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Simultaneous stochastic optimization of mining complexes - mineral value chains: an overview of concepts, examples and comparisons

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ABSTRACT
This paper overviews the simultaneous stochastic optimisation of mining complexes or mineral value chains where raw materials mined from mineral deposits in an area are transformed into a set of sellable products. The supply of materials extracted from available mines represents a major source of uncertainty and technical risk that needs to be managed, along with market demand. An overview of the main concepts, case studies and comparisons show how the approach manages risk and capitalises on synergies between the components of the mining complex and major differences from conventional methods. Results lead to strategic plans with larger amounts of metal produced from the same mineral resources, a substantially improved ability for operations to meet production forecasts, and a significantly higher net present value.

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1 Introduction

A mining complex or mineral value chain is an engineering system that manages the extraction of raw materials from a group of mineral deposits (mines) in a geographic region, followed by the treatment of the extracted materials through different processing facilities that are interconnected by various material handling and transportation methods. This system leads to sellable products delivered to various customers and/or the spot market, along with different types of waste materials that need to be managed. Underlying uncertainties (stochasticity) relate to the quantity, quality and spatial distribution of the materials supplied from the mines, capital investment options, technical aspects (mining, processing, environmental), and the metal's spot market prices. An example of a mining complex is shown in Figure 1. In this example, three mines share multiple downstream facilities including stockpiles, concentrators, leach pads, smelters, tailings, slag, and waste dumps. At the same time, the products generated, as well as the material handling and transportation processes, connect upstream mining activities with final customers and markets.

Conventional strategic or life-of-mine planning [1–3] considers and optimises each component of a mining complex independently, an approach that ignores the synergies between various components, leading to sub-optimal plans, forecasts and evaluations. Early efforts to advance related approaches, concepts and technologies to jointly optimise different components of a mining complex are introduced in Hoerger et al [4], showing the application at

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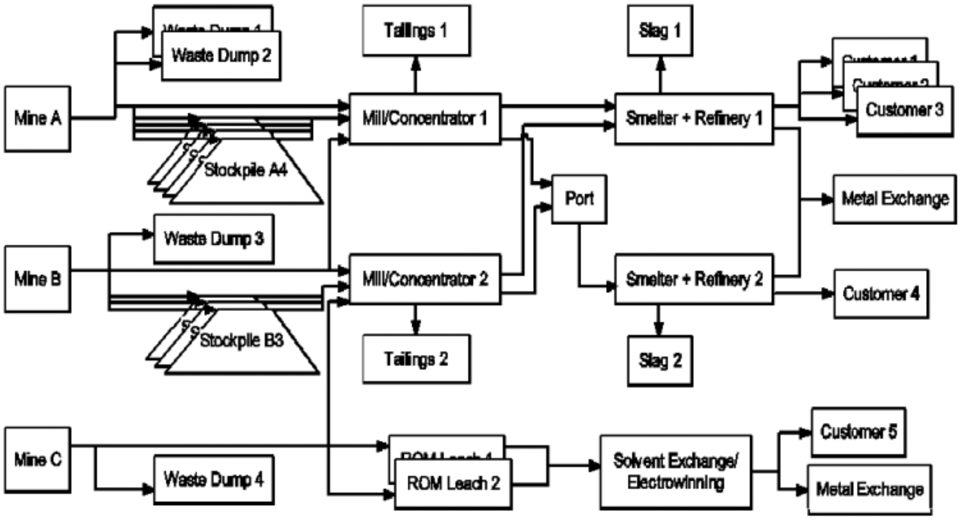

Figure 1. Example of a mining complex/mineral value chain; materials flow from mines on the left to products delivered to customers on the right of the figure.

Newmont's Nevada operations. Their multi-period mixed integer programming model considers downstream decisions in operations, including several mines, stockpiles and processing facilities, and takes advantage of the synergies between the related components, leading to considerable production improvements. Further efforts to increase the components and capabilities of a mining complex to be integrated and simultaneously optimised are shown in Stone et al [5]; the authors present BHP's optimisation framework, software Blasor, and applications in simultaneous optimising multi-pit operations with complex product requirements. Whittle [6,7] introduces the software Prober that was developed to perform a global optimisation of the main components of mining complexes based on an interactive approach. While these constitute major developments and all case studies available document substantially improved performance when compared to the traditional optimisation approaches, they also have major limitations stemming from: (i) significant simplifications and assumptions made to reduce computational complexity, as well as (ii) the inability to deal with the inherent critical uncertainties. Computational complexities are due to the exceptionally large size of mathematical programming optimisation formulations needed to address practical strategic mine production planning applications.

Simplifications and assumptions include pre-decided long-term schedules in the mines considered, the large-scale aggregation of mining blocks misrepresenting the selectivity of mined materials, the exclusion of parts of mining complexes, the inability to deal with non-linear relations, including stockpiles, the inability to integrate all interactive components, such as mineral processing plants and others. The additional major assumption that all previously mentioned approaches rely upon is that the mineral deposit model used as input is the actual deposit, as represented by an estimated orebody model [1]. The latter models, whether they are geostatistically estimated or not, misrepresent the in-situ spatial variability of pertinent geological attributes of the materials extracted from the ground and, importantly, do not quantify the uncertainty of metal grades, material types and their quality, all leading to sub-optimal production forecasts [8–14].

The supply of materials extracted from mines in a mining complex represents a major source of uncertainty (or stochasticity) and technical risk that needs to be accounted for and managed in strategic mine planning (similarly to the case of individual mines [15]). The development of new digital technologies facilitates the simultaneous stochastic optimisation of mining complexes in a single optimisation model [16–30]. This advancement overcomes the major limitations of previous approaches noted above and adds substantial value. Simultaneously optimising all inter-related components and aspects of a mining complex integrates production scheduling, blending, stockpiling, equipment capacities and related capital investments. It further includes the integration of non-linear transformations occurring in processing streams, waste management, the utilisation of processing streams, and, finally, the transportation of products to customers, as well as the assessment of market contract options. Resulting improvements to life-of-asset(s) strategic planning and valuation have been shown to: Markedly improve the reliability in a mining operation's ability to meet production forecasts, lead to larger amounts of metal produced from the same mineral resource and a substantially higher net present value (NPV) than existing approaches. These are all a direct outcome of the ability of new smart technologies to capitalise on directly managing technical risk and synergies between the components of a mineral value chain. The present manuscript provides an overview of the above developments, starting from the general aspects of the simultaneous stochastic optimisation framework and continuing with a comprehensive presentation of main insights from case studies in different mining complexes, including comparisons to conventional industry optimisation practices.

