



# Simultaneous production scheduling and transportation optimization from mines to port under uncertain material supply

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## ABSTRACT

Industrial mining complexes can be optimized using simultaneous stochastic optimization (SSO), which manages the risks associated with meeting production targets while capitalizing on the synergies that exist between the various components of the related mineral value chain. This paper introduces an extension of past SSO approaches for the long-term, allowing to simultaneously optimize the schedule of the production and the mines-to-port transportation of mining complexes under uncertain material supply. The inclusion of mine-to-port transportation scheduling facilitates the analysis of the mines-to-port equipment usage, while generating suitable mine production schedules. The proposed stochastic mathematical program formulation is applied to a two-mine, single-port iron ore mining complex. In doing so, it is shown that the related model is capable of producing optimal production schedules, minimizing deviations from products requirements, and delineating the yearly use of the mine-to-port transportation equipment.

## 1. Introduction

Industrial mining complexes or mineral value chains consist of various components such as mines, stockpiles, waste dumps, transportation systems, and processing facilities, among others. The simultaneous stochastic optimization (SSO) approach for long-term (strategic) planning and scheduling capitalizes on the synergies of these components to generate an optimal production schedule while managing technical risks, using several material supply (mineral deposit) simulations (Goodfellow and Dimitrakopoulos, 2016, 2017; Montiel and Dimitrakopoulos, 2015, 2018; Del Castillo and Dimitrakopoulos, 2019). The SSO approach is an extension of previous stochastic integer programming (SIP) approaches, optimizing a single mine under material supply uncertainty (Ramazan and Dimitrakopoulos, 2007, 2013; Dimitrakopoulos and Ramazan, 2008). However, mine-to-port transportation might be important for the extraction of certain commodities, yet it is not included in the SSO approaches. Indeed, in some situations, such as in iron ore mining complexes, the mine-to-port transportation is a key element ensuring that the products extracted at the mines reach their respective clients. These types of complexes can include several mines, stockpiles, and ports connected by complex railway systems, while accounting for material supply uncertainty, a critical source of technical

(geological) risk (Baker and Giacomo, 1998; Vallée, 2000). The interactions between the locations and the mine-to-port transportation system can be included in the optimization process to ensure that the value of the operation is maximized (Everett, 2001), while managing technical risk (Gomes Leite et al., 2019).

Developments in mine-to-port transportation scheduling optimization have been limited to short-term (operational) production planning where the mine extraction schedules are optimized beforehand and exclude sources of material supply uncertainty. Liu and Kozan (2011) propose a model to schedule trains on a single-track railway connecting two mines to a port using a job-shop problem representation. Singh et al. (2014) present a model optimizing the mine-to-port transportation scheduling over the medium term for a large iron ore mining complex. These two models require a pre-determined extraction schedule at the mines and, therefore, a fixed tonnage and product quality to be transported by the mine-to-port transportation system. This approach ignores the interdependencies of the two components of the mining complex. A change in the schedule of the mine-to-port transportation system affects the amount and quality of material that can be delivered, which in turn affects which material should be extracted at a given time, and vice versa. Additionally, neither study considers uncertainty in the amount

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or quality of the extracted material, further limiting the reliability of the optimization forecasts. [Bodon et al. \(2011\)](#) propose a combination of optimization and a discrete event simulation method to optimize the extraction sequence, the mine-to-port transportation scheduling, and the port operations simultaneously. The approach is applied to a case study to analyze different capital expenditure options and operation modes, evaluating their impact on the quantity and quality of the delivered products. The approach successfully combines mine production and mine-to-port transportation scheduling in a single model. [Kalinowski et al. \(2020\)](#) present a deterministic model to schedule maintenance on a mine-to-port transportation network while minimizing the impacts of the networks' ability to transport material from mines to ports. Although the model considers the effects of maintenance on the mine-to-port transportation capacity, it does not consider the effects that a reduction in capacity would have on the mine extraction schedules. Advancements have also been made to short-term supply chain optimization, which could potentially be applied to short-term mine-to-port transportation optimization ([Vahdani and Shams, 2020](#); [Buakam and Wisittipanich, 2020](#)).

[Belov et al. \(2020\)](#) develop a method for the short-term scheduling of trains and vessels, as well as the management of port stockpiles, to guide long-term infrastructure capacity planning. This approach allows the model to maintain the level of detail of a short-term optimization while covering a longer scheduling period. The approach does not include extraction scheduling at the mines, nor does it directly incorporate sources of uncertainty. [Montiel and Dimitrakopoulos \(2015\)](#) incorporate transportation alternatives for material output from processing facilities in long-term production scheduling within a stochastic optimization framework. These alternatives allow some flexibility in managing the transportation equipment by determining the proportion of material types being transported by the different methods. However, these proportions are predetermined. [Gomes Leite et al. \(2020\)](#) present a Markov decision process model for the mine-to-client supply chain for various time horizons. Though their approach considers multiple components of the mining supply chain and solves them simultaneously, it does not model the mine-to-port transportation system.

The existing technical literature does not include any attempts to integrate mine-to-port transportation scheduling into the overall long-term (strategic) production planning optimization framework for mining complexes under supply uncertainty. As mentioned above, past work focuses on short-term (operational) deterministic approaches. To address this gap, this paper introduces a new long-term (strategic) stochastic integer programming model that incorporates mine-to-port transportation constraints into the mine production scheduling optimization under material supply uncertainty. This uncertainty is represented by stochastic simulations of the pertinent attributes (e.g. grade and material types) of the related mineral deposits ([Goovaerts, 1997](#); [Boucher and Dimitrakopoulos, 2009](#); [Gómez-Hernández and Srivastava, 2021](#); [Minniakhmetov and Dimitrakopoulos, 2021](#)). The proposed SIP model includes multiple mines, stockpiles, waste dumps, loading facilities, different transportation system layouts, and a single port. Extracted materials can be sent to stockpiles, waste dumps, or to the port via the mine-to-port transportation system. At the port, fixed yearly demand (quantity and quality) for the multiple products is considered for the material extracted from the related mines. The overall aim of the model is to minimize the costs associated with meeting the product demand at the port as well as to manage the risks associated with meeting these targets. The model presented herein produces long-term extraction schedules for the related mines, as well as a schedule for the mine-to-port transportation equipment utilization. This schedule can be used to guide overall strategic mine planning decisions. In the following sections, the proposed mathematical programming model is presented, as well as an overview of the solution method. Then, a case study for a two-mine, single-port iron ore mining complex is presented. Finally, conclusions and directions for future work are given.

## 2. Method

This section presents a new stochastic mathematical programming model. The model was developed to simultaneously optimize mine production scheduling and mine-to-port transportation for the long-term (strategic) planning of rail-connected mining complexes, while accounting for uncertain material supply. It was also developed for mining complexes with a single port at which a yearly demand is specified for several products.

### 2.1. Definitions and notation

This section includes the definitions and the notation used to specify the proposed SIP model.

#### 2.1.1. Indices and sets

- $s$ : Stochastic orebody simulation,  $s \in S$
- $t$ : Time period,  $t \in T$
- $i, j$ : Nodes in the graph representing the mining complex,  $i, j \in N$
- $m$ : Mine,  $m \in M$
- $\ell$ : Loading area at a mine  $m$ ,  $\ell \in \mathcal{L}_m \subset N$
- $h$ : Stockpile at a mine  $m$ ,  $h \in \mathcal{H}_m \subset N$
- $w$ : Waste dump at a mine  $m$ ,  $w \in \mathcal{W}_m \subset N$
- $d$ : Destination at a mine  $m$ ,  $d \in D_m = \mathcal{L}_m \cup \mathcal{H}_m \cup \mathcal{W}_m$
- $b$ : Mining block at a mine  $m$ ,  $b \in B_m$
- $r$ : Final product,  $r \in R$
- $e$ : Element making up the final products,  $e \in E$
- $w$ : Mine-to-port transportation equipment,  $w \in W$
- $\theta$ : Path, starting and ending at the port, followed by mine-to-port transportation equipment,  $\theta \in \Theta$
- $\mathbb{P}_b$ : Set of extraction predecessors of block  $b$
- $\mathbb{W}_b^{\text{smooth}}$ : Set of blocks within block  $b$  smoothing window
- $\mathcal{S}_h$ : Randomized order in which the blocks sent to stockpile  $h$  can be reclaimed

