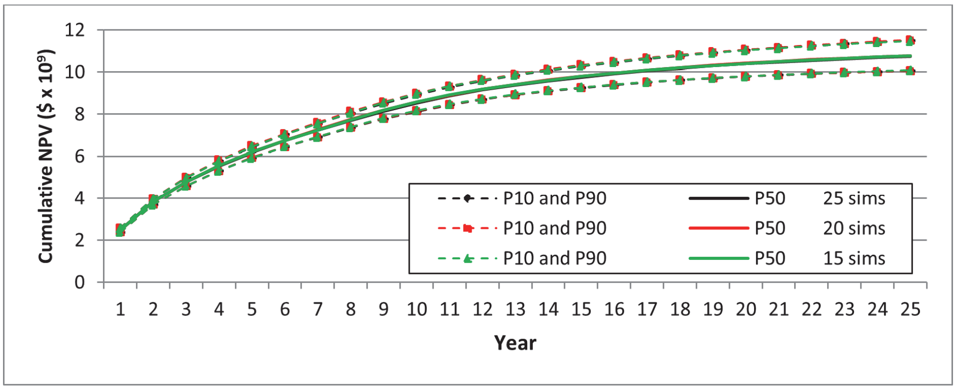


Figure 30: Risk profile for NPV, considering 15, 20 and 25 simulations.

Results presented in Section 3.5 consider a set of 25 simulations as input for risk profiles. A different set of 25 realizations was considered here for risk analysis over the same physical schedule obtained to assess the sensitivity of results to another group of simulations. Risk profiles are presented in Figures 31 and 32.

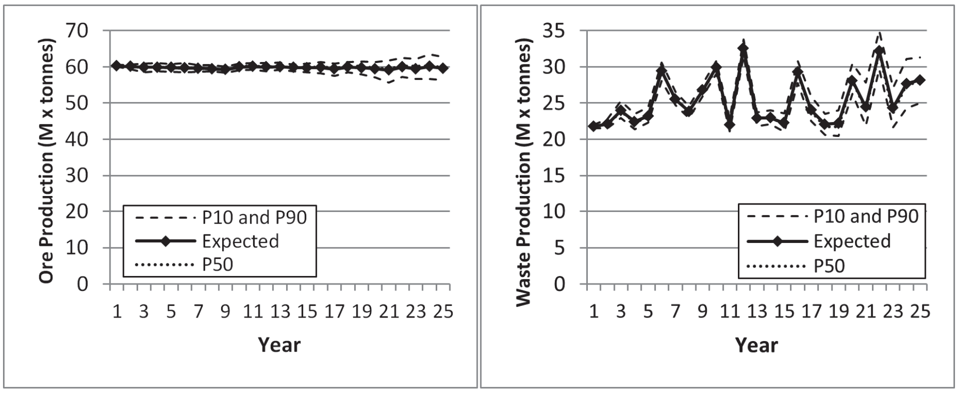


Figure 31: Risk profile for ore and waste for another set of simulations.

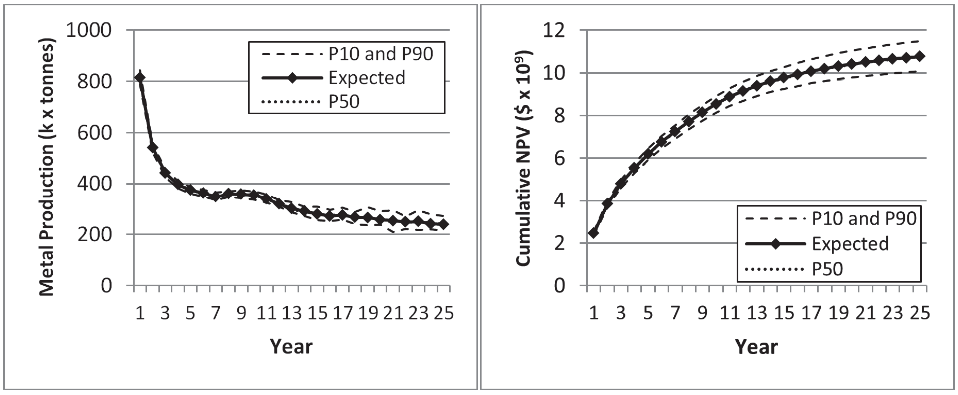


Figure 32: Risk profile for metal and NPV for another set of 25 simulations.

Risk profiles over ore and metal production (Figures 31 and 32) are following the same pattern described previously (Figures 25 and 26). Waste and NPV profiles are, in practical terms, identical, including the two picks above 30Mt in the same years. Therefore, it can be affirmed that the method, when applied to this deposit, returns robust results for 25 simulations. Figure 33 presents the risk profile for the average grade over periods, also considering another set of 25 simulations:

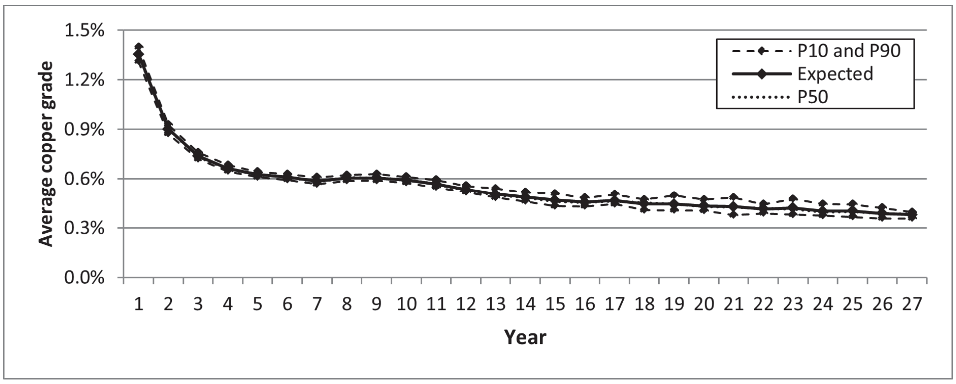


Figure 33: Risk profile for average grade for another set of 25 simulations.

Note that period 21 has “relatively” higher uncertainty in terms of expected average copper grade, but low variability is observed over all periods scheduled.

Chapter 4 – TWO-STAGE STOCHASTIC SURFACE CONSTRAINED MINE PRODUCION SCHEDULING WITH PIT DISCRETIZATION

4.1 Introduction

Mining is an activity based on extracting underground materials in a given sequence, such as to maximize the net present value (NPV) of the project. Planning the sequence of extraction, by the definition of a life-of-mine production schedule, requires understanding the uncertain metal quantities available throughout the deposit. Conventional estimation methods do not capture the intrinsic geological variability and uncertainty, returning single and possibly misleading production forecasts (Dimitrakopoulos et al, 2002; Dowd, 1994), which may underestimate potential metal production and project value (Godoy, 2003). A review of these issues as well as an overview of recent developments in dealing with uncertainty (stochasticity) in optimizing mine design and production scheduling can be found in Dimitrakopoulos (2011).

Stochastic optimization for long-term mine production scheduling using simulated annealing was a concept introduced by Godoy and Dimitrakopoulos (2004) and was further explored in Leite and Dimitrakopoulos (2007) and Albor and Dimitrakopoulos (2009), currently being applied to scheduling with multiple rock types and processing streams (Montiel and Dimitrakopoulos, 2012). Stochastic integer programming with recourse is introduced in Ramazan and Dimitrakopoulos (2005, 2012) to maximize total discounted cash flows, while minimizing deviations from production targets (ore tonnage, grade and metal), as well as deferring risk to latter production periods considering the concept of orebody risk discounting introduced by Dimitrakopoulos and Ramazan (2004), which penalizes deviations from production targets differently over mining periods. The SIP framework considers stockpiles and allows for controlling grades, ore and metal productions. The well-known drawback of any mixed-integer programming based approach in the mine scheduling context is computational due to the number of binary variables (Hustrulid and Kuchta, 2006). Notable variations of the SIP framework include: long- and short-term mine production scheduling based on simulated future grade control data (Jewbali, 2006; 2010); Albor and Dimitrakopoulos (2010) use the SIP formulation for pushback design, demonstrating that stochastically generated pit limits are larger than the corresponding conventional ones; Menabde et al. (2007) propose an alternate formulation that uses a variable cutoff grade and relies on aggregations of blocks to ensure the problem is computationally tractable; Boland et al. (2008) propose a multi-stage stochastic programming approach that considers both processing and mining decisions. To address the computational and size limits of SIP mine scheduling formulations, Lamghari and

Dimitrakopoulos (2012) introduce Tabu and Variable Neighborhood Search, bypassing the need to solve SIP formulations with conventional integer programming solvers and assisting computationally efficient solutions. Regarding stochastic pit space discretization, something which can facilitate the scheduling of large deposits, Asad and Dimitrakopoulos (2012) proposed a graph structure to consider geological and market uncertainties and solve the problem using a parametric maximum flow algorithm integrated with Lagrangian relaxation and the subgradient method. The approach considers grade and material uncertainty in order to reduce risks of misclassification over different processes.