In the following sections, firstly, the basic parts of the stochastic mathematical programing model considered herein are outlined and related concepts are elaborated upon. Subsequently, examples from applications are presented to demonstrate key practical aspects and contributions of the approaches discussed. Conclusions follow.

2 Method overview

The simultaneous stochastic optimisation of a mining complex – mineral value chain is outlined in this section, while the reader is referred to Goodfellow and Dimitrakopoulos [18,19] for detailed descriptions. Emphasis is placed on explaining the main conceptual differences between established modelling approaches, while briefly indicating efficient solution approaches to the corresponding exceptionally large optimisation problem.

2.1. Basic mathematical model

Consider that a mining complex integrates C components, as shown in the example in Figure 1 (mines, stockpiles, processing streams, concentrators and plants, waste dumps, and so on); each component is denoted as $i \in C$. There are A properties of the materials mined (metal, grade, tonnage, geometallurgy, and so on) tracked along the value chain and each one is denoted by $a \in A$. S stands for the number of stochastically simulated scenarios (e.g. [31–34]) of the pertinent rock properties of the mineral deposits mined, while each scenario is $s \in S$, and composed of mining blocks reflecting expected mining selectivity. Note that S may as well include commodity prices incorporating market uncertainty, when applicable. Production periods, $t \in T$, are in years and the life-of-mining-complex optimised has T production years; the extraction sequence of blocks mined per year per mine are outputs of the optimisation process.

The optimisation approach developed is based on two-stage stochastic integer programming (SIP) detailed in Birge and Louveaux [35], which is a framework used in past approaches to life-of-mine planning optimisation of single mines [36–39]. The objective of the simultaneous optimisation is to maximise the net present value of products sold, whilst managing and deferring risk, as well as minimising deviations from production targets at different destinations in the value chain model. The objective function is:

$$\underbrace{\max\frac{1}{|S|}\sum_{t\in T}\sum_{s\in S}\sum_{i\in C}\sum_{a\in A}p_{a,i,t}\cdot v_{a,i,t,s}}_{\text{Part1}}$$
$$\underbrace{-\frac{1}{|S|}\sum_{t\in T}\sum_{s\in S}\sum_{i\in C}\sum_{a\in A}\left(c^{+}_{a,i,t}\cdot u_{a,i,t,s}+c^{-}_{a,i,t}\cdot l_{a,i,t,s}\right)}_{\text{Part2}} \qquad (1)$$
$$\underbrace{-\sum_{t\in T}\sum_{K\in K}p_{k,t}\cdot w_{k,t}}_{\text{Part3}}$$

Part 1 of the objective function accounts for discounted net profit from the products generated by a mining complex. The term $v_{a,i,t,s}$ stands for the value of a pertinent property a in a given scenario s, at period t at a component i of the mining complex, and $p_{a,i,t}$ is the related time-discounted revenue (or expense) generated, based on a given discount rate. Part 2 is used to manage and minimise deviations from various targets, and includes, mines, stockpiles, blending, processing, deleterious, elements, and so on. It is used to reduce risk in forecasts as well as risk deferral (geological risk discounting). $u_{a,i,t,s}$and $l_{a,i,t,s}$ represent the upwards and downwards deviations, respectively, from production requirements and targets for property a, at time t, for scenario s, and $c^{+}_{a,i,t}$and$c^{-}_{a,i,t}$ are the cost (penalty) of deviation. This cost is used to defer the risk of not meeting production targets to later periods by using geological risk discounting factors [15,38]. Lastly, Part 3 includes K capital expenditure (CapEx) options, such as equipment purchase, infrastructure expansion and so on, where $p_{k,t}$ represents the discounted purchase price of the capital expenditure option k ($k \in K$) and $w_{k,t}$ is the decision variable that defines the number of CapEx options k that are exercised in period t. For a detailed description of related constraints, including capacity, reserve and mining block access, destination policy, mine extraction, processing stream flow and others, please refer to Goodfellow and Dimitrakopoulos [18].

2.2. Mining aspects and conceptual differences from past approaches

To elaborate on the main concepts related to the simultaneous optimisation model discussed above, Figure 2 shows an example, which includes: two mines with the two related mineral deposits represented by different simulated scenarios, one of the existing processing streams of the related mineral value chain, and two customers for the product(s) generated from the processing stream. The example aims to stress that the proposed simultaneous stochastic optimisation with the objective function in (1) will output *(i)* the production schedule for each mine and the destination policies for the materials extracted from the mines; *(ii)* the choice of a processing stream is decided not when scheduling a mine's production, but at the potential processing streams, and *(iii)* the value of products is decided at the customer level (again, not at the mine scheduling stage), where also joint supply and demand uncertainty can be accounted for. These are major differences from all the past approaches to mine planning optimisation. It is notable that, in this framework, unlike with all past strategic mine planning optimisation approaches, *(a)* no economic values of mining blocks are used for the scheduling process; financial assessments are done at the stage of products produced for different customers or the spot market or a combination of both, and all production costs of raw materials from mines to products are accounted for. In addition, *(b)* no pre-decided 'optimal' cut-off grades are needed as an input; the final schedules generated to maximise net present value for the value chain provide the optimal cut-offs. Furthermore, notable is *(c)* the ability to deal with the properties of the materials mined at different locations at the potential processing streams, while blending and destinations are decided. This allows for the optimisation process to deal not only with metal, tonnes and grades, but also with non-linearly behaving properties such as throughput and recovery, as well as geometallurgical properties [25,40,41], and so on. This provides