#### 2.1.2. Parameters

- $M$ : big M (scalar with large value)
- $d$ : Economic discount rate
- $\mathcal{G}$ : Geological risk discount rate
- $c_m^{\text{mine}}$ : Cost of extracting a block at mine  $m$
- $c_d^{\text{trans}}$ : Cost of transporting material from mine  $m$  to a mine destination  $d$
- $c_h^{\text{rec}}$ : Material reclamation cost in stockpile  $h$
- $c_w^{\text{fixed}}$ : Fixed cost associated with using equipment  $w$
- $c_\theta^{\text{path}}$ : Travel cost of equipment on path  $\theta$
- $c_i^{\text{load}}$ : Loading cost at node  $i$
- $c_0^{\text{load}}$ : Unloading cost at port
- $c^{\text{cap}}$ : Cost of using equipment under capacity
- $c_r^{O-}, c_r^{O+}$ : Penalty cost of deviating below or above, respectively, from the ore tonnage demand for product  $r$
- $c_r^{e-}, c_r^{e+}$ : Penalty cost of deviating below or above, respectively, from the element  $e$  grade demand for product  $r$
- $c^{\text{smooth}}$ : Penalty cost associated with mining blocks within block  $b$ 's smoothing radius in different periods  $t$
- $Q$ : Tonnage of a block
- $Q_d^{\text{dest}}$ : Capacity of mine destination  $d$
- $Q_m^{\text{min } t}$ : Minimum mining rate at mine  $m$  in period  $t$
- $Q_m^{\text{max } t}$ : Maximum mining rate at mine  $m$  in period  $t$
- $Q_w$ : Single-trip capacity of equipment  $w$
- $H_{wt}^{\text{time}}$ : Maximum time available to equipment  $w$  in period  $t$
- $H_{rt}^{\text{ore}}$ : Ore tonnage demand of product  $r$  in period  $t$

$H_{rt}^{-}, H_{rt}^{+}$ : Lower and upper bound, respectively for element  $e$  content in product  $r$  in period  $t$   
 $\psi_{bs}^e$ : Grade of element  $e$  in block  $b$  in geological scenario  $s$   
 $o_{ij}^{\theta}$ : Indicates whether or not arc  $(i, j)$  is included in path  $\theta$   
 $\alpha_{ij}$ : Indicates whether or not nodes  $i$  and  $j$  are connected in the graph  
 $\tau_{ij}$ : Time required to travel from node  $i$  to node  $j$   
 $Q_{ij}$ : Maximum number of equipment which can travel on arc  $(i, j)$  in a period  
 $T$ : Time required to load a tonne of material onto a piece of equipment  
 $n^{trips}$ : Minimum number of trips to be completed by a piece of equipment per period

$d_{rst}^{-}, d_{rst}^{+}$ : Deviations below or above, respectively, from the element  $e$  grade target of product  $r$  in period  $t$  and scenario  $s$

$d_{bt}^{smooth}$ : Number of blocks in block  $b$  smoothing radius which are mined in a different period  $t$

## 2.2. Optimization model

### 2.2.1. Objective function

$$\min \left( \underbrace{\sum_{t \in T} \sum_{m \in Md} \sum_{d \in D_m} \sum_{b \in B_m} \frac{c_m^{mine} Q_b^{td}}{(1+d)^t}}_{\text{Part I}} + \underbrace{\sum_{t \in T} \sum_{m \in Md} \sum_{d \in D_m} \sum_{b \in B_m} \frac{c_d^{trans} Q_b^{td}}{(1+d)^t}}_{\text{Part II}} + \underbrace{\sum_{t \in T} \sum_{m \in Mh} \sum_{h \in H_m} \sum_{b \in B_m} \frac{c_h^{rec} Q_b^{th}}{(1+d)^t}}_{\text{Part III}} \right. \\
+ \underbrace{\sum_{t \in T} \sum_{w \in W} \frac{c_w^{fixed} O_{wt}}{(1+d)^t}}_{\text{Part IV}} + \underbrace{\sum_{t \in T} \sum_{w \in W} \sum_{\theta \in \Theta} \frac{c_{\theta}^{path} z_{\theta}^{wt}}{(1+d)^t}}_{\text{Part V}} \\
+ \underbrace{\sum_{t \in T} \sum_{m \in Ml} \sum_{l \in L_m} \sum_{b \in B_m} \frac{(c_l^{load} + c_0^{load}) Q_b^{tl}}{(1+d)^t}}_{\text{Part VI}} + \underbrace{\sum_{t \in T} \sum_{m \in Mh} \sum_{h \in H_m} \sum_{b \in B_m} \frac{(c_h^{load} + c_0^{load}) Q_b^{th}}{(1+d)^t}}_{\text{Part VII}} + \underbrace{\sum_{t \in T} \sum_{w \in W} \sum_{\theta \in \Theta} \frac{c_w^{cap} d_{\theta w}^{-}}{(1+D)^t}}_{\text{Part VII}} \\
+ \underbrace{\sum_{t \in T} \sum_{r \in R} \frac{c_r^{D-} d_{rt}^{D-} + c_r^{D+} d_{rt}^{D+}}{(1+D)^t}}_{\text{Part VIII}} + \underbrace{\frac{1}{S} \sum_{s \in S} \sum_{t \in T} \sum_{r \in R} \sum_{e \in E} \frac{c_t^{e-} d_{rst}^{e-} + c_t^{e+} d_{rst}^{e+}}{(1+D)^t}}_{\text{Part IX}} + \underbrace{\sum_{t \in T} \sum_{m \in Mb} \sum_{b \in B_m} \frac{c^{smooth} d_{bt}^{smooth}}{(1+D)^t}}_{\text{Part X}} \left. \right) \quad (1)$$

### 2.1.3. Decision variables

#### 2.1.3.1. Discrete decision variables.

$x_b^{td}$ : Indicates whether or not block  $b$  is extracted and sent to destination  $d$  in period  $t$

$\xi_b^{th}$ : Indicates whether or not block  $b$  is reclaimed from stockpile  $h$  in period  $t$

$\rho_b^{tr}$ : Indicates whether or not block  $b$  is assigned to product  $r$  in period  $t$

$\mathcal{O}_{wt}$ : Indicates whether or not equipment  $w$  is used in period  $t$

$z_{\theta}^{wt}$ : Number of times equipment  $w$  travels on path  $\theta$  in period  $t$

#### 2.1.3.2. Continuous decision variables.

$y_{w\theta b}^t$ : Proportion of block  $b$  loaded onto equipment  $w$  travelling on path  $\theta$  in period  $t$

$d_{t\theta w}^{-}$ : Unused capacity of equipment  $w$  travelling on path  $\theta$  in period  $t$

$d_{rt}^{D-}, d_{rt}^{D+}$ : Deviations below or above, respectively, from the ore demand target of product  $r$  in period  $t$

The proposed model is a two-stage stochastic integer program (SIP); its objective function (1) is a minimization function aiming to reduce mining and mine-to-port transportation costs, as well as to reduce the risks associated with meeting product demand at the port. The objective function has four main sections. Section I minimizes the overall production scheduling costs at the mines: Part I involves the extraction costs of blocks at the mines; Part II involves the transportation costs to mine destinations, and Part III involves the stockpile reclamation costs. Section II includes the mine-to-port transportation costs: Part IV involves the equipment fixed cost; Part V involves the equipment's path-dependent travels costs; Part VI involves the equipment's loading and unloading costs at the different locations; and Part VII involves the cost of underutilizing equipment. Section III is related to the risk of deviating from product demand targets at the port: Part VIII involves the ore tonnage product demand target deviation penalty costs; and Part IX involves the ore product quality target deviation penalty costs. Section IV aims to generate a mineable schedule by ensuring a minimum mining width. The model aims to mine block  $b$  in the same period as the blocks within a smoothing window. A penalty is applied to the blocks within this window that is not mined out (Dimitrakopoulos and Ramazan, 2004; Ramazan and Dimitrakopoulos, 2007, 2013). Parts VII to X include a geological discount rate (GDR),  $\mathcal{D}$ . Like the economic discount rate ( $d$ ), which reduces the value of costs over time, the GDR aims to reduce the cost of deviating over time. The inclusion of the GDR makes it more costly to deviate from targets in earlier periods than later periods, hence deferring the risk of not meeting production targets at the port (Dimitrakopoulos and Ramazan, 2004).