Chapter 3 introduced a SIP formulation based on mining surfaces, building upon previous work from Goodwin et al. (2005), where the objective is to maximize discounted cash flows and control the risks of not achieving ore production targets. The work proposed a hybrid approach combining the mathematical formulation with a sequential implementation. Each mining period is subdivided and the schedule is sequentially optimized until finding an initial solution for the entire mining period, which is included in the long-term schedule and optimized jointly with periods previously defined. This formulation does not consider recourse actions for risk management which is included herein.

The concept of surfaces in mine production schedule optimization, first defined in Goodwin et al. (2005), is based on the fact that mining blocks describing a deposit are not independently distributed in space and can be grouped into vertical columns. Surfaces are defined as sets of elevations in which mining periods in the production schedule are divided. Each column of blocks can be partitioned by T surfaces into $T+1$ groups of blocks. Surfaces are divided into small pieces called cells. For each surface (or period), a cell is defined as a continuous variable carrying the elevation associated with a fixed pair of coordinates (x, y). Block attributes are accumulated starting from the topography down to the last block over each column, with cumulative values being stored at each level, which allows calculations by taking differences between surfaces. A key aspect of this approach is the need to associate blocks with surface cells and it is performed by comparing their elevations in space, as discussed herein.

The present paper builds upon previous work by considering the modelling with surfaces and the sequential approach from Chapter 3 and a two-stage SIP formulation with recourse actions as in Ramazan and Dimitrakopoulos (2012). In the next section, the mathematical formulation is proposed with a review of the sequential implementation. Then, a case study in a gold deposit shows the application of the method to a highly variable and relatively large deposit. The approach is applied first to discretize the pit space and, later, to define the yearly-based production schedule. Conclusions follow.

4.2 Two-Stage Stochastic Surface Based Scheduler

4.2.1 Notation

- M : number of cells in each surface.
- Z : number of levels in the orebody model.
- T : number of periods over which the orebody is being scheduled.
- S : number of simulated orebody models considered.
- E_c^z : elevation of the centroid for a given block (c, z).
- H_x : maximum difference in elevation for adjacent cells in contact laterally in the *x* direction, calculated by $H_x = \Delta x \times \tan(\theta)$, where Δx is the block size in x and θ is the maximum slope angle.
- H_y : maximum difference in elevation for adjacent cells in contact laterally in the y direction, calculated by $H_y = \Delta y \times \tan(\theta)$, where Δy is the block size in y.
- H_d : maximum difference in elevation for adjacent cells in contact diagonally, calculated by $H_d = \sqrt{(\Delta x)^2 + (\Delta y)^2} \times \tan(\theta)$.
- X_c, Y_c and D_c: equivalent to H_x, H_y and H_d concept, the sets of adjacent cells, laterally in x, in y and diagonally, for a given cell c, respectively.
- T_c^z : cumulative tonnage of block (c, z) and all blocks above it.
- $O_{c,s}^z$: cumulative ore tonnage of block (c, z) and all blocks above it in scenario s.

- $\underline{T_t}$ and $\overline{T_t}$: lower and upper limits, respectively, in total tonnage to be extracted during period t.
- $\underline{O_t}$ and $\overline{O_t}$: lower and upper target limits, respectively, in ore tonnage to be extracted during period t.
- C_t^- and C_t^+: costs associated to unit shortage and surplus, respectively, in tonnage of ore processed over period *t*.
- $V_{c,t,s}^z$: cumulative discounted economic value of block (c,z) and all blocks above it in scenario s and period t.
- $e_{c,t}$: continuous variables associated with each cell c for each period t, representing cell elevations.
- $x_{c,t}^z$: binary variables that assumes 1 if block (c,z) is the last block being mined in period t over c, and 0 otherwise. $x_{c,0}^z$ is defined as constant equal to 0, $\forall(c,z)$.
- $d_{t,s}^-$ and $d_{t,s}^+$: deviation variables measuring shortage and surplus, respectively, in the tonnage of ore processed over period t under scenario *s.*

4.2.2 Mathematical Model

The mathematical model proposed in Chapter 3 is extended herein to a risk management framework with a two-stage SIP formulation with recourse actions (Ramazan and Dimitrakopoulos, 2012). The objective function (1) maximizes the expected net present value from mining and processing selected blocks over all considered mine production periods, and manages the risk of not achieving ore production targets through the definition of a risk profile.

$$\max \frac{1}{S}\left(\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{c=1}^{M}\sum_{z=1}^{Z} V_{c,t,s}^{z}\left(x_{c,t}^{z}-x_{c,t-1}^{z}\right)-\sum_{s=1}^{S}\sum_{t=1}^{T}\left(C_t^- d_{t,s}^- + C_t^+ d_{t,s}^+\right)\right) \qquad (1)$$

The constraints presented in Equations (2) to (8) are scenario-independent, while constraints in Equations (9) and (10) are scenario-dependent stochastic constraints.

Surface constraints: the following set of constraints (2) guarantee that each surface t has, at maximum, the same elevation as surface $t-1$, which is used to avoid crossing surfaces and blocks being mined more than once. $e_{c,0}$ are constant elevations defined by the actual topography of the deposit.

$$e_{c,t-1} - e_{c,t} \geq 0 \qquad c = 1,\ldots,M; \qquad t = 2,\ldots,T \qquad (2)$$

Slope constraints: the maximum surface slope angle is guaranteed herein by Equations (3) to (5). Each cell elevation is compared to the elevation of the 8 adjacent cells, which therefore represents a set of $8 \times nx \times ny \times T$ continuous constraints. Note that adjacent cells are compared twice, guaranteeing upward and downward slopes. The number of slope constraints controlled by surface relations does not depend on slope angles and requires fewer constraints than conventional formulations. This is noted as follows:

$$e_{c,t} - e_{x,t} \le H_x \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T; \quad x \in X_c \tag{3}$$

$$e_{c,t} - e_{y,t} \le H_y \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T; \quad y \in Y_c \tag{4}$$

$$e_{c,t} - e_{d,t} \le H_d \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T; \quad d \in D_c \tag{5}$$

Link constraints: mining blocks and surfaces are linked in the formulation by comparing the elevation of each block centroid with the elevation of each surface. Variables $x_{c,t}^{z}$ will assume value 1 only for block centroids that are in the same elevation or exactly above the correspondent surface (index t). Constraints (6) guarantee this link and constraints (7) guarantee that there is only one block defining the end of each period t over each column c of blocks z.

$$0 \le \sum_{z=1}^{Z} \left(E_c^z x_{c,t}^z \right) - e_{c,t} \le \Delta z \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T \tag{6}$$

$$\sum_{z=1}^{Z} x_{c,t}^z = 1 \qquad c = 1,\ldots,M; \quad t = 1,\ldots,T \tag{7}$$

Mining constraints: constraints (8) ensure ore and waste production requirements are respected during each mining period:

$$\underline{T_t} \le \sum_{c=1}^{M} \sum_{z=1}^{Z} T_c^z \left(x_{c,t}^z - x_{c,t-1}^z \right) \le \overline{T_t} \qquad t = 1,\ldots,T \tag{8}$$