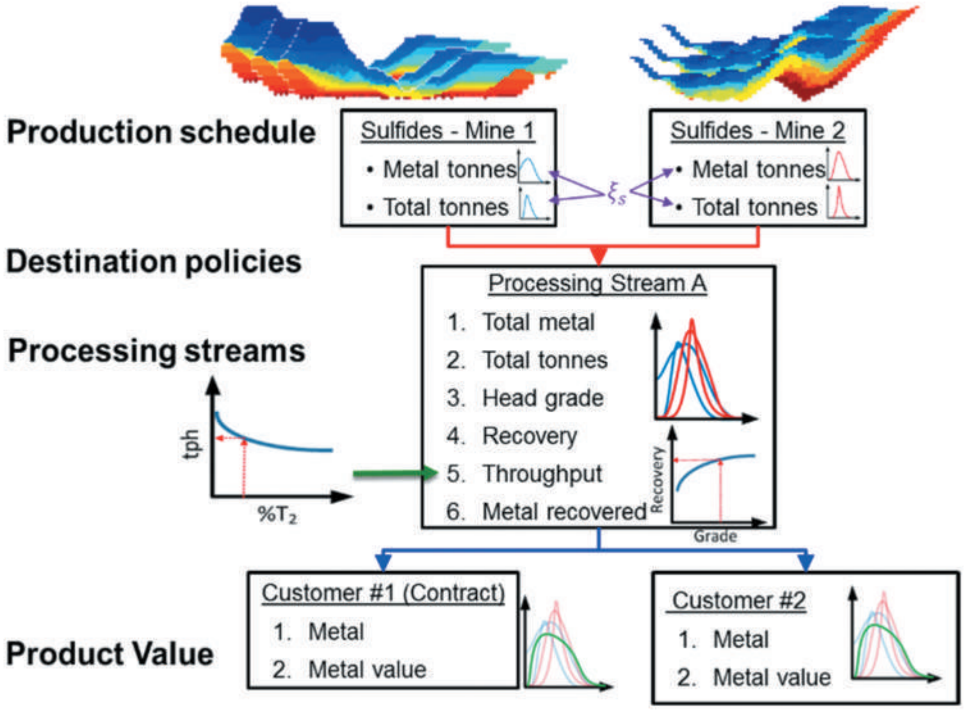

Figure 2. Example of a mining complex/mineral value chain and one of its material processing streams.

a substantially more effective framework to deal with critically important aspects in the performance and, therefore, strategic planning and valuation of a mining complex, including the utilisation and operating modes of the various processing streams.

It is critically important to stress that, as Figure 2 shows, the uncertainty and variability of all pertinent aspects of the materials that flow through the value chain are quantified at any stage. In addition to the ability to quantify uncertainty and manage risk with respect to the supply of materials, the stochastic simultaneous optimisation approach allows the integration of commodity price (demand) uncertainty when product value is assessed at the customer level. An example is shown in Section 3.3.

2.3. Solution approaches

It is apparent that the size and computational requirements needed to apply the approach discussed herein in any real-world case study requires solving very large-scale mathematical models. For example, in order to provide a sense of the corresponding optimisation problem size, consider a mining complex with three mines, two ore processing streams, a stockpile and a waste dump, life of mines between 10 and 30 years, 15 to 20 simulated scenarios per mineral deposit for one attribute (grade) and an average deposit size of a few hundred thousand mining blocks; in such a case, scheduling decision variables will be in the order of 20 million. While such a case is common and certainly not at the most demanding level for the approach discussed herein, this size is greater than what any of the existing mathematical programming solvers can handle in a reasonable amount of time. Furthermore, the optimisation model discussed also incorporates non-linear constraints and belongs to the notoriously difficult class of mixed-integer non-linear programs. These intrinsic solution difficulties have led to new algorithmic developments. The most widely used solution methods have been metaheuristics, for they provide an efficient methodology with

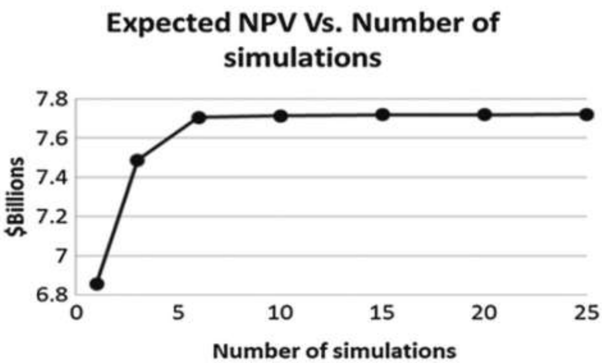

Figure 3. Example showing the sensitivity of the NPV forecast to the number of simulated realisations used in the simultaneous optimisation of a mining complex (modified from [17]).

a use that is not restricted by the size of the problem or by linearity. Related methods are presented in the references provided at the beginning of this section, while the reader who is interested on this topic is referred to Lamghari and Dimitrakopoulos [42–45], Gilani and Sattarvand [46], Paithankar et al [27], Fathollahzadeh et al [47], and others. In the examples presented in the following section, a multi-neighbourhood simulated annealing approach is used [18]. More specifically, the related algorithm uses three neighbourhoods. The first neighbourhood deals with the extraction sequence and is based on a constraint relaxation strategy. Slope constraints are dropped from the search space to create a larger space, and the neighbourhood structure involves changing the period in which a block is extracted. The selected neighbour solution is afterwards transformed into a feasible one by applying a repair heuristic that moves the predecessors or the successors of the selected block. The second and third neighbourhoods deal with the downstream processing streams by modifying either the destination of a cluster of blocks or the amount of material sent from one processor to another. Using multiple neighbourhoods along with constraint relaxation have proved an effective strategy that ensures both diversification and intensification and moves the search away from local optima.

Computational challenges also relate to the number of simulated scenarios of mineral deposits used. It has been documented [17] that, in a mining complex with a single mine and several processing streams, 10 to 15 simulations are sufficient to generate stable results; see for example Figure 3. This result repeats the findings of previous experiments [48] and is attributed to the related support-scale effect, given that several hundred to a few thousand mining blocks are eventually grouped to represent a year of production in the related mining schedules. Recent experiments further show that, in a mining complex that comprises two mines and 20 simulations per mine (i.e. 400 combined scenarios), stable results will be generated when about 70, randomly chosen, of these scenarios are used, showing that strategies with respect to selecting different scenarios support improvements in computational efficiency.

3 Examples from applications and comparisons

This section elaborates on pertinent aspects of the method outlined in the previous section based on case studies at different mining complexes. General major aspects, comparisons and main differences with conventional approaches are first discussed, then applications dealing with specific topics, such as waste management, CapEx investments and the integration of joint supply and demand uncertainty, are visited.

3.1. General aspects of mining complex optimisation and comparisons

The first example presented in this section is from a nickel laterite mining complex that includes two deposits [19] and focusses on orebody material supply risk management and minimising deviations from the required production targets. The chemical elements of interest are nickel, silica, magnesia and iron, and five material types are considered, namely, bedrock, limonite, saprolite waste, low-grade saprolite and high-grade saprolite. Materials from the two mines are sent to intermediate stockpiles of the same capacity, three for low-grade saprolite and another three for high-grade saprolite materials; bedrock, limonite and saprolite waste are sent to the same waste dump. All intermediate stockpiles feed two homogenisation piles, each having a fixed capacity that then feed a pyrometallurgy processing plant. While maximising NPV, the simultaneous stochastic optimisation method aims to generate materials flow starting from the mines that meet the plant's requirement of iron grade blending between 12% and 16%, the requirement that the feed must meet silica-to-magnesia ratio (SiO_2:MgO) between 1.5 and 1.8 between 1.5 and 1.8, and the plant's capacity requirements. Note that, for the study, all pertinent geological attributes of the two deposits (nickel, silica, magnesia and iron) are stochastically simulated [49], as needed. Figure 4 makes several comparisons. The top of the figure shows the plant's feed over production periods in terms of SiO_2:MgO ratio (left) and tonnage (right) from a production plan generated using conventional industry mine planning optimisation practices (estimated orebody models and deterministic optimisation, as noted in Section 1). The conventional forecast is in red, while the risk profiles of the same forecasts are shown (dark blue is the P50 with the related P10 and P90 in green; note that P10, P50 and P90 represent the 10%, 50% and 90% probability of obtaining values below the