## 2.2.2. Constraints

### 2.2.2.1. Production scheduling constraints

$$\sum_{t \in T} \sum_{d \in D_m} x_b^{td} \leq 1 \quad \forall m \in \mathcal{M}, b \in B_m \quad (2)$$

$$\sum_{d \in D_m} x_b^{td} \leq \sum_{\tau \leq t} x_b^{\tau d} \quad \forall t \in T, m \in \mathcal{M}, b \in B_m, \bar{b} \in \mathbb{P}_b \quad (3)$$

$$\sum_{b \in B_m} Qx_b^{td} \leq Q_d^{dest} \quad \forall t \in T, m \in \mathcal{M}, d \in D_m \quad (4)$$

Constraint (2) ensures that a block cannot be extracted more than once, and that it can only be sent to a single destination. Constraint (3) ensures that the slope constraints and that the block precedence are satisfied. Constraint (4) ensures that the amount of material sent to a destination does not exceed its capacity in any period.

$$\sum_{b \in B_m} \sum_{d \in D_m} Qx_b^{td} \leq Q_{mi}^{max} \quad \forall t \in T, m \in \mathcal{M} \quad (5)$$

$$\sum_{b \in B_m} \sum_{d \in D_m} Qx_b^{td} \geq Q_{mi}^{min} \quad \forall t \in T, m \in \mathcal{M} \quad (6)$$

Constraints (5) and (6) ensure that the maximum and minimum mining rates, respectively, at each mine are respected throughout the life of the operation.

$$|\mathbb{W}_b^{smooth}| \left| \sum_{d \in D_m} x_b^{td} - \sum_{d \in D_m} \sum_{\bar{b} \in \mathbb{W}_b^{smooth}} x_b^{\tau d} \right| \leq d_{bt}^{smooth} \quad \forall t \in T, m \in \mathcal{M}, b \in B_m \quad (7)$$

Constraint (7) counts the number of surrounding blocks, which are mined in a different period; these blocks incur a cost in Section IV of the objective function (1). This ensures a certain connectivity between the mined blocks, producing a more mineable schedule.

### 2.2.2.2. Stockpile constraints

$$\xi_b^{th} \leq \sum_{\tau < t} x_b^{\tau h} \quad \forall t \in T, m \in \mathcal{M}, h \in \mathcal{H}_m, b \in B_m \quad (8)$$

$$\sum_{\tau \leq t} \xi_b^{\tau h} \geq \xi_b^{(h+1)} \quad \forall t \in T, m \in \mathcal{M}, h \in \mathcal{H}_m, b \in \mathcal{S}_h \quad (9)$$

Constraint (8) ensures that blocks are sent to a stockpile before they can be reclaimed. Constraint (9) implements a random block removal order policy. Indeed, when blocks are sent to a stockpile, they are randomly placed in a list indicating the order in which they are reclaimed. Accordingly, a first-in-first-out rule is applied in order to remove all the blocks introduced in previous periods before those introduced in the current one.

It should be noted that the above policy avoids the disadvantages associated with the assumptions of standard stockpile modelling approaches. The perfect blending approach assumes that all material in a stockpile is homogenous, while the perfect selection approach assumes that the material's location within a stockpile is well known. Typically, stockpiles are heterogeneous and highly variable, therefore, neither assumption is realistic (Dirkx and Dimitrakopoulos, 2018). Moreover, the perfect blending approach requires non-linear constraints, adding significant complexity to the model, which cannot be solved with linear programming commercial solver. The random block removal order strategy applied to the stockpiles overcomes the previously listed disadvantages because it does not make assumptions about a stockpile's material, and it also ensures that the model remains linear.

### 2.2.2.3. Linking constraints

$$\sum_{w \in W} \sum_{\theta \in \Theta} y_{w\theta b}^t = \sum_{\ell \in \mathcal{L}_m} x_b^{t\ell} + \sum_{h \in \mathcal{H}_m} \xi_b^{th} \quad \forall t \in T, m \in \mathcal{M}, b \in B_m \quad (10)$$

$$x_b^{t\ell} \leq \sum_{h \in \mathcal{H}_m} \sum_{w \in W} \sum_{\theta \in \Theta} y_{w\theta b}^t o_{\ell h}^\theta \quad \forall t \in T, m \in \mathcal{M}, \ell \in \mathcal{L}_m, b \in B_m \quad (11)$$

$$\xi_b^{th} \leq \sum_{\ell \in \mathcal{L}_m} \sum_{w \in W} \sum_{\theta \in \Theta} y_{w\theta b}^t o_{\ell h}^\theta \quad \forall t \in T, m \in \mathcal{M}, h \in \mathcal{H}_m, b \in B_m \quad (12)$$

Constraint (10) ensures that the blocks that are sent directly to a loading area or that are reclaimed from a stockpile are loaded onto mine-to-port transportation equipment in the same period. Constraints (11) and (12) ensure that the blocks that are sent to a loading area or are reclaimed from a stockpile are loaded onto equipment travelling on a path including that destination.

### 2.2.2.4. Mine-to-port transportation constraints

$$\sum_{w \in W} \sum_{\theta \in \Theta} z_\theta^{wt} (o_{ij}^\theta + o_{ji}^\theta) \leq Q_{ij} \alpha_{ij} \quad \forall t \in T, i < j \in N \quad (13)$$

Constraint (13) ensures that a path segment's capacity is respected. A path is defined as the route taken by mine-to-port transportation equipment travelling to the different locations within the mining complex. For the purpose of this model, each path starts and ends at the port. A path segment is defined as a portion of the path, connecting two different locations. Each segment has a maximum number of equipment travelling on it within a period.

$$\sum_{\theta \in \Theta} \sum_{i < j \in N} \tau_{ij} z_\theta^{wt} (o_{ij}^\theta + o_{ji}^\theta) + \sum_{m \in \mathcal{M}} \sum_{b \in B_m} \sum_{\theta \in \Theta} T Q y_{w\theta b}^t \leq H_{wt}^{time} \quad \forall t \in T, w \in W \quad (14)$$

Constraint (14) ensures that each piece of equipment has a limited availability time in each period, and that the resulting transportation schedule is operationally feasible. This constraint allows the inclusion of the planned equipment maintenance in the long-term schedule.

$$\sum_{m \in \mathcal{M}} \sum_{b \in B_m} Q y_{w\theta b}^t + d_{t\theta w}^- = Q_w z_\theta^{wt} \quad \forall t \in T, w \in W, \theta \in \Theta \quad (15)$$

$$\sum_{t \in T} \sum_{w \in W} \sum_{\theta \in \Theta} y_{w\theta b}^t \leq 1 \quad \forall m \in \mathcal{M}, b \in B_m \quad (16)$$

Constraint (15) ensures that the equipment's capacity is never exceeded. In addition, the unused capacity of the equipment in each period is penalized in the objective function (see Section 2.2.1). Note that the capacity constraint is specified over the total number of times the equipment is used. Constraint (16) ensures that a mining block cannot be transported more than once.

$$z_\theta^{wt} \leq M \mathcal{C}_{wt} \quad \forall t \in T, w \in W, \theta \in \Theta \quad (17)$$

$$\mathcal{C}_{wt} n^{trips} \leq \sum_{\theta \in \Theta} z_\theta^{wt} \quad \forall t \in T, w \in W \quad (18)$$

$$\mathcal{C}_{w(t+1)} \geq \mathcal{C}_{wt} \quad \forall t \leq T-1, w \in W \quad (19)$$

Constraint (17) ensures that the equipment fixed costs are paid by activating the binary variable  $\mathcal{C}_{wt}$ . Once it is activated, the equipment's use is subject to constraints (18) and (19). Constraint (18) ensures that used equipment will complete a minimum number of trips while constraint (19) ensures that once a piece of equipment is used in one period, it will continue to be used in the following periods. Together, these constraints reduce the number of equipment in use at any time.