Constraints (7) and (8) measure deviations in processed ore tonnages, considering upper and lower bounds, $\overline{O_t}$ and $\underline{O_t}$, in order to penalize in the objective function.

$$\sum_{c=1}^{M}\sum_{z=1}^{Z} O_{c,s}^{z}\left(x_{c,t}^{z}-x_{c,t-1}^{z}\right)-d_{t,s}^{+}\leq\overline{O_t} \qquad s=1,\ldots,S; \qquad t=1,\ldots,T \qquad (9)$$

$$\sum_{c=1}^{M}\sum_{z=1}^{Z} O_{c,s}^{z}\left(x_{c,t}^{z}-x_{c,t-1}^{z}\right)+d_{t,s}^{-}\geq\underline{O_t} \qquad s=1,\ldots,S; \qquad t=1,\ldots,T \qquad (10)$$

The variables involved in the formulation are defined as following. Note that elevations are continuous and can be initially constrained by the limits of the orebody models.

$$e_{c,t}\in\Re \qquad c=1,\ldots,M; \quad t=1,\ldots,T \qquad (11)$$

$$x_{c,t}^{z}\in\{0,1\} \qquad c=1,\ldots,M; \quad t=1,\ldots,T; \qquad z=1,\ldots,Z \qquad (12)$$

$$d_{t,s}^{-},d_{t,s}^{+}\in\Re^{+} \qquad s=1,\ldots,S; \qquad t=1,\ldots,T \qquad (13)$$

4.2.3 Sequential Implementation

It was shown in Chapter 3 that solving a similar mathematical formulation based on surfaces using an exact method is impractical. The sequential implementation proposed therein is also considered herein, replacing the mathematical formulation by the one presented in Section 4.2.2. The approach is used first to discretize the pit space into phases of similar ore tonnages, where the formulation forces the scheduling of more profitable materials to initial stages of development. These defined phases are used later in order to reduce the complexity of the yearly-based production scheduling, which is optimized using the same approach, but following the pre-defined phases. The sequential implementation works as follows, where periods can be considered as years or phases. The case study to be presented in Section 4.3.2 solved more than 100 smaller optimization processes before finding the final solution, therefore steps are explained here in general terms and schematic illustrations can be found in Chapter 3.

Periods are included in the process in a stepwise fashion. A new period $t+1$ is added to the optimization process only after the method returns its best solution for a mining schedule of t periods. The proposed formulation is executed successive times and results for one optimization process are used as limiting assumptions for the next processes. The steps of the sequential implementation are:

1. Take the actual topography of the deposit as a top limiting surface; no other surface can go above this limit.
2. Similarly, define a bottom limit by eliminating external waste volumes, according to slope angles, but guaranteeing that every block with some probability of being ore is above this limit. No surface in any period can go deeper than such limits and blocks below are not considered in optimization processes.

3. If no phases are provided, this step should be skipped; otherwise, each period must be associated with one phase beforehand. Each surface cannot go deeper than its phase limits and blocks below are not considered in optimization processes for this period.
4. Find an initial solution for period 1 with successive runs of the mathematical formulation proposed in Section 4.2.2, given pertinent operational considerations: fractional periods, bench and maximum bench limits.
5. Iteratively improve the initial solution, using the same formulation in a Local Search approach with neighborhood definition.
6. Find an initial solution for period 2, as in Step 4. The results of Steps 5 and 6 will give a feasible schedule of 2 mining periods.
7. Improve this schedule considering the same Local Search approach from Step 5 but for all mining periods jointly.
8. Loop over Steps 6 and 7, including new periods, until there is no more profitable material available.
9. Freeze all periods except for the last and look for extra deeper blocks to be included, still respecting limits defined in Steps 2 and 3.

In order to use this approach for pit discretization, targets and limits required for each phase should be defined, including a rescaled economical discount rate per phase. The approach is then performed including phases until there is no more profitably mineable ore available. Finally, Step 9 of the implementation is performed, guaranteeing all profitable material is included. The schedule of phases is performed more efficiently, due to the reduced number of phases, if compared to the number of mining periods. The economical discounting per phase forces the method to mine the best material available under physical constraints in the first phase, and the same occurs sequentially for the next phases. The resulting surfaces for this schedule of phases represent an optimized discretization of the pit space into pieces with controlled conditions in terms of ore tonnages. These surfaces are later used to limit the complexity of the yearly-based mine production schedule optimization. Each mining period is associated to one phase; for example, if two phases were defined with ~100Mt of ore production and the schedule has to provide mining periods with ~40Mt of ore, Periods 1 and 2 can be assigned to Phase 1 and the remaining periods to Phase 2. The scheduling method limits the bottom of each mining period surface (Step 3), eliminating blocks below the assigned phase prior to the optimization processes, which improves efficiency and allows scheduling of larger deposits with longer life-of-mine.

4.3 Case Study at a Gold Deposit

The conventional approach and the proposed method are applied to a gold orebody, represented by 98,081 mining blocks of $20x20x20m^3$ size, to demonstrate the performance of the proposed formulation and its sequential implementation in comparison to industry best practices. Table 3 shows the parameters considered for the yearly-based schedule, which were taken as approximated values from actual large gold projects for testing purposes that illustrate the method.

Gold price, $/g	40.0
Mining cost, $/t	6.0
Processing cost, $/t	20.0
Recovery, %	90.0
Discounting rate per period, %	10.0
Mining capacity, Mt/year	85.0
Expected ore production, Mt/year	15.0
Slope angle, degrees	54.0

Table 3: Economic and technical parameters for the case study.

4.3.1 Conventional Mine Production Scheduling

The Whittle Software was used for mine production scheduling according to industry best practices based on a single estimated model (deterministic approach). In order to assess the geological uncertainty involved, this schedule was evaluated for each simulated orebody model considered, generating the risk profiles presented over Figures 34 to 36.

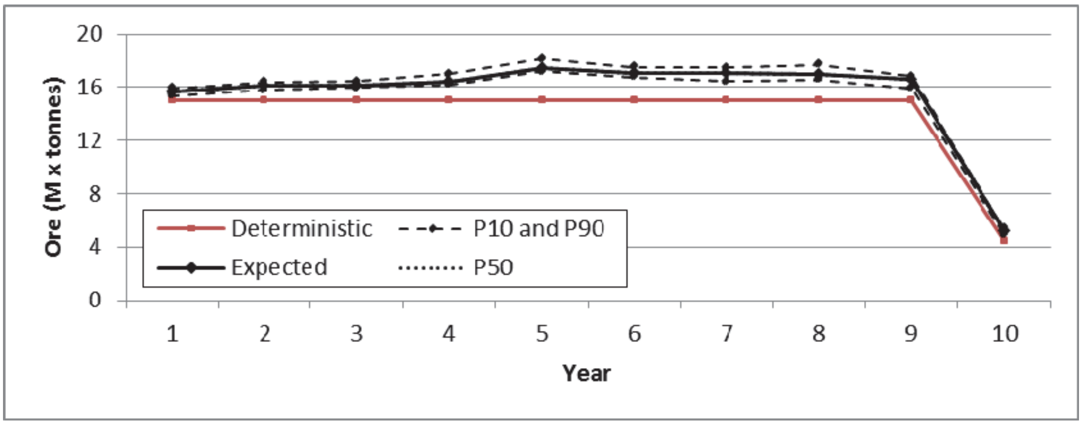


Figure 34: Risk profile for ore production in the conventional schedule.