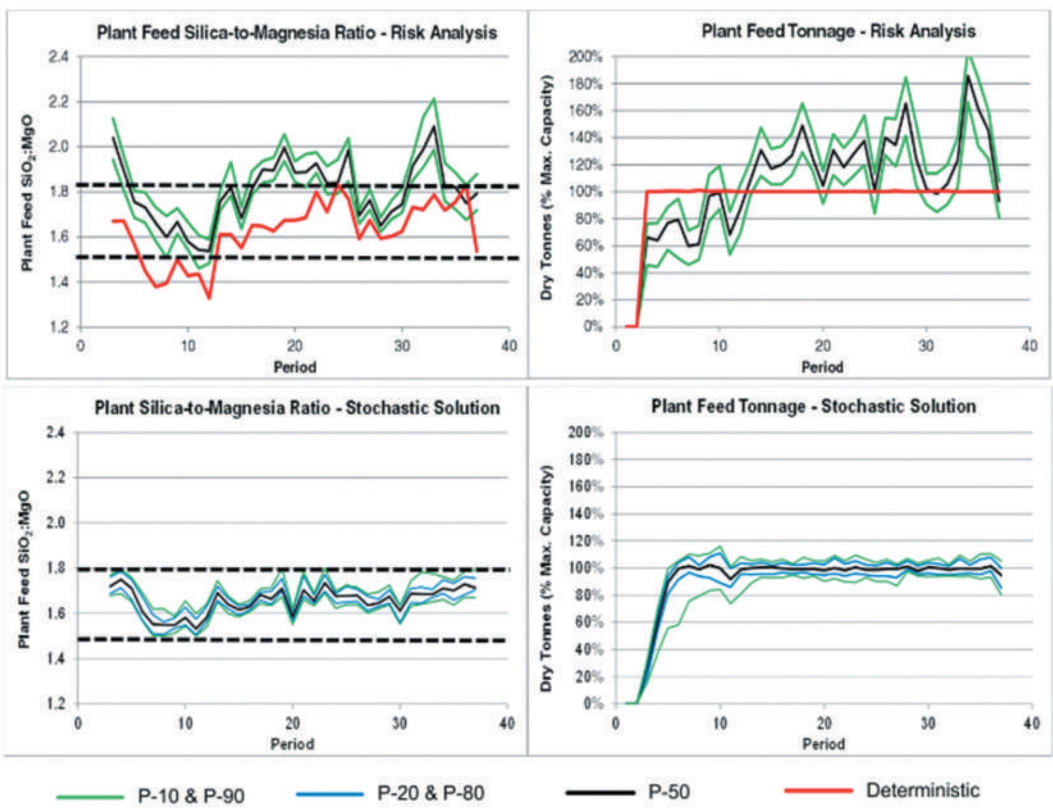

Figure 4. Nickel laterite mining complex: Plant feed SiO_2:mgo ratio (left) and tonnage (right) for a conventional production plan and its risk profile (top), compared to the plant feed performance of the proposed simultaneous stochastic optimisation (bottom).

corresponding forecast) and are generated by testing the performance of the conventional schedules against the geostatstically simulated scenarios of the materials extracted from the two mines. The risk profiles show that the SiO_2:MgO ratio in several periods (e.g. 15 to 25, 30 to 35) will exceed the upper limit allowed, while the plan does not feed the plant up to year 12 with the tonnages of iron content expected. Contrary to these results, the bottom graphs in Figure 4 show that the method presented herein will meet expectations (see average, P10 and P90) for both SiO_2:MgO ratio and tonnages of materials delivered to the plant, to the best of the information available at the time of the study. It should be noted that the two bottom right graphs in Figure 4 reflect the effect of Part 2 in (1) that manages risk and minimises deviations from production targets and requirements.

While managing supply risk, as shown in the previous example, is a core aspect of the stochastic simultaneous optimisation, a case study at a copper-gold mining complex with various mines and processing streams documents further critically important features. Figure 5 depicts results aiming to provide a comparison between high-end conventional practices that are typically deterministic