### 2.2.2.5. Demand and blending constraints

$$\sum_{w \in W} \sum_{\theta \in \Theta} y_{w\theta b}^t = \sum_{r \in R} \rho_b^{tr} \quad \forall t \in T, m \in \mathcal{M}, b \in B_m \quad (20)$$

$$\sum_{m \in \mathcal{M}} \sum_{b \in B_m} Q \rho_b^{tr} + d_{rt}^{O-} - d_{rt}^{O+} = H_{rt}^{ore} \quad \forall t \in T, r \in R \quad (21)$$

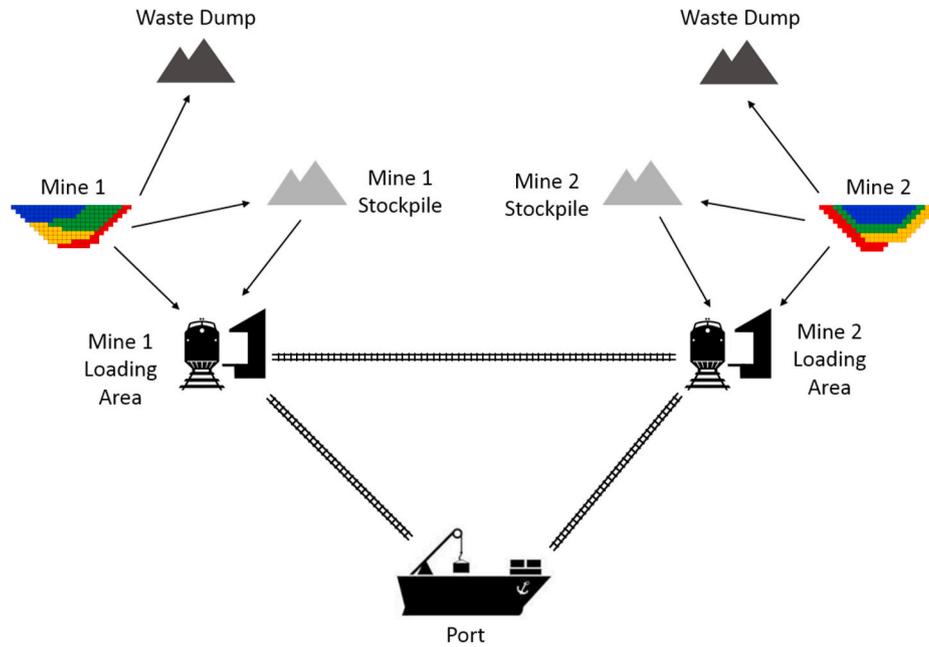


Fig. 1. Components and layout of the mining complex.

$$\sum_{m \in \mathcal{M}} \sum_{b \in B_m} Q \rho_b^{rs} (\psi_{bs}^e - H_{rs}^{e+}) - d_{rst}^{e+} \leq 0 \quad \forall s \in S, t \in T, r \in R, e \in E \quad (22)$$

$$\sum_{m \in \mathcal{M}} \sum_{b \in B_m} Q \rho_b^{rs} (\psi_{bs}^e - H_{rs}^{e-}) + d_{rst}^{e-} \geq 0 \quad \forall s \in S, t \in T, r \in R, e \in E \quad (23)$$

Constraint (20) ensures that every block delivered to the port is assigned to a final product. Constraint (21) sets the deviations from the ore tonnage target for each product. Moreover, for each material uncertainty scenario considered, constraints (22) and (23) set the deviations from the upper and lower bound targets of the different elements considered. These constraints allow the optimization process to make the best decisions to reduce the overall risk of missing demand targets.

2.2.2.6. Integrality and non-negativity constraints

$$x_b^{td} \in \{0, 1\} \quad \forall t \in T, m \in \mathcal{M}, d \in D_m, b \in B_m \quad (24)$$

$$\xi_b^{th} \in \{0, 1\} \quad \forall t \in T, m \in \mathcal{M}, h \in \mathcal{H}_m, b \in B_m \quad (25)$$

$$\rho_b^{rs} \in \{0, 1\} \quad \forall t \in T, r \in R, m \in \mathcal{M}, b \in B_m \quad (26)$$

$$\mathcal{C}_{wt} \in \{0, 1\} \quad \forall t \in T, w \in W \quad (27)$$

$$z_{\theta}^{wt} \geq 0, \text{ integer} \quad \forall t \in T, w \in W, \theta \in \Theta \quad (28)$$

$$y_{w\theta b}^t \geq 0 \quad \forall t \in T, w \in W, \theta \in \Theta, m \in \mathcal{M}, b \in B_m \quad (29)$$

$$d_{\theta w}^- \geq 0 \quad \forall t \in T, \theta \in \Theta, w \in W \quad (30)$$

$$d_{rt}^{O-}, d_{rt}^{O+} \geq 0 \quad \forall t \in T, r \in R \quad (31)$$

$$d_{rst}^{e-}, d_{rst}^{e+} \geq 0 \quad \forall s \in S, t \in T, r \in R, e \in E \quad (32)$$

$$d_{bt}^{smooth} \geq 0 \quad \forall t \in T, m \in \mathcal{M}, b \in B_m \quad (33)$$

Constraints (24) to (28) enforce integrality for the variables while constraints (29) to (33) enforce non-negativity.

2.3. Comments

The model presented in this section contributes to the joint optimization of mine-to-port transportation and long-term production scheduling of rail-connected mining complexes under material supply uncertainty from the related mines. This, in order to meet yearly demand in quantity and quality for multiple products at the related port.

The model presented in this section contributes the joint optimization of mine-to-port transportation and long-term production scheduling of rail-connected mining complexes, while considering material supply uncertainty from the mines. The joint optimization aims to meet yearly demand for multiple products at the port in terms of their quantity and

Table 1  
Ore and grade targets for each product.

	Year	Ore Tonnage	Fe (%)	SiO <sub>2</sub> (%)	Al <sub>2</sub> O <sub>3</sub> (%)	P (%)	LOI (%)
Product 1	1	5,000,000	57.9–59.4	4.6–5.2	1–1.05	0.033–0.04	8.8–11
	2	5,000,000					
	3	4,000,000					
	4	4,000,000					
	5	4,000,000					
Product 2	1	4,000,000	57.1–58.5	4.9–5.5	0.9–1.05	0.031–0.038	9.5–13
	2	4,000,000					
	3	4,000,000					
	4	3,000,000					
	5	2,000,000					