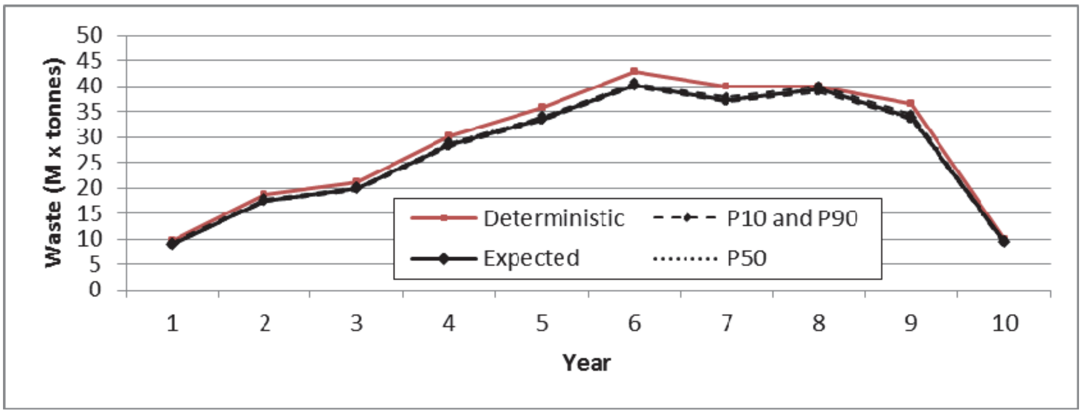


Figure 35: Risk profile for waste production in the conventional schedule.

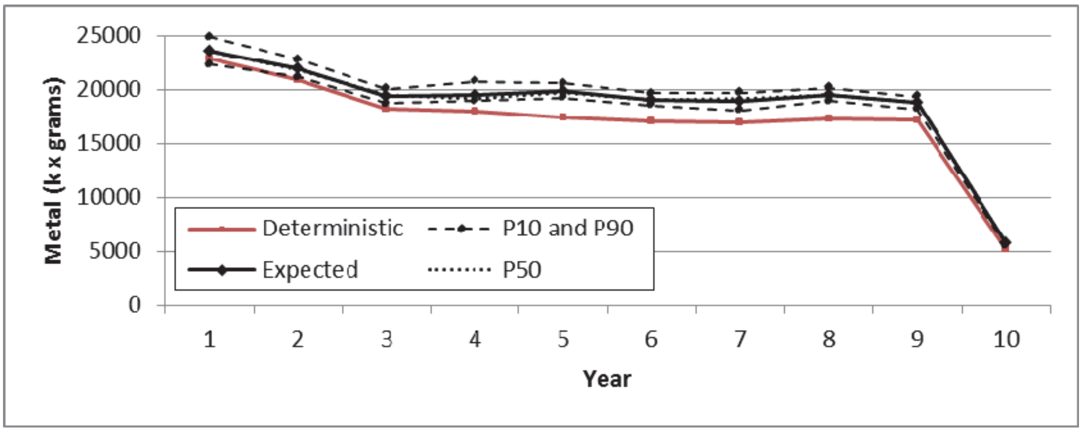


Figure 36: Risk profile for metal production in the conventional schedule.

All profiles illustrate that the answers provided by the deterministic approach are not likely to be achieved in practice when evaluated over equally probable scenarios. Figures 34 and 35 show that a representative amount of ore is being misclassified as waste, with consequent overestimation of the required fleet to handle inexistent extra quantities of waste. This schedule, if performed in practice, would require a stockpile with capacity above 15Mt to handle the ore not forecasted by the plan that has to be reclaimed in one extra year of production. Figure 36 shows the respective metal content available in each scheduled period with risk profile also above deterministic expectations. Net present value profiles are not comparable as the provided schedule is not realistic in terms of processing capacities.

4.3.2 Stochastic Pit Discretization

The method proposed in Section 4.2 has no requirements in terms of time frame for each period being scheduled; hence, it can be used to schedule weeks or decades of production and conclusions will be valid for the time frame chosen and the economic discounting rate considered. As the time frame increases more material is smoothed out inside periods and the economic discounting rate should be rescaled. For the same reason that yearly-based schedules are considered as a fair pit discretization to allow for later scheduling of months of production, the scheduling of phases is a fair discretization of the pit space to constrain a yearly-based schedule. The advantage of this kind of pit discretization is that geological uncertainty is taken into account while controlling ore tonnages per phase, which are more suitable limiting assumptions for a stochastic mine scheduler. Note that the concept here is different than the conventional definition of pushbacks where nested pits are defined by maximizing the undiscounted cash flow for a set of increasing metal prices. The effect of economic discounting considered here forces blocks with higher value to be mined as soon as possible with opposite effect for blocks with lower values; working with undiscounted values is just a shortcut for methods that are incapable to work within time frames.

The sequential algorithm was first applied to discretize the pit space into phases with similar ore production, but there is no operational parameter being controlled, such as minimum mining width, for example. The target ore production was setup to 50Mt in order to allow for 3 years of schedule (15Mt each) inside the first phase, leaving 10% (or 5Mt) of flexibility for risk management. Figures 37 and 38 show sections of the resulting 4 phases, presenting shapes of increasing size with more waste being mined in later phases.

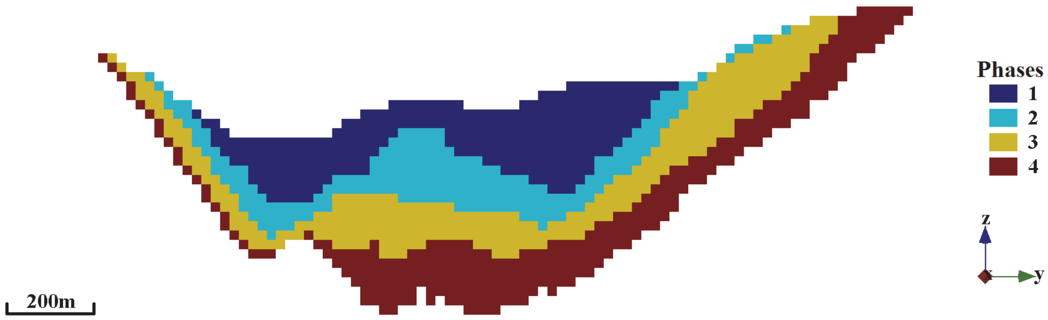


Figure 37: North-South vertical section E1030 for the mining phases.

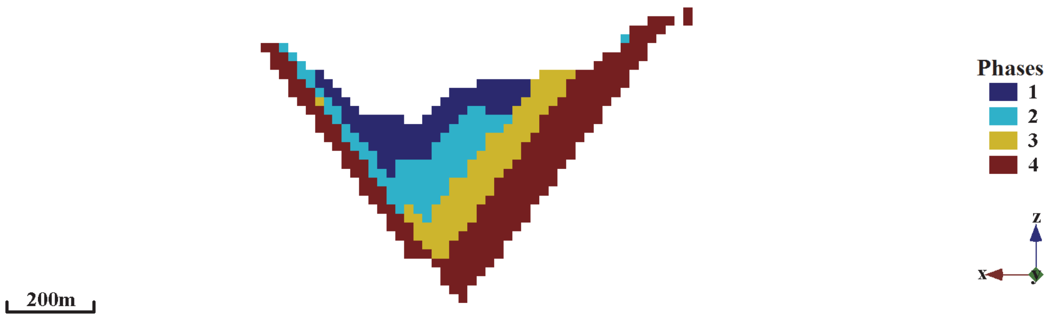


Figure 38: East-West vertical section N2250 for the mining phases.

The associated risk profiles over cumulative NPV, ore, waste and metal productions, along with the corresponding deciles P10, P50 and P90, are shown over Figures 39 and 40. The ore production profile shows higher production over the last phase, as the material of Phase 5 (~10Mt of ore) was incorporated into Phase 4. The waste production profile shows increasing behavior throughout phases. Metal production is always between 55 and 70 tonnes per phase and all production profiles demonstrate low uncertainty around expected values. The profile over cumulative NPV shows increasing behavior until the last phase and a small increment given by Phase 4; this profile cannot be evaluated in terms of absolute values, as the discounting rate applied is a long term approximation.

The amount of ore contained up to Phase 4 can be schedule at most in 14 periods. Hence, mining periods 1 to 3 are assigned to Phase 1; 4 to 6 to Phase 2; 7 to 9 to Phase 3; and the remaining to Phase 4. The surface of each period cannot cross below the surface of its assigned phase during the mine production schedule optimization.