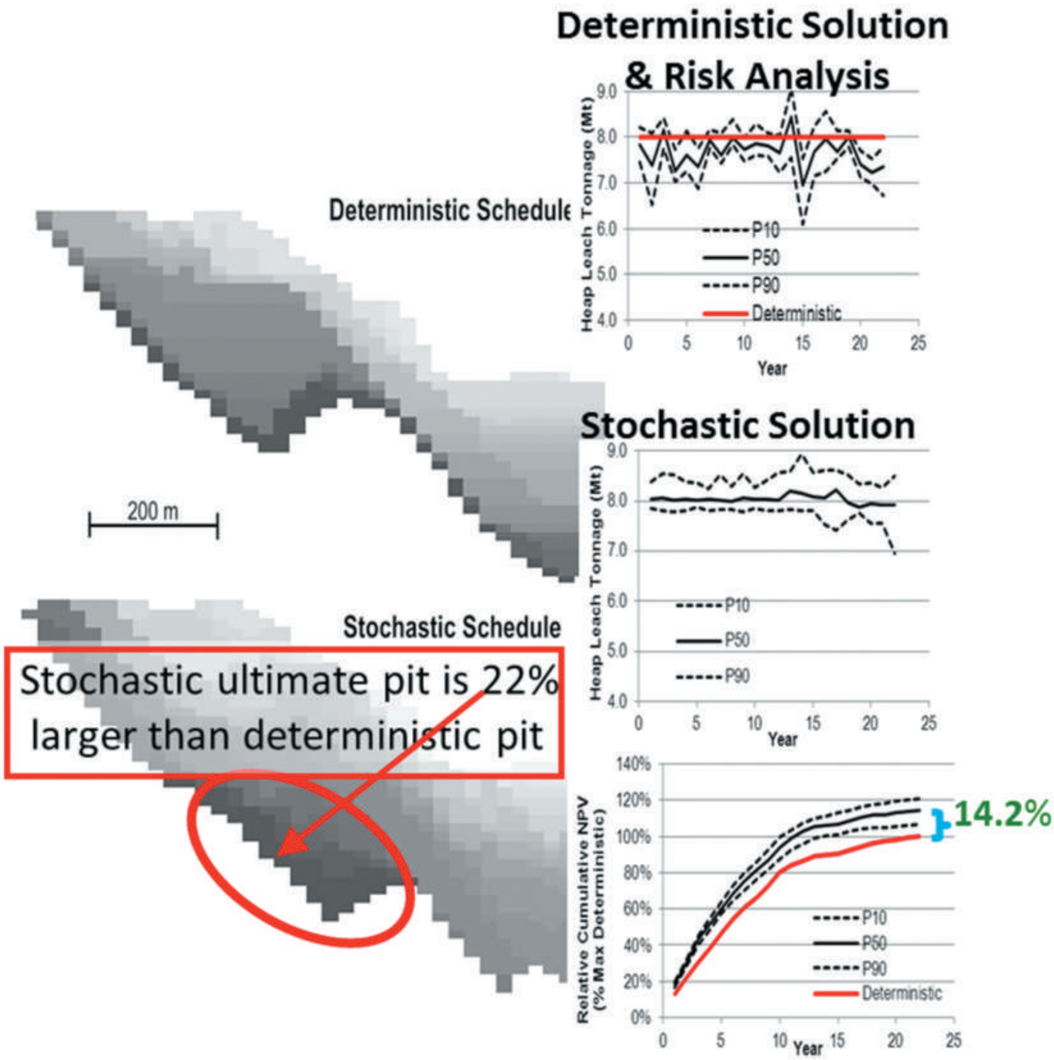

Figure 5. A copper-gold mining complex: Stochastic pit limits are larger than the deterministic (left); risk analysis of deterministic of heap leach tonnage shows targets will not be met (top right), while the stochastic solution will (middle right); total NPV of stochastic solution is 14.2% higher than the deterministic one (bottom right).

(as noted before, a single estimated representation of the related mineral deposits is used along with deterministic mine planning optimisation) and the approach discussed herein. As also shown in the previous example, Figure 5 shows that risk analysis of conventional production forecasts documents that these forecasts will not be met (e.g. tonnage to processing streams in the top right part of the figure), while the corresponding forecasts for the proposed method indicates that they will be met (middle-right side of figure). Two major issues are also shown in Figure 5: *(a)* the NPV of the simultaneous stochastic optimisation of the mining complex leads to substantially higher NPV (14.2%), corresponding to a substantial increase in metal production, and *(b)* the optimal pit limit in the mine shown is not only physically different, but also larger (22%) than the conventional, which is one of the reasons that more metal from the same mineral deposit is recovered. It should be noted that these results are not exclusive to this case study; they have been consistently documented for a wide range of mining complexes that supply various commodities [23–28,50–54].

In the next example, the simultaneous stochastic optimisation framework for long-term mine planning is applied at an operating gold mining complex that consists of two open-pit mines, three external sources supplying additional materials, 11 stockpiles, three ore processing facilities (autoclave, oxide mill and leach pad) and one waste dump [55]. The quantification of uncertainty and variability associated with the diverse sources of material used by this mining complex include not only the pertinent attributes of the two mineral deposits, but also the existing stockpiles and external sources. To provide the required global uncertainty quantification for the materials used in this mining complex and for the pertinent elements from the two mines (Au, SS, CO_3 and C_{org}) simulated using related methods [49], the existing stockpiles are geostatistically simulated using either collected samples assessing their quality and variability or are simulated using related production grade control data. The uncertainty of the material supplied by the three external sources is quantified using Monte Carlo simulations using the information available from previous production years. An important component of this mining complex is its autoclave, which is used in full capacity by blending materials from different sources, as required. Typically, blending requirements are satisfied by adding acid to sulphide ore to reduce the CO_3 concentration and allow the SS/CO_3 ratio to remain within the required range. Regarding this aspect, Figure 6 shows the risk profile of the acid comparison at the autoclave over the life-of-asset plan of this mining complex, which is generated by the simultaneous optimisation. The latter plan consistently meets the legally

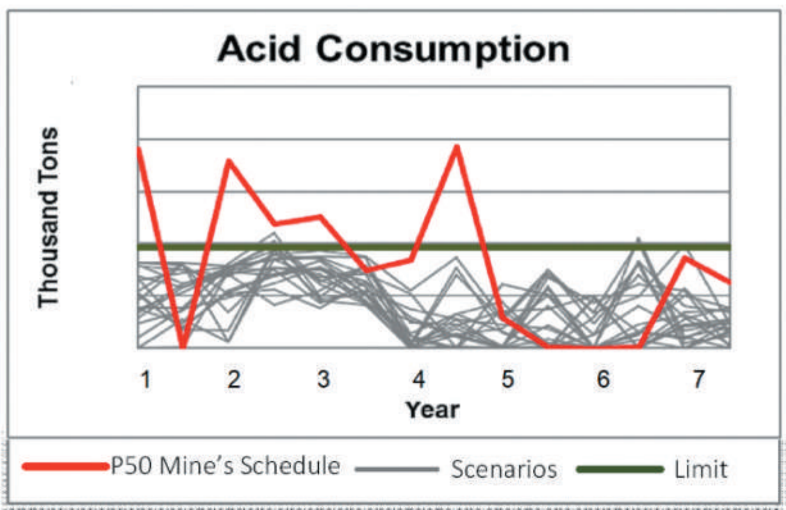

Figure 6. Risk profile for acid consumption (grey lines) below the maximum allowed by regulations (dark green line) for the stochasic optimisation production plan and P50 of the risk profile (red line) for the conventional deterministic long-term schedule (modified from [55]).

required acid consumption limit, unlike the mine's conventional long-term plan where the uncertainty of the different sources of material is not considered (as the P50 of the corresponding risk profile shows).

In addition to the ability to meet important operational production requirements, the stochastic solution for the strategic production plan for this gold mining complex also considers issues of the physical aspects of related schedules. Figure 7 shows the practical life-of-mine schedule for two benches in one of the mines from the mining complex to demonstrate that the actual production schedules are physically very different. Finally, similar to the previously noted case studies, the stochastic optimisation approach discussed herein outperformed the conventional plan of the mining complex not only in terms of acid consumption at the autoclave and other material blending requirements, but also showed a significant increase in recoverable gold and NPV while managing risk and increasing the probabilities for achieving production targets.