**Table 2**  
Fleet characteristics.

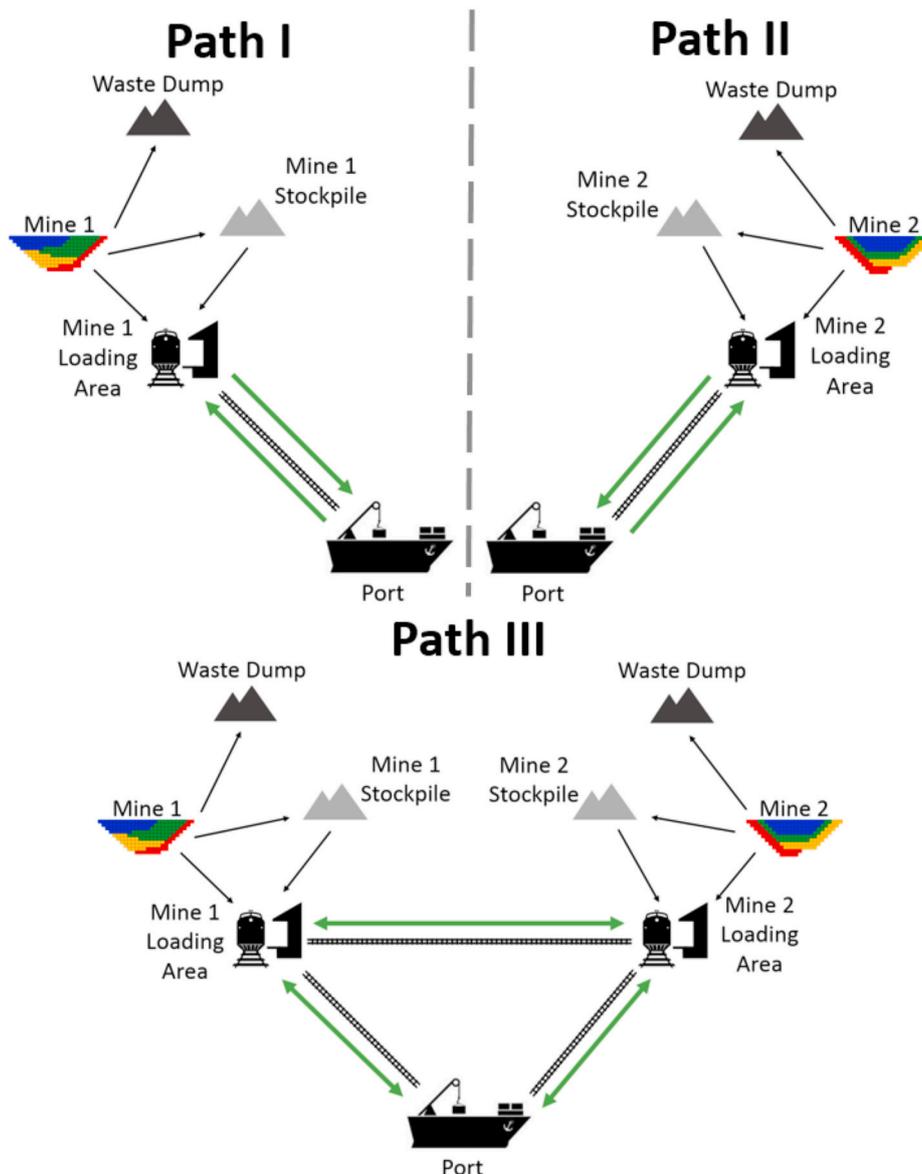
Type	Number Available	Capacity per Trip (Tonnes)
I	1	23,000
II	1	32,000

quality. The proposed model is formulated as a two-stage stochastic integer program. The objective function is threefold. It includes the reduction of the mining and mine-to-port transportation costs and, most importantly, the management and reduction of the risk of not meeting product targets at the port under material supply uncertainty. The model is driven by the common mining extraction constraints, including mining rates and connectivity between the blocks of the mineral deposits mined, during the related time horizon. Mined ore blocks are sent to loading areas or stockpiles. Several constraints are specified for the mine-to-port transportation equipment related to their capacity and maintenance, while their number is adjusted during the time horizon considered. The loading of blocks from the loading areas and stockpiles to the transportation equipment has a direct impact on the ore extraction

planning and production scheduling from the related mines. Meanwhile, the blending constraints allow for the connection between the blocks delivered and the port and the demand targets. The potential target deviations are managed and minimized for the tonnage and quality of each product, thus accounting for material supply uncertainty. The proposed model offers the required strong connection between the extraction of materials from the mines and the mine-to-port transportation, while managing related product risks. This is the first such approach available in the technical literature; previous models developed for the strategic optimization of industrial mining complexes do not include mine-to-port transportation, an activity typically completed as a standalone step.

**3. Case study**

The formulation presented in Section 2 is applied to study an iron ore mining complex where components of a mine-to-port transportation system consist of a railway system with a fleet of trains. An overview of the operation as well as key parameters are first introduced, and the results obtained are then presented.



**Fig. 2.** Representation of the possible paths taken by the mines-to-port trains.

**Table 3**  
Train path definitions.

Path	Definition	Cost per Trip (\$)
I	Port – Mine 1 – Port	800
II	Port – Mine 2 – Port	640
III	Port – Mine 1 – Mine 2 – Port or Port – Mine 2 – Mine 1 – Port	960

**Table 4**  
Economic parameters.

Parameter	Value
Mining cost (\$/t)	3
Transportation costs within the mine (\$/t)	2
Reclamation costs (\$/t)	0.1
Economic discount rate (%)	10
Geological risk discount rate (%)	12

3.1. Overview

This case study considers an iron ore mining complex with two mines and a single port. Each mine has a waste dump, a loading area, and a stockpile, as shown in Fig. 1. In the figure, the arrows depict the flow of the extracted material, and the railway tracks show the existing railway system that connects the mines to the port. At the mines, a total of approximately 2000 mining blocks are available, having dimensions of 25 m by 25 m by 12 m and a mass of 22,500 tonnes. Material supply uncertainty is included using fifteen geostatistically simulated scenarios (Boucher and Dimitrakopoulos, 2009, 2012) to quantify the uncertainty and variability of the five different elements considered: iron, silica, aluminum oxide, phosphorus, and loss-on-ignition (LOI). At the port, the demand for two products is considered. Each product is characterized by

a fixed yearly tonnage target as well as by product quality constraints for the elements considered, as shown in Table 1.

The transportation system of the mining complex includes a fleet of two trains as described in Table 2. Each train is available for 6300 h per year. Three paths are available for the trains to follow when transporting material, as shown in Fig. 2 and described in Table 3. The mine-to-port transportation costs depend on the travel distance between the locations on each path. The economic parameters used in the optimization model are listed in Table 4. For this case study, the transportation costs within the mine (therefore, from a mining face to a loading area, a stockpile, or a waste dump) are the same for all destinations and for both mines.

3.2. Results

The model in the case study described previously is solved using the branch and cut algorithm implemented in CPLEX v.12.6.1.0 in a Visual Studio 15 (C++) environment. The number of binary and integer variables (in the order of 70,000) and the number of constraints (in the order of 175,000) in the model are too large to obtain results in a reasonable span of time and the rolling time horizon approach is applied (Dimitrakopoulos and Ramazan, 2008; Ramazan and Dimitrakopoulos, 2013). The time horizon chosen is two years, with a one-year overlap; each horizon is solved to an optimality gap of less than 1%.

3.2.1. Production schedules

Production schedules are generated for both mines, as shown in Fig. 3. The cross-sections show that, during the first two years, Mine 2 is mined more extensively than Mine 1. Since the extraction and mine transportation costs are identical for both mines, the results indicate that either Mine 2 provides better supply to meet the product demand at the port, or that Path II's smaller cost (Table 3) results in a less expensive extraction for Mine 2 in earlier years. Perhaps it is a combination of both reasons.

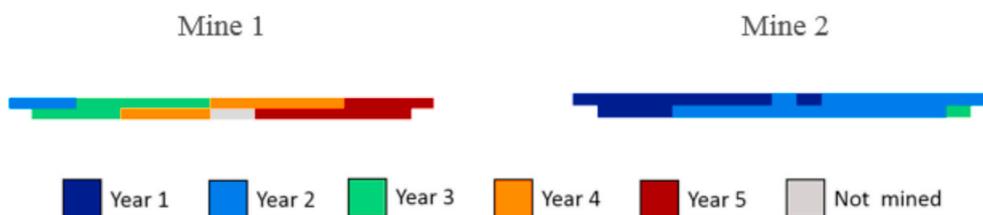


Fig. 3. Cross-sections of the mining schedules for the two mines.

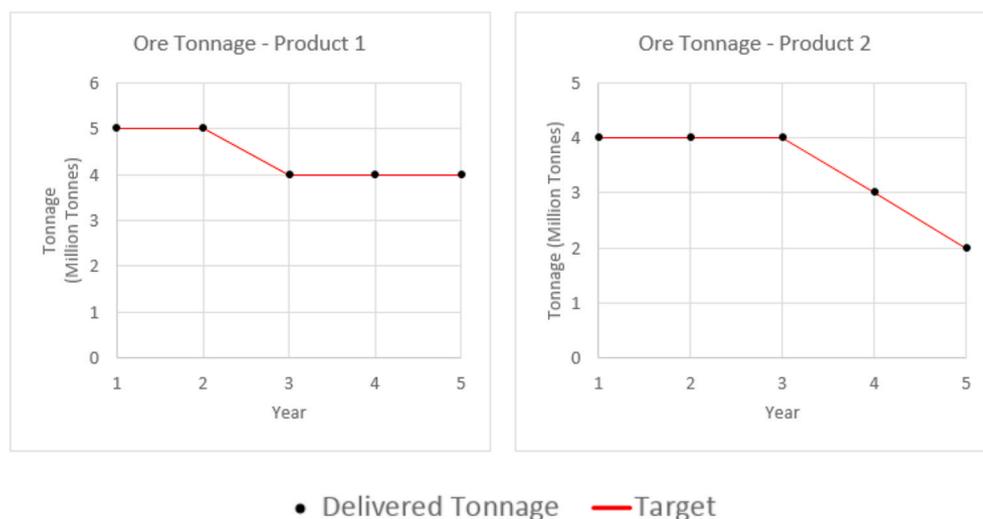


Fig. 4. Yearly tonnage of products delivered to the port.