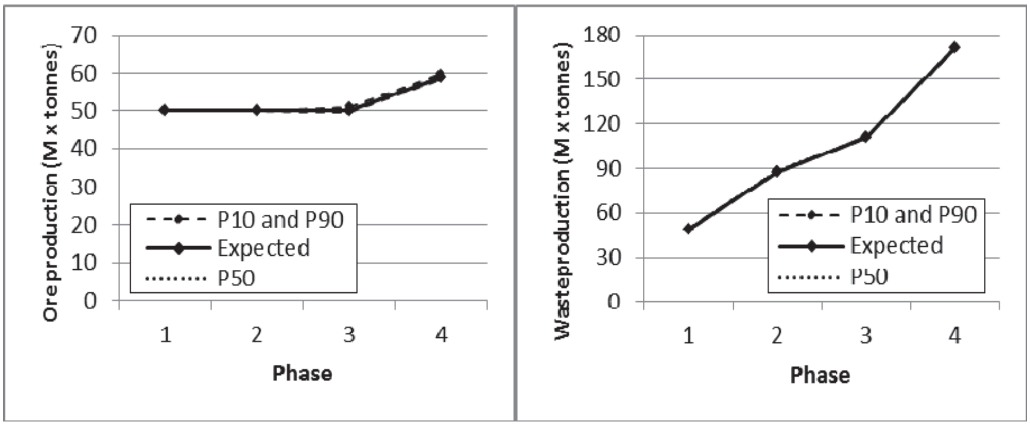


Figure 39: Risk profiles for ore and waste production for mining phases.

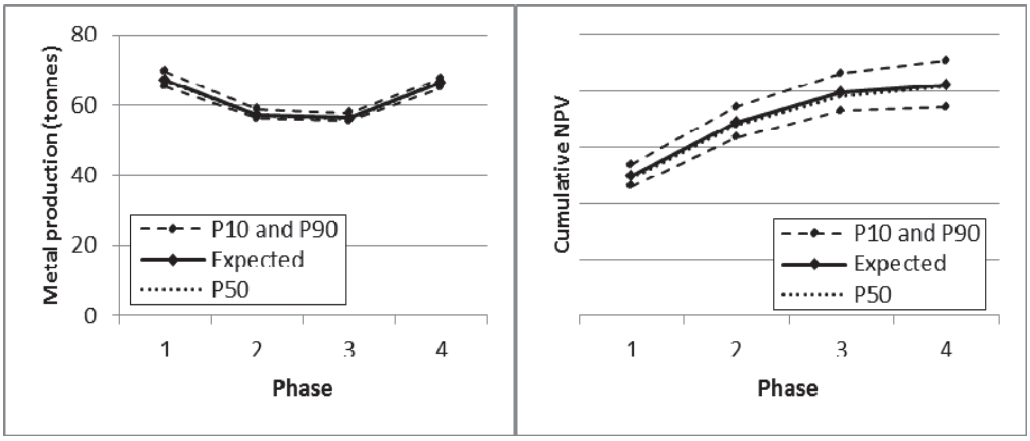


Figure 40: Risk profiles for metal and cum. NPV for mining phases.

4.3.3 Schedule Results

The resulting physical schedule is presented over Figures 41 and 42 with mining periods respecting phases previously defined. Note that results are not expected to be smoothed out as operational constraints are not taken into account in the mathematical formulation.

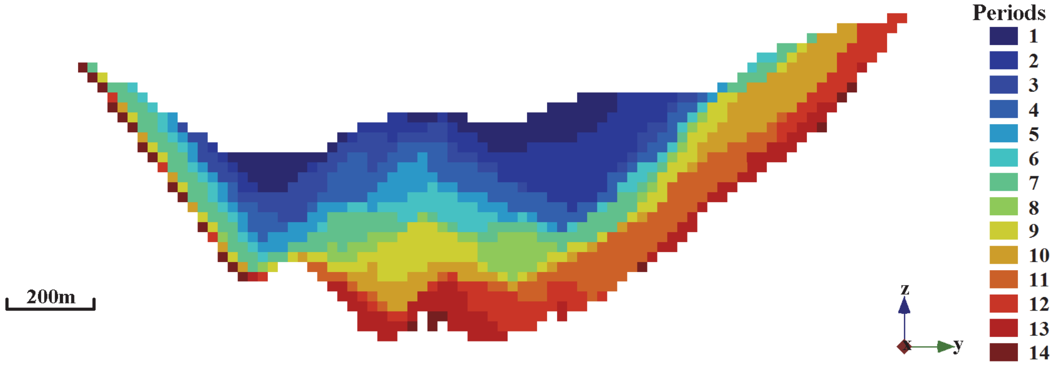


Figure 41: NS vertical section E1030 for the stochastic yearly schedule.

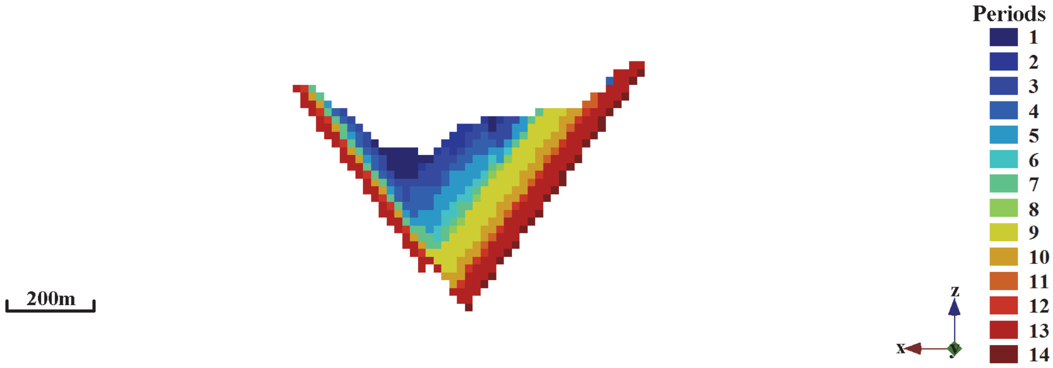


Figure 42: EW vertical section N2250 for the stochastic yearly schedule.

The associated risk profiles over cumulative NPV, ore, waste and metal productions, along with the corresponding deciles P10, P50 and P90, are shown over Figures 43 to 46. When considering the stochastic optimizer, four extra mining periods are scheduled. Figure 43 shows production controlled close to the target, except by a 10% shortfall in Year 9, but stable production over the remaining years. The shortfall represents a situation where it was worthy to pay for the incurred penalties instead of mining more ore to complete the capacity, probably due to the amount of overlaying waste. The profile shows lower risk in initial periods if compared to later periods, as proposed by the formulation. Figure 44 shows profile with increasing behavior, similarly to predefined phases, and almost no risk throughout the whole life-of-mine. Figure 45 shows higher metal production during the first two years, stabilizing after that. There is a low risk associated and it does not vary throughout the life-of-mine. Figure 46 presents the risk profile for the cumulative NPV with a total expected value of 1.35 billion dollars with 90% chances of being above 1.18 billion dollars. There is less than 3% increment in expected NPV after period 10 for the stochastic case.

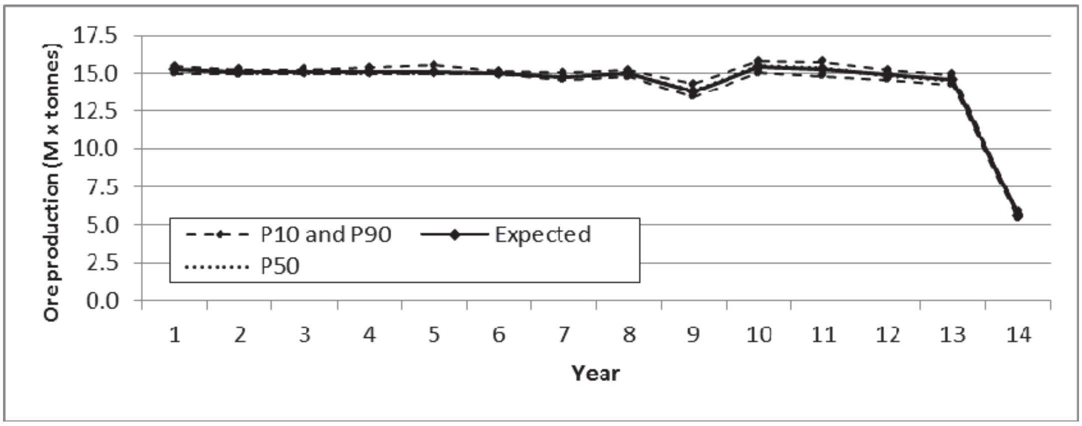


Figure 43: Risk profile for ore for the stochastic yearly schedule.