The use of the simultaneous stochastic optimisation framework for strategic planning at a large copper–gold mining complex to integrate uncertainty regarding non-additive geometallurgical attributes, such as the semi-autogenous power index and bond work index, is detailed in Kumar and Dimitrakopoulos [25]. The related mining complex is composed of two mines, five crushers, two stockpiles, three mills, two leach pads and one waste dump. The main sellable product is copper, however, gold, silver and molybdenum are also produced. To ensure a consistent throughput of material to comminution circuits and a reduction of energy consumption and related costs, material with different hardness properties must be combined. This requirement was handled by introducing constraints controlling the hard-soft ratio at the mills (geometallurgical targets). Furthermore, and as needed, in addition to grade and material types, the semi-autogenous power index and the bond work index are geostatistically simulated in order to integrate the hardness uncertainty of the processed material into the stochastic optimisation framework. Figure 8 shows an example of the hard/soft ratio and its related risk profile for one mill for the stochastic plan.

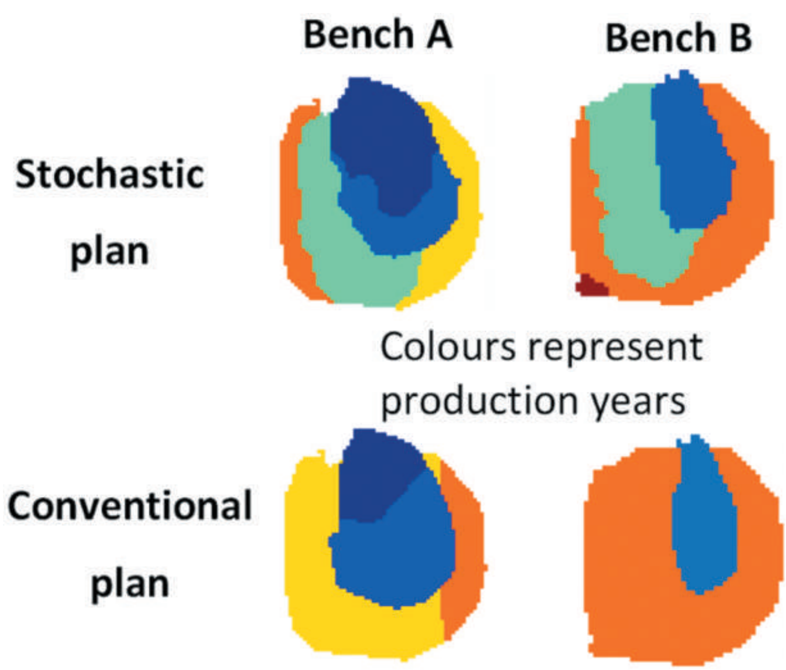

Figure 7. Life-Of-Mine production schedule for two benches in one of the open-pit mines in the gold mining complex; stochastic solution shown at the top and the mines conventional deterministic plan at the bottom, where colours reflect years of production (modified from [55]).

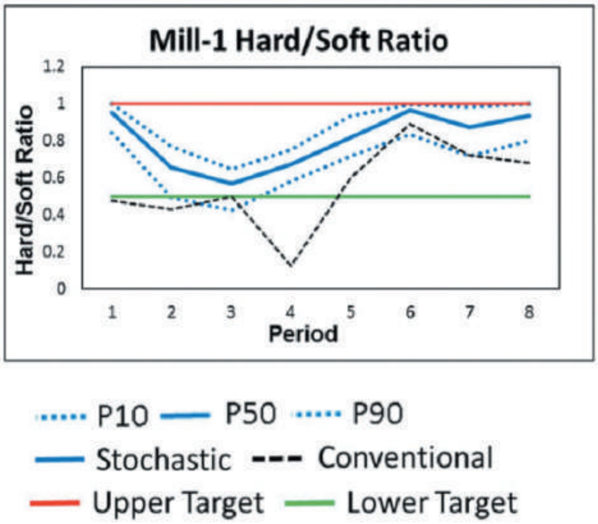

Figure 8. An example of geometallurgical target forecasts (hard/soft ratio) for one mill for the stochastic and conventional plans at a copper–gold mining complex (modified from [25]).

Compared to, and unlike the performance of the conventional plan, also shown in Figure 8, the stochastic one shows very small chances of deviation from the hard-soft ratio targets. In general, when compared to the conventional strategic production plan of the mining complex, the stochastic approach is shown to significantly reduce the risk of not meeting capacity and geometallurgical targets. Other substantial improvements are increases of 12.5% in additional copper, 22.9% in gold, 32.4% silver and 34.7% additional molybdenum, leading to a 19.3% increase in NPV.

3.2. Addressing waste management issues

Waste management within the strategic mine planning and production scheduling process is a critical aspect of the related optimisation, in addition to the delivery of valuable products to the market. Within the simultaneous stochastic optimisation framework, the inclusion of uncertainty and variability in the pertinent properties of the materials mined and the waste produced both impacts and improves the life-of-asset planning, with respect to environmental and economic aspects. This is because the related mine production schedules (including blending, storage and disposal of waste material) directly manage waste and contribute to the reduction of remediation costs, site monitoring and further environmental impacts, thus improving operational sustainability. A recent study at a gold mining complex [56] considers material supply uncertainty based not only on gold, but also on co-simulated carbon and sulphur grades, which is used to then determine the neutralisation and acid generation potentials. The former potential relates to the quantity of carbonates that can reduce acid mine drainage potential, while the latter depends on the content of sulphates derived from pyrite. The materials mined contain both non-acid-generating and potentially acid-generating rock. Managing potentially acid-generating rock is critical in this mining complex, in order to avoid the production of harmful contaminants, minimise surface disturbance and to satisfy permitting constraints. The case study shows that simultaneous stochastic optimisation generates production plans and forecasts that balance the mineral processing facility requirements and waste management by simultaneously optimising the cut-off grade policy and considering uncertainty. A comparison of the results with the conventional life-of-asset production plan shows substantial improvements in terms of satisfying environmental, permitting and processing targets. The extraction rate of potentially acid-generating waste materials is obtained by optimising production scheduling that jointly considers waste