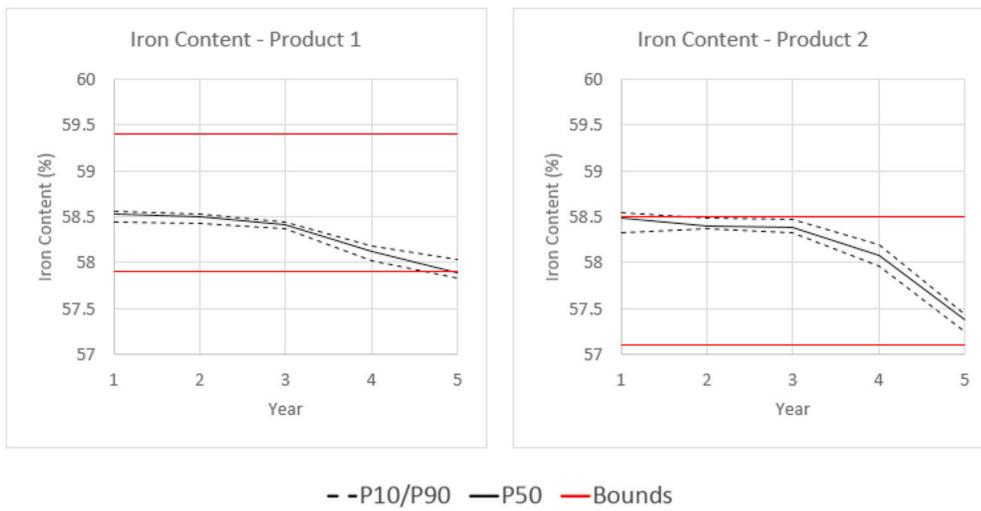


Fig. 5. Yearly iron grade of products at the port.

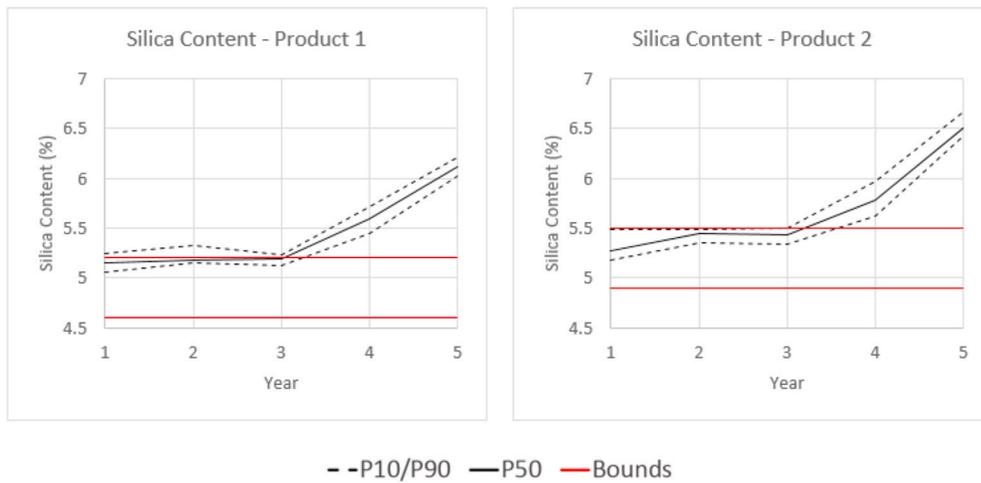


Fig. 6. Yearly silica grade of products at the port.

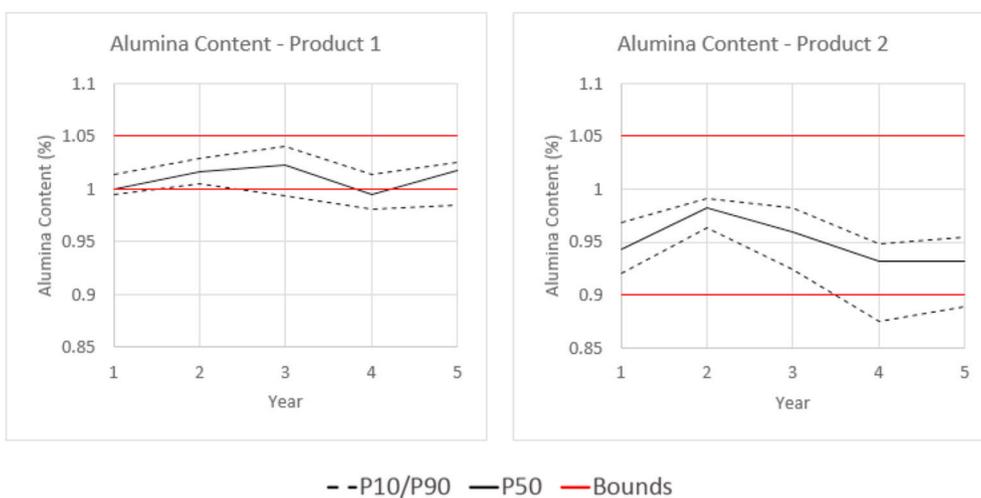


Fig. 7. Yearly alumina grade of products at the port.

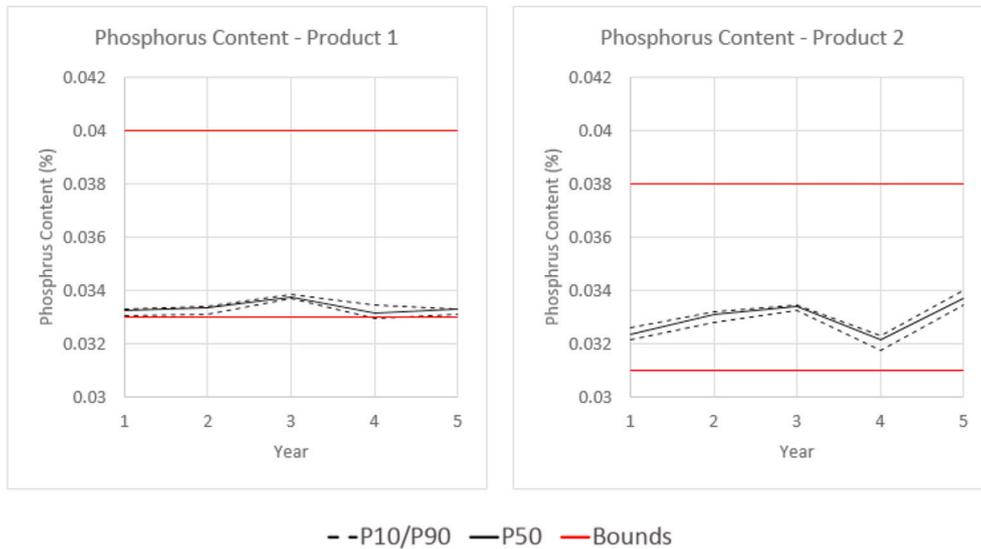


Fig. 8. Yearly phosphorus grade of products at the port.