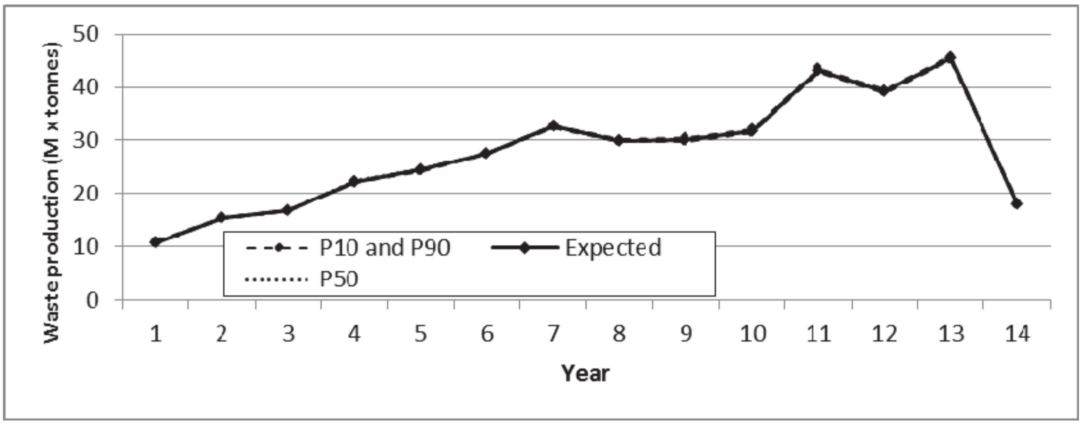


Figure 44: Risk profile for waste for the stochastic yearly schedule.

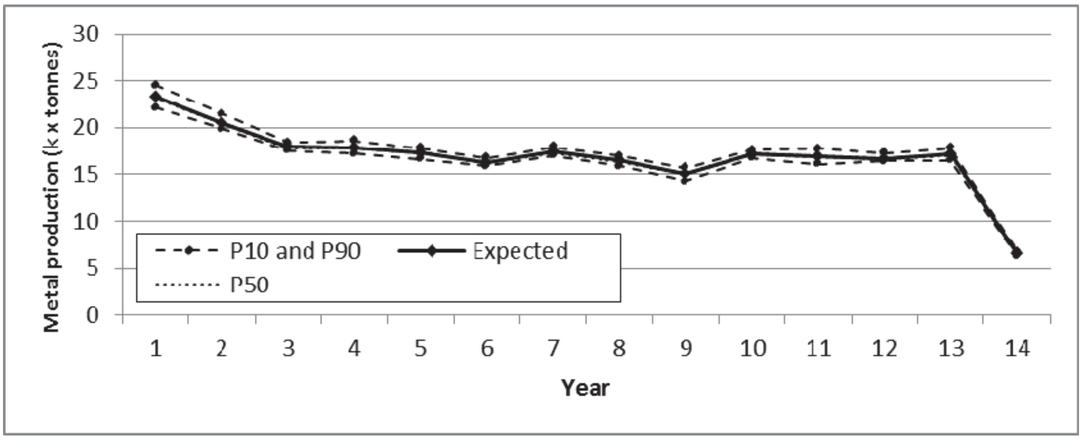


Figure 45: Risk profile for metal for the stochastic yearly schedule.

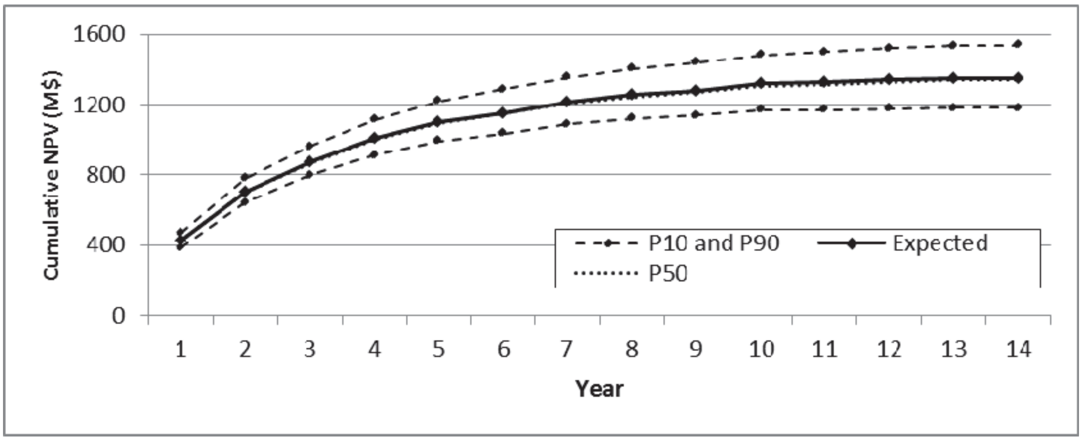


Figure 46: Risk profile for NPV for the stochastic yearly schedule.

4.4 Conclusions

The present work proposes a new mathematical programming formulation making use of limiting surfaces in the context of SIP for mine production scheduling optimization, adding the benefits of easier and general slope angle management, with simultaneous maximization of discounted cash flows and minimization of risks of not achieving production targets. The sequential implementation considered divides the scheduling problem into sub-problems, using surfaces based on relevant engineering aspects of multiple-period scheduling. Periods are included in a stepwise fashion by first defining one initial solution and later improving this solution using a Local Search strategy based on the same mathematical formulation. The approach is first applied to discretize the pit space into mining phases, which are then considered to reduce the complexity of the yearly-based mine production schedule optimization. A case study over a full field gold deposit addresses the question of size and returns 19% higher expected NPV with 43% more ore processed, if compared to industry best practices.

Chapter 5 – CONCLUSIONS AND FUTURE WORK

This thesis starts by presenting a literature review on conventional and stochastic life-of-mine production schedule optimization methods. Developments have shown that the use of stochastic frameworks allows finding mine production schedules with higher value and lower risk, simultaneously, while the introduction of (meta-)heuristic approaches allows for efficient implementations with results close to the optimal solution. The review also presents previous works which introduced the concept of surfaces in this context addressing the issues of efficiency. Chapter 3 proposes a SIP formulation based on surfaces to address the optimization of life-of-mine production scheduling, whereby the supply of metal is uncertain and described by a set of equally probable orebody representations. The proposed formulation maximizes discounted cash flows, controls risk of deviating from production targets and facilitates a divide-and-conquer approach, where scheduling can be performed sequentially, facilitating production scheduling for relatively large mineral deposits. Applications of the proposed mathematical formulation show computational efficiency as well as the equivalence of solutions generated sequentially. Chapter 4 extends this formulation to a two-stage SIP that manages the risk of deviating from production targets. The sequential implementation is considered for a pit space discretization followed by a yearly-based mine production scheduling at a relatively large gold deposit.

All case studies returned results in a reasonable amount of time, with production targets controlled, risk postponed to later stages of development and improvements in expected NPV if compared to deterministic industry best practices. Results proved the efficiency and suitability of the method for real sized instances, including the benefits of accounting for uncertainty in metal content.

Other possible improvement regarding the proposed method:

- Replace the CPLEX optimizer by a meta-heuristic approach, looking for improvements in efficiency.
- Introduce the use of parallel algorithms implemented over the CPU or the GPU (central/graphics processing unit). Neighborhood definitions and the choice of the meta-heuristic approach should take into account the feasibility of a parallel code.
- Adapt the method to consider simulations of prices and costs forecasts.