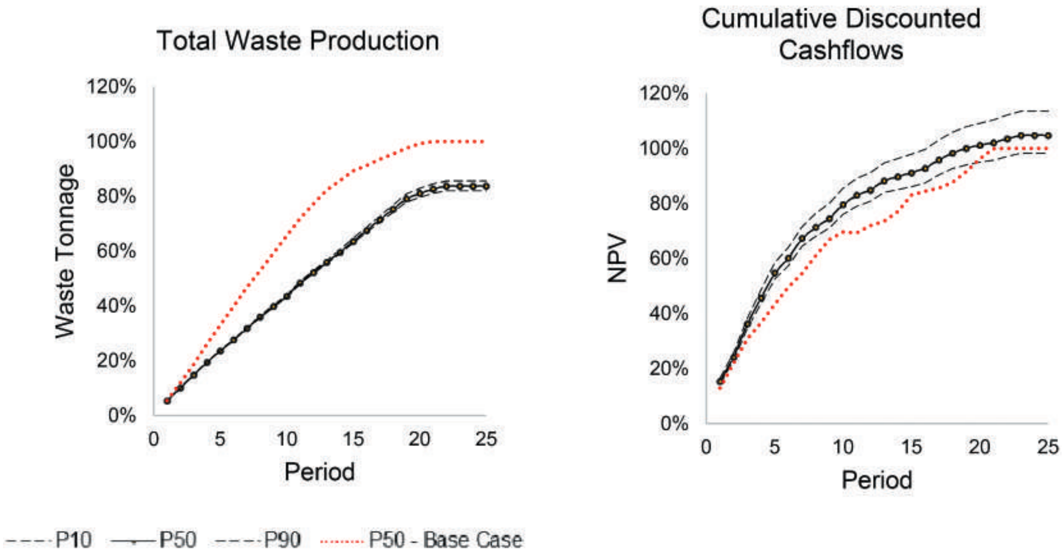

Figure 9. Simultaneous stochastic optimisation results and corresponding risk profiles (black lines) compared to the conventional forecast (red dashed line) for cumulative waste production on the left, and net present value on the right (modified from [56]).

management, cut-off grades to be used, processing stream destinations and stockpiling options. Figure 9 shows that the stochastic approach leads to *(a)* 16% less waste production compared to the conventional mine plan, and *(b)* a 6% increase in the NPV, when compared to a conventional approach, while minimising the likelihood of deviating from production targets and ensuring environmental constraints are satisfied.

Tailings management at a gold mining complex with copper and silver by-products considering a capital expenditure (CapEx) investment for expanding its existing tailings storage facility, the main bottleneck in a potential extension of the asset's life, is detailed in a case study presented by Saliba and Dimitrakopoulos [57]. The gold mining complex includes two open-pit mines, eight stockpiles, an autoclave processing facility that recovers metal from the sulphide ore, one non-acid generating waste dump and one tailing storage facility that handles potential acid-generating waste products. The simultaneous stochastic optimisation framework is employed to assess the option of expanding the tailings storage facility and to extend the production life of the mining complex, by optimising the scheduling of materials extraction from the two mines, destination policies and downstream material flow along with CapEx investment decisions. A core aspect in this case study, unlike the examples presented above, is the consideration of CapEx (Part 3 of (1)); notably, a tailings storage facility expansion or the construction of a new facility is a critical capital-intensive consideration and is in the order of hundreds of million dollars. Results show that accounting for the effects of material supply uncertainty and variability on the components of the related mineral value chain provides important results. Within the stochastic framework, an investment is proposed that increases the capacity of the tailings facility by 25% and leads to an overall 14% increase in gold production, positive cashflows in all production years and a 4% higher NPV than the simultaneous stochastic optimisation plan without expansion. The comparison also indicates a possibility for a longer mining complex production life, if a larger tailings facility is constructed. In addition, and as expected, there are major differences from the mine's conventional long-term mining and reclamation plan. The stochastic framework documents a substantial upside in terms of metal produced (gold, copper and silver) and an improved utilisation of resources (open-pits, stockpiles, existing tailings facility), as well as two additional years of production prior to considering the tailings facility expansion. Figure 10 shows the tailings volume forecasts for different approaches considered.

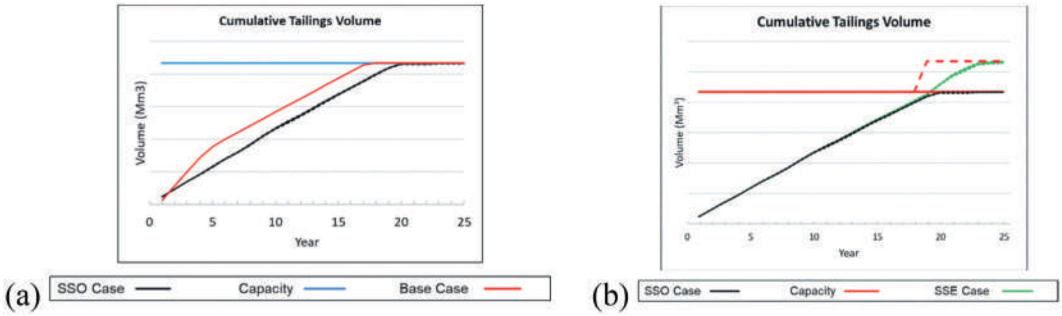

Figure 10. Cumulative tailings volume forecasts: (a) conventional production forecast (red) vs stochastic simultaneous optimizaton (black; dotted lines represent P10/P90 of forecasts); (b) stochastic optimisation with CapEx (green) and without (black), while the expanded capacity is shown (dotted red); (modified from [57]).

3.3. Joint supply and demand uncertainty

Accounting for the joint supply (orebody) and demand (commodity price) uncertainties are additional aspects of the simultaneous stochastic optimisation framework, and they are based on the joint use of stochastic orebody and commodity price simulations [58], as Part 1 of (1) indicates. This is demonstrated in the strategic life-of-asset planning of a multi-pit, multi-stockpile, multi-processor gold mining complex with an exceptionally large number of operating constraints related to both capacities and complex geochemical material blending needs [59]. Figure 11 shows a graph from this case study presenting the cumulative NPV versus production years for both the strategic plan that considers only supply uncertainty, as well as the plan that considers the joint supply and demand uncertainty. The figure shows that (a) on average, the NPV forecasts that consider only the supply uncertainty and those that account for joint supply and demand uncertainty are comparable, with an over all difference of 3%. However, (b) the related risk profiles are different with the P10 to P90 range, and in the case of the joint uncertainty to be overall about double from that of considering only the supply uncertainty, reflecting the uncertainty in commodity prices.