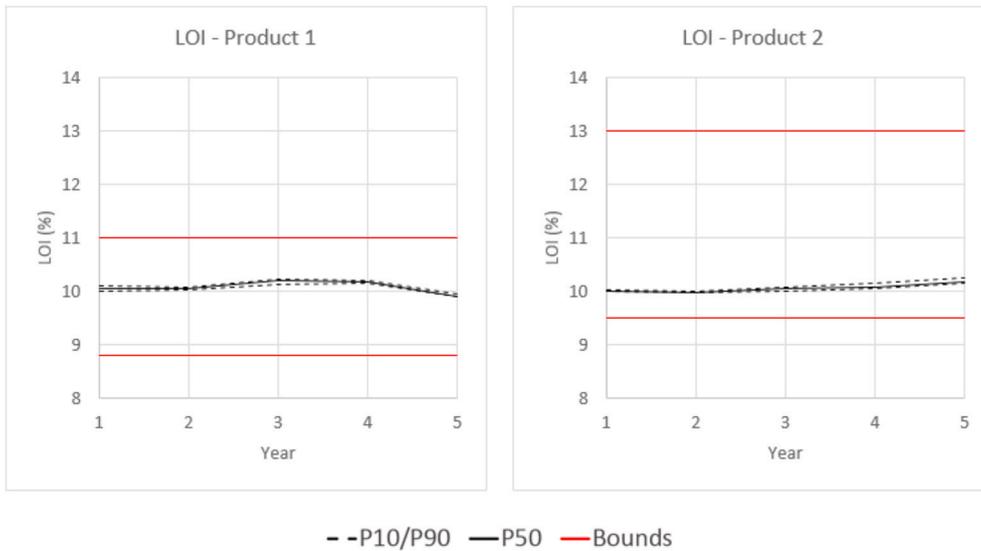


Fig. 9. Yearly LOI of products at the port.

Fig. 4 compares the tonnage delivered at the port for each product relative to its demand. Note that the decisions relative to the distribution of the material between the products at the port are scenario-independent. Therefore, there are no risk profiles. For both products, the demand is well met, with deviations of less than 0.5%. Fig. 5 to Fig. 9 present the yearly forecasts (P50 of the results) for meeting each quality constraint for both products, along with the associated risk profiles (P10 and P90 of the related forecasts). The P10, P50 and P90 respectively represent the 10%, 50% and 90% probability of obtaining values below the corresponding amount. Note that the risk profiles shown are created using a set of simulations that differ from those used in the optimization process. Fig. 5 presents the yearly iron grade of the final products along with the related risk profiles. Product 1's forecasted iron grades are well within the given bounds for years 1 through 4, however, a small deviation can be seen in year 5; the P10 value is slightly lower than the required lower bound. For Product 2, the forecasted iron grades are within the given bounds for years 2–5. In year 1, there is a slight deviation from the upper bound; the P90 value exceeds the upper bound limit marginally. Overall, the iron demand is expected to be well met for both products.

Fig. 6 illustrates the yearly forecasted silica content of the final products along with the related risk profiles. Both products exhibit significant deviations from the upper bound. For Product 1, there are deviations during all years. In years 1–3, the deviations are relatively small (the P90 deviates less than 3%) before increasing in year 4 (the P90 deviates almost 10%) and reaching a maximum in year 5, where the P10, P50 and P90 deviate by approximately 20%. There are also deviations for Product 2 during years 3–5; they are small (less than 1%) but increase over time to reach a maximum in year 5 where they reach over 20%. These results indicate that, for the years in which there are larger deviations, the material that can be extracted from the deposit may not have the silica properties required to meet the demand for these products. Hence, blending the ore extracted from the deposits with ore from other sources may be necessary to meet demand. Additionally, Fig. 6 illustrates the effects of the geologic discount rate (Section 2.2.1): the deviations, and thus the risk, are generally smaller in earlier years than in later years. Deferring risk to later years may allow an operation to consider other sources of material allowing to meet demand with higher certainty. Fig. 7 shows the yearly forecasts and the related risk profiles of the alumina content of the final products. Product 1 exhibits minor

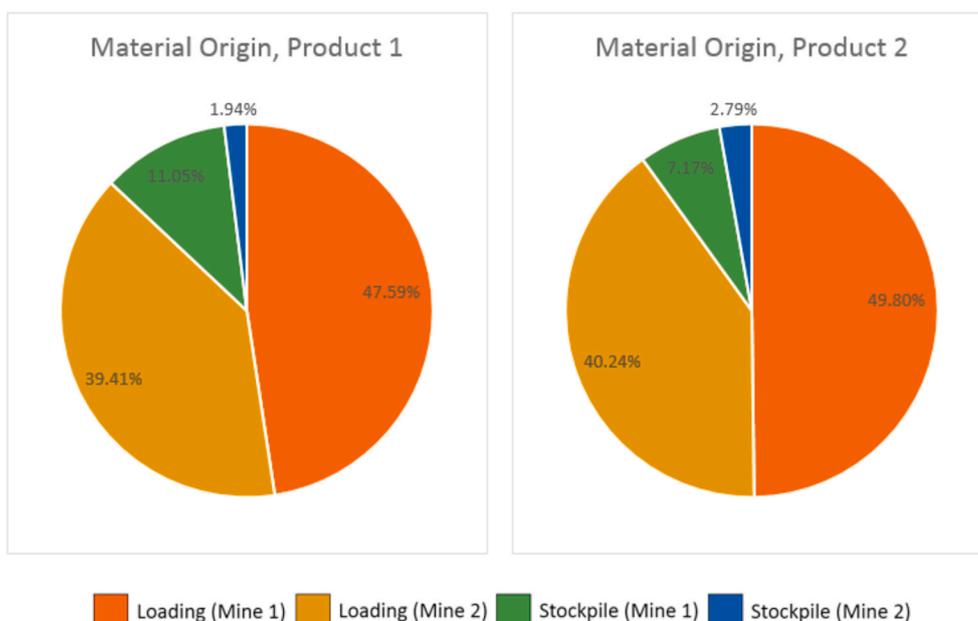


Fig. 10. Provenance of material making up the products at the port.

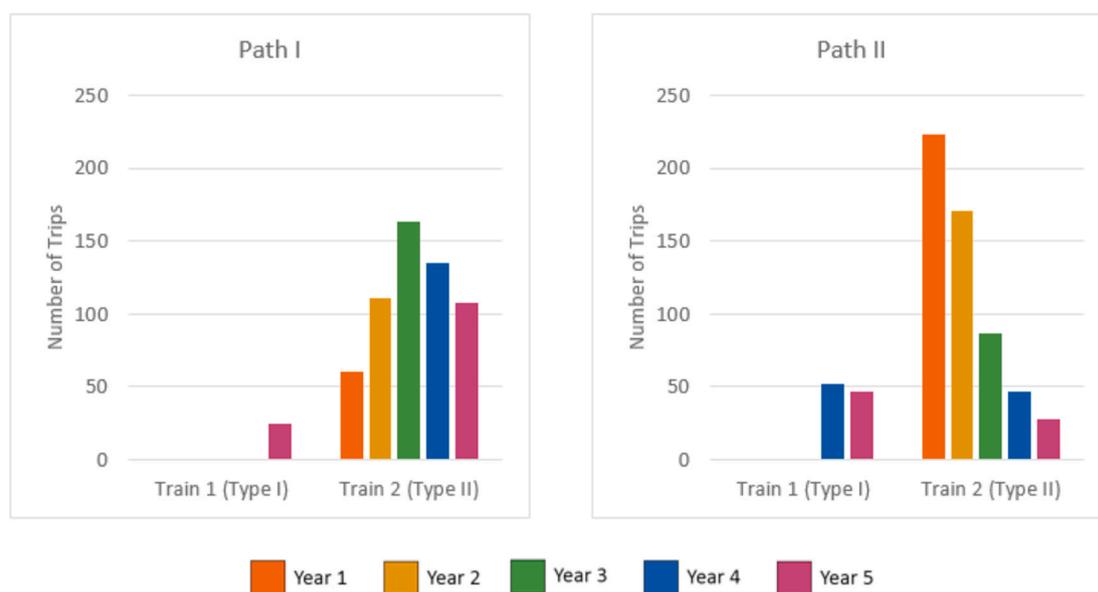


Fig. 11. Train use per path and year.

forecasted deviations in all years except for year 2; the P10 deviates from the lower bound by less than 2% in those four years, and the P50 deviates by less than 1% in year 4. As for Product 2, minor forecasted deviations occur in periods 4 and 5, where the P10 deviates by less than 3%.