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Appendix A – Implementation Details

This thesis focused on explaining and exemplifying the main parts of the proposed method. Details of the implementation are discussed in this appendix.

A.1 Basic Procedures

The proposed formulations guarantee that there is exactly one block (binary variable) assuming value 1 over each column and mining period, and each of these blocks is located immediately above its respective surface. Therefore, one extra layer of air blocks above the initial surface must be created in order to avoid infeasibility in places where there was no mining.

The sequential approach should provide reports/files throughout its execution and allow for recovering intermediate solutions for tests with different sets of parameters; this remains a valid comment for individual surfaces as well, which could be used at any moment to limit other processes.

The definition of the number of fractional periods is case specific and has to be tested until the most efficient setup is found. Less fractional periods mean more tonnage for each, which may imply infeasibility or penalties, due to bench and maximum depth limits. Defining fractions dynamically combined with the maximum depth parameter was tested without success. Details will not be reported here, but the conclusion is that fractions should have equal size, otherwise processes become unstable and less efficient.

A.2 Gap Setup

One important parameter intrinsic to integer programming models is the maximum gap accepted, which is the percentage distance between the actual integer solution and the best solution found by the linear relaxation. The proposed method essentially has two different kinds of problems to solve: finding initial solutions through optimizations of fractional periods and improving these solutions through multiple-period optimizations. For the first case, due to the reduced size and highly constrained rules, CPLEX is efficient, returning solutions below 1% gap in a few seconds or minutes for the deposits studied. However, for the multiple-period steps, maximum gap definition becomes a sensitive parameter.

A.3 Bench Limits

Bench limits were presented as a tool to reduce size and complexity of the optimization process, but constrained bench limit configurations could drive processes to infeasibility or forbid the optimizer to find deeper (maybe more profitable) solutions. Another case over the same testing deposit illustrates one solution found, considering the following bench limits configuration: 7, 4, 4, 4, 4, 3, 1, *free*. The results of the optimized schedule are presented in Figures 47 to

49, showing that risk profiles are still similar to previously presented cases, except by a small peak in metal production over period 5. The physical schedule is significantly different which is a result of stronger bench limits that do not allow mining to go deeper. Tighter bench constraints allow for finding different schedules that may achieve similar economical results and risk profiles but with operational consequences and benefits in processing time, as this case ran 2 times faster than the others. Results indicate that mining could go deeper, but it is forced to advance laterally, leaving more blocks isolated over the boundaries of the pit.

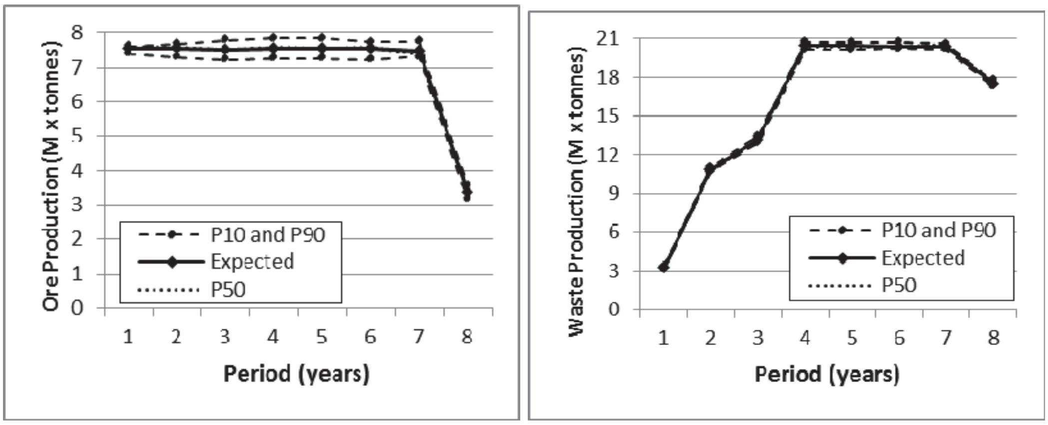

Figure 47: Risk profile for ore and waste tonnages for Case 4.

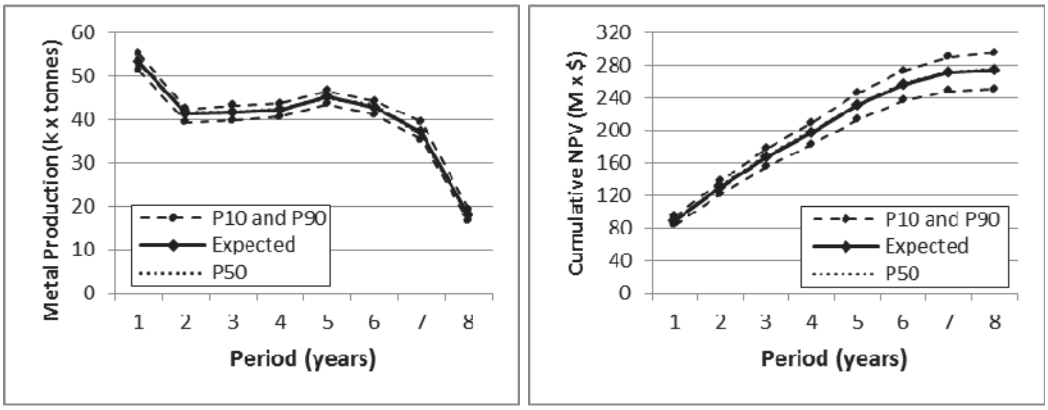

Figure 48: Risk profile for metal tonnages and NPV for Case 4.

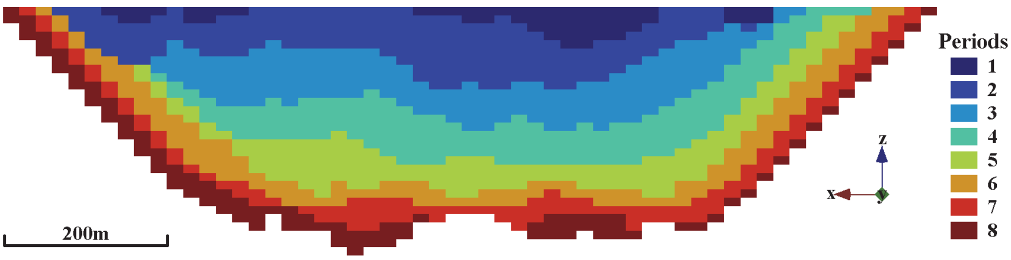

Figure 49: East-West vertical section (N10290) for Case 4 schedule.

A.4 Further Computational Improvements

Regarding the maximum depth parameter, recall that the methodology presented in chapter 3, after finding the solution for the first fractional period, gives a range over the whole orebody model area. However, areas that were not mined in the first fractional period have a lower chance of being mined in the second. Hence, for dealing with larger deposits, such range was defined only for mined areas and surroundings. For the first fractional period, there was no change in the implementation. Figure 50 compares this step of the method before and after changes. The same idea was also implemented for tolerance ranges.

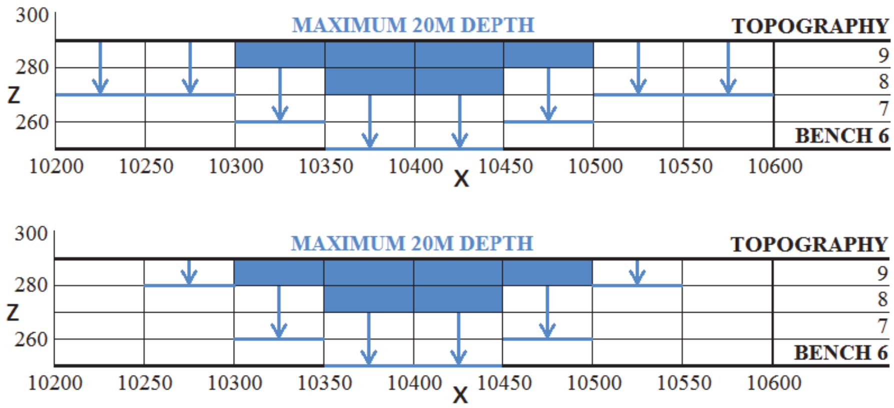

Figure 50: Maximum depth step before (top) and after (bottom).

Regarding multiple-period optimizations, considerable improvement can be obtained by modifying only the last surface included, which works faster and reduces the complexity of next multiple-period steps.

Modifying the surface of period t tends to force bigger changes in period $t-1$ than in period $t-2$; therefore, the tolerance range given for later periods should be bigger than for earlier ones. The implementation considered a factor of 2 to reduce the tolerance depending on the period, that is: tolerance x is given to period t, $x/2$ to period $t-1$, $x/4$ to $t-2$, and so on. This strategy reduced significantly the complexity of multiple-period problems, without forbidding iterative improvements.

For dealing with bigger deposits, prior to scheduling years, additional steps had to be performed. Following the concept of reserves parametrization, using a series of factors over the price of the metal (revenue factors), the same approach presented in chapter 3 was performed, but dropping production and variability constraints. The algorithm was executed once for each provided factor, generating a set of nested pits, for the larger copper deposit presented in section 3.5. Figure 51 shows the 21 nested pits generated with factors varying from 0.20 to 1.00 in step size of 0.04. The case study over chapter 4 did not require this step, due to its reduced size and shorter life of mine.

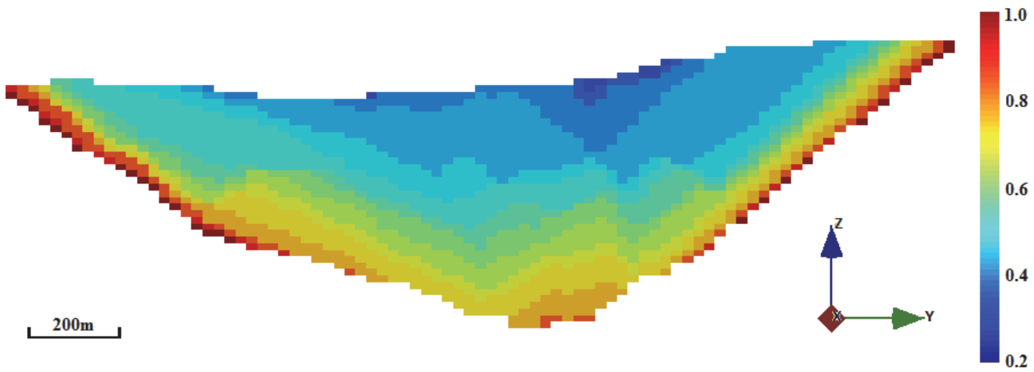

Figure 51: Section E19200 for nested pits applying revenue factors.

Using those nested pits as guide, the full amount of rock was divided into phases, similarly to the pit discretization of chapter 4. Considering expected ore production per phase close to 150Mt, no limit in total production, no variability control and a rescaled discounting rate, it was profitable to schedule 151,486 blocks into 15 phases, using the same algorithm presented previously. The risk profiles and physical design of those phases are presented in Figures 52 to 54.

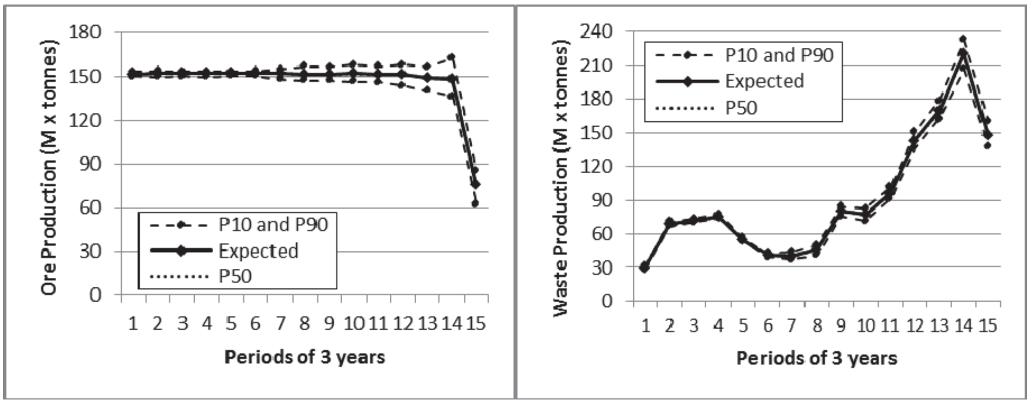

Figure 52: Risk profile for ore and waste tonnages for phases.

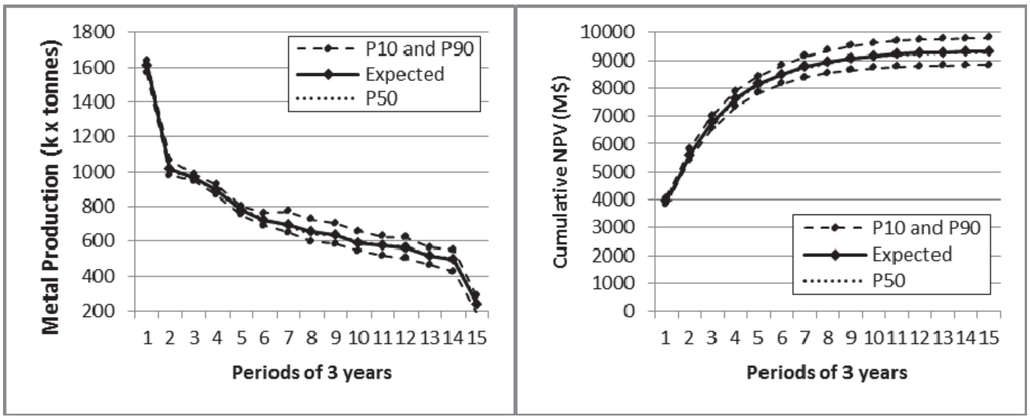

Figure 53: Risk profile for metal tonnages and NPV for phases.

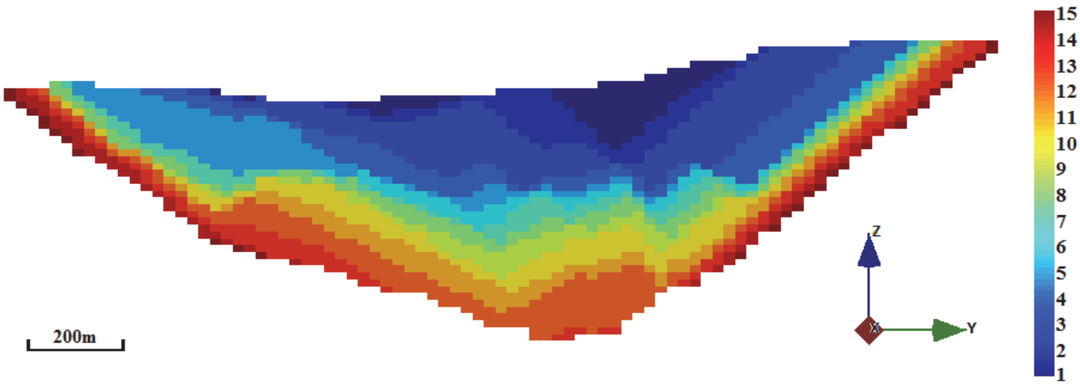

Figure 54: North-South vertical section (E19200) for phases.

The NPV risk profile is meaningless, considering the discounting rate defined over phases, not years, but the remaining results define a reasonable division of the pit space from more profitable to less profitable volumes. Ore production is achieved for all phases with low risk, even though there is no imposed control; this is due to the very low cutoff grade defined, which classified most of the valued blocks as ore. Waste production seems possible to be controlled up to the 10th phase, increasing absurdly after that. Metal production follows the expected behavior, decreasing over time. Results obtained here guided the presented yearly-based schedule, keeping mining just up to year 25, as there is an explosion on waste management with negligible improvements in NPV during later periods.

Appendix B – DVD with Data and Programs Used

Attached to this Master thesis is a DVD with the database for tests used in Chapter 3 and the last version of the C++ code.