The same study also investigates the cut-off grade decisions of the simultaneous stochastic optimisation approach that does not utilise the prior and deterministic cut-off grade optimisation [60–62] as input to production planning and scheduling, unlike the conventional approaches. The

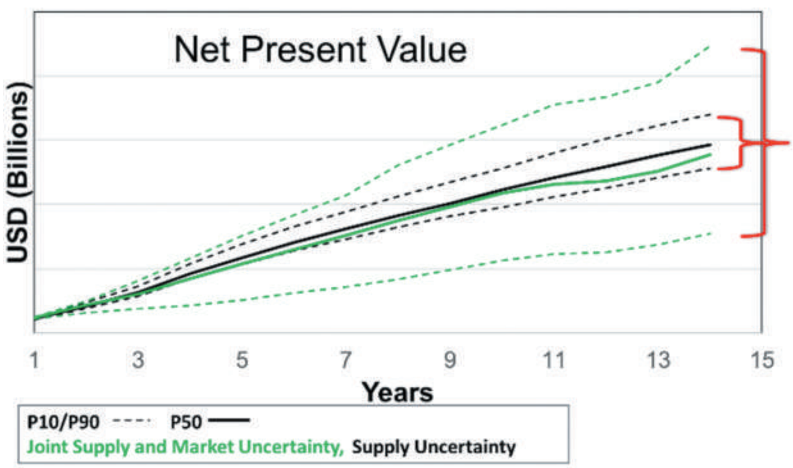

Figure 11. A gold mining complex and NPV comparison between the strategic plan based on stochastic simultaneous optimisation accounting for only supply (orebody) uncertainty (in black lines) and joint supply and demand (commodity price) uncertainty (in green lines). (Modified from [59]).

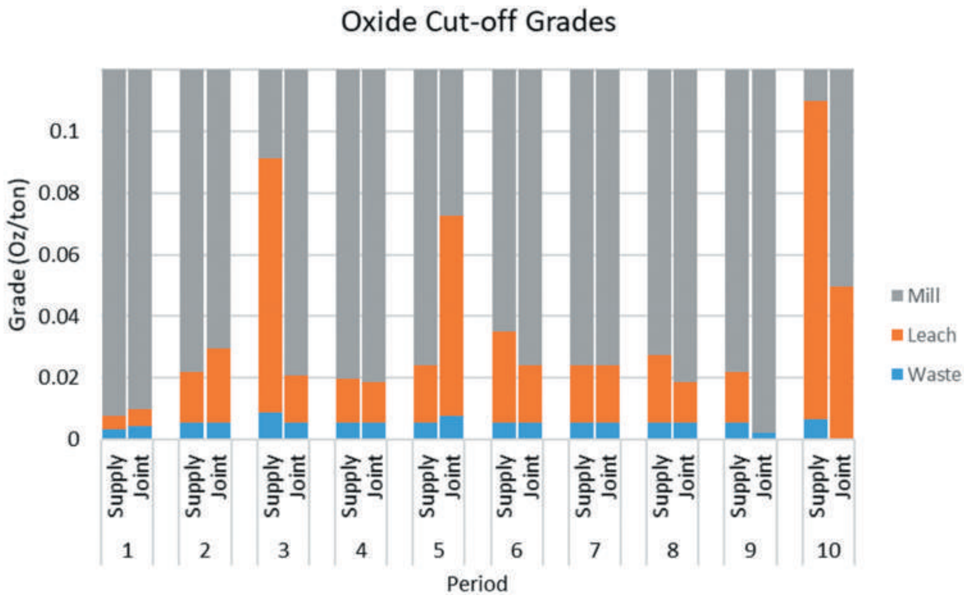

Figure 12. Comparison of optimal cut-off grades from a gold mining complex considering supply uncertainty versus joint supply and demand uncertainty. (Modified from [59]).

approach considers the actual value of generated products, the non-linear blending of materials mined, and the extraction sequences from the related mines, which subsequently provide the ability to calculate the truly optimal cut-off grades [63]. Figure 12 shows the output of the optimal cut-off grades for oxide materials that are to be sent to the mill, leach pad or waste in the mining complex. As seen in this figure, the case considering the supply uncertainty for oxide materials accounting for fluctuations in material availability and quality from one mining period to another generates the optimal production schedules and corresponding cut-off grades that maximise NPV. The results from the case considering the joint supply and demand uncertainty show the ability of the simultaneous stochastic optimisation approach to adapt the production schedules to the mine and to process additional material during periods of elevated price forecasts, and, at the same time, to be relatively conservative when commodity price drops become more prominent. As this example demonstrates, simultaneous stochastic optimisation can account for, manage and quantify risk, including commodity price fluctuations, which supports strategic planning and related decision making. Additional aspects and examples are available in [23,24].

4 Comments and conclusions

This paper overviews underlying components of a new advanced framework for the simultaneous optimisation of mining complexes under uncertainty, which aims to maximise shareholder value and manage risk intelligently, as well as address pertinent aspects of sustainability. To support strategic planning, the proposed framework optimises all the components of a mineral value chain, from mines to products, in an integrated fashion that capitalises on the synergies between the components of the mining complex. The approach deals with challenging technical issues due to uncertainty in the key parameters involved, its large-scale and computational needs, as well as the intricacies of the related data analytics and optimisation. Based on stochastic integer programming and efficient solutions through metaheuristics, the method described has been tested through several case studies in different mining complexes and commodities, and has been shown to consistently outperform conventional approaches for strategic planning. As the examples presented herein also show, the proposed method (a) generates substantially higher NPV; (b) typically produces substantially more metal(s) from the same mineral deposits considered in the related studies; (c) delivers production forecasts with substantially improved reliability; (d) integrates core aspects of waste production, quality and management as part of production planning optimisation; and (e) can directly integrate the joint supply and demand uncertainty, unlike any other approach. These observations are due to the ability of the new smart digital technologies overviewed here to capitalise on the explicit management of technical risk and on the synergies between the integrated components of a mineral value chain in one mathematical model. Additionally, unlike with all past strategic mine planning optimisation approaches, no economic values of mining blocks are considered in the scheduling process of mining blocks in the mines of a mining complex. Financial assessments are not performed at the mine scheduling stage but when products are generated for different customers or the spot market or a combination of both, and all production costs of raw materials from mines to products are accounted for. Similarly, the choice of a processing stream is determined not when scheduling a mine's production, but at the potential processing streams, while blending and destinations are decided. Finally, no pre-decided 'optimal' cut-off grades are used as inputs; the final schedules generated to maximise net present value for the value chain provide the optimal cut-offs.

As always, the framework of the simultaneous stochastic optimisation of mining complexes overviewed here can be further extended to advance several aspects related to strategic planning, ranging from advanced alternative operational modes, additional waste management and rehabilitation components, the interaction of strategic and operational production planning, advancing components for the transportation to client(s) and so on. Similarly, improving methods for the stochastic simulation of the properties of mineral deposits, such as new high-order simulations that have been shown to further improve results in mine production planning [52], need to be developed further [64]. Dealing with the inherent complexities of the optimisation problem, such as large sizes and non-linearity, is important. Consequently, approaches that integrate machine learning techniques for the effective and computationally efficient handling of the related optimisation problem [65] are further needed. Finally, It seems natural that the strategic mining complex optimisation framework is extended and adapted to address short-term planning aspects and requirements [66].

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