Fig. 8 illustrates the yearly forecasts and the related risk profiles of the phosphorus grade of the final products. For Product 1, the phosphorus grades are within the bounds in all years, with the exception of year 4, where the P10 deviates marginally from the lower bound. Also, for Product 2, the phosphorus grades are well within the bounds during all years, with no forecasted deviations. The risk profiles for the phosphorus grades of each product are very tight; i.e., there is very little difference between the P10, P50, and P90 values. Fig. 9 shows the yearly forecasts and the risk profiles of the loss on ignition (LOI) of the final products. There are no forecasted deviations during any years for either product; therefore, the demand is expected to be met.

The origin of the material included in the final products is presented in Fig. 10. Most of the material delivered to the port is taken directly from the mines, with limited contribution from the stockpiles. Thus, the stockpile material left could be used in the future to deal with future demand. Fig. 10 highlights the need to optimize all components of a mineral value chain simultaneously. Indeed, both products require blending material from both mines; the demand would not have been met as well if the mines had been optimized individually.

### 3.2.2. Transportation schedules

The optimization process provides a schedule for the mine-to-port transportation fleet indicating the forecasted use of each train each year. Fig. 11 shows the number of trips for each path completed by each train yearly. Path III is not shown as it was used only once by Train 2 in year 3. Moreover, only the largest train (Table 2) is used during the first 3 years, and it is used to a greater extent than Train 1 in periods 4 and 5.

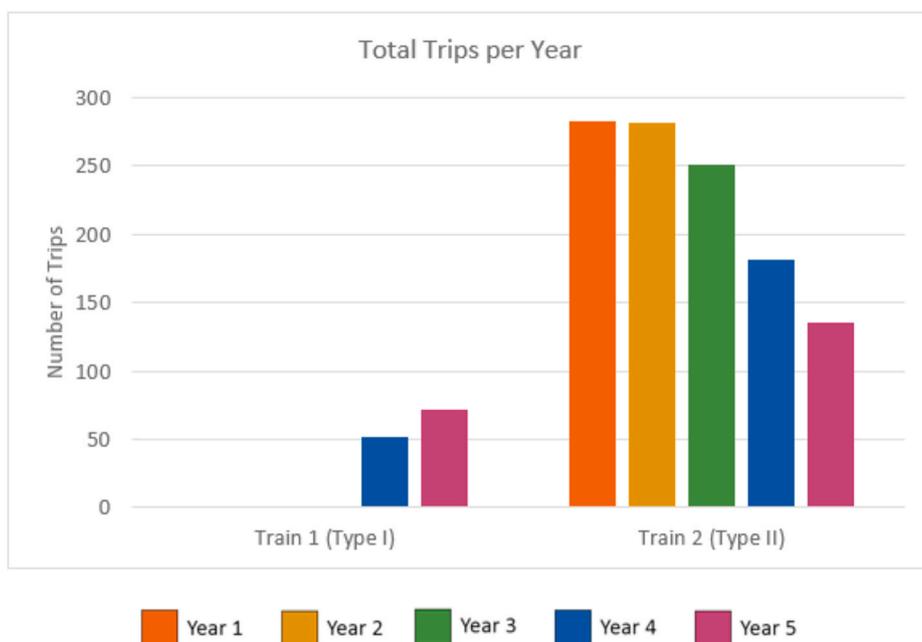


Fig. 12. Overall number of trips completed by each train.

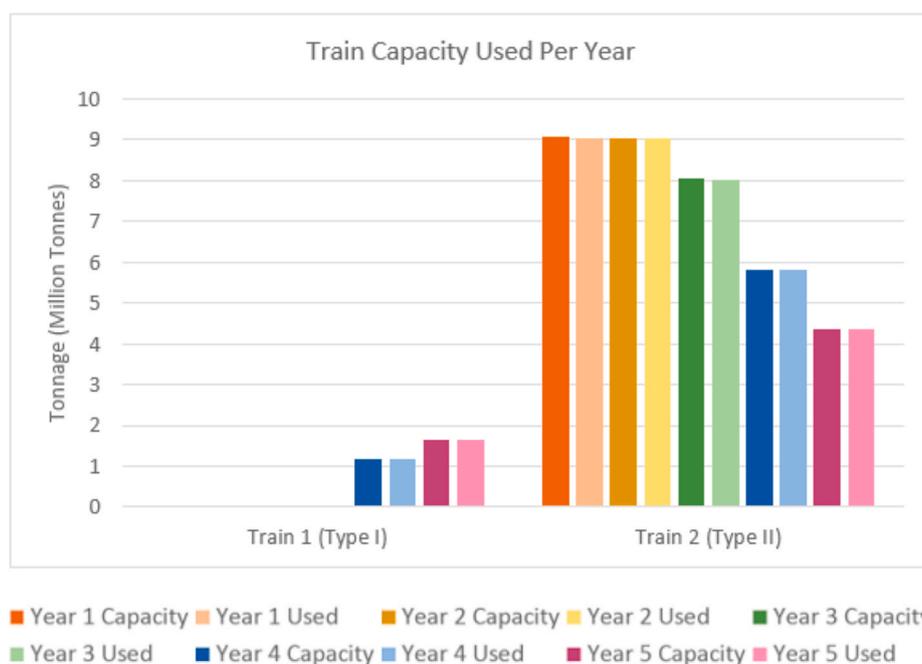


Fig. 13. Summary of the capacity of each train used.

These results indicate that the smaller train can be allocated to other operations during this time, or, in the case of a future project, that it should not be purchased before year 4. This result highlights the need to incorporate mine-to-port transportation scheduling into the optimization of the mining complex. Indeed, traditionally, if only the fixed yearly capacity of the mine-to-port transportation system is specified, then only a percentage use of the system is obtained, rather than the expected utilization of each equipment considered. In addition, Fig. 11 indicates that Train 1 is scheduled to use Path II to a higher extent in years 1 and 2 since, as mentioned in the previous section, Mine 2 is scheduled to produce more material than Mine 1 in this period. When the production shifts over to Mine 1 during years 3–5, the number of trips on Path I increases accordingly. Fig. 12 summarizes the yearly number of trips on

all paths completed by each train. It also indicates that the number of trips decreases over time, in accordance with the decrease in the total demand and the delivered tonnage for the products at the port over time (Table 1 and Fig. 4).

Finally, Fig. 13 allows the comparison of the total capacity of the trains available (obtained by the product of the number of trips completed and the single-trip capacity (Table 2) and the yearly amount of material transported. As shown in the figure, the trains are being used near capacity during each year. This follows from penalizing the unused capacity in the objective function (Eq. (1)) as well as from reducing the overall cost. This shows that simultaneously optimizing the mine-to-port transportation schedule and production schedules of the mines, allows for the adjustment of the transportation schedule according to the

tonnage of material extracted at each mine. Doing so also optimizes train travel.

#### 4. Conclusions

In this paper, a new stochastic mixed integer program is formulated to simultaneously optimize the long-term schedule of the production and the mines-to-port transportation of mining complexes under uncertain material supply. It is the first formulation in the technical literature to date that integrates mine-to-port transportation scheduling in the long-term (strategic) optimization of mining complexes. This formulation can include different numbers of mines, stockpiles, waste dumps, loading areas, trains, and railway layouts; however, only a single port is allowed. The mining complex is subject to multiple operating constraints related to capacities, transportation, and to the geochemical blending of uncertain material supply to meet product targets at the port. A case study is introduced to apply the proposed model to an iron mining complex composed of two mines, each having a stockpile, a loading area, and a waste dump, as well as a single port. The results indicate the model's ability to meet product demand and quality constraints while minimizing the risk of not meeting the targets. The inclusion of the mine-to-port transportation scheduling in the long-term optimization of a mining complex allows for the analysis of how the fleet is used over time.

Future research could address limitations of the model presented in terms of size and complexity, by developing suitable metaheuristic approaches to facilitate the optimization of large-scale instances. As the example presented herein demonstrates, the proposed approach has the potential to increase the value of mining operations by capitalizing on synergies between the components considered simultaneously, particularly mine-to-port transportation. In addition, the formulation presented could be extended to mining complexes with multiple ports, each having different stockpiles, as well as integrate port-to-client transportation as a new aspect to extend decision support for related strategic planning.